# Dispersion phenomena in optical fibers
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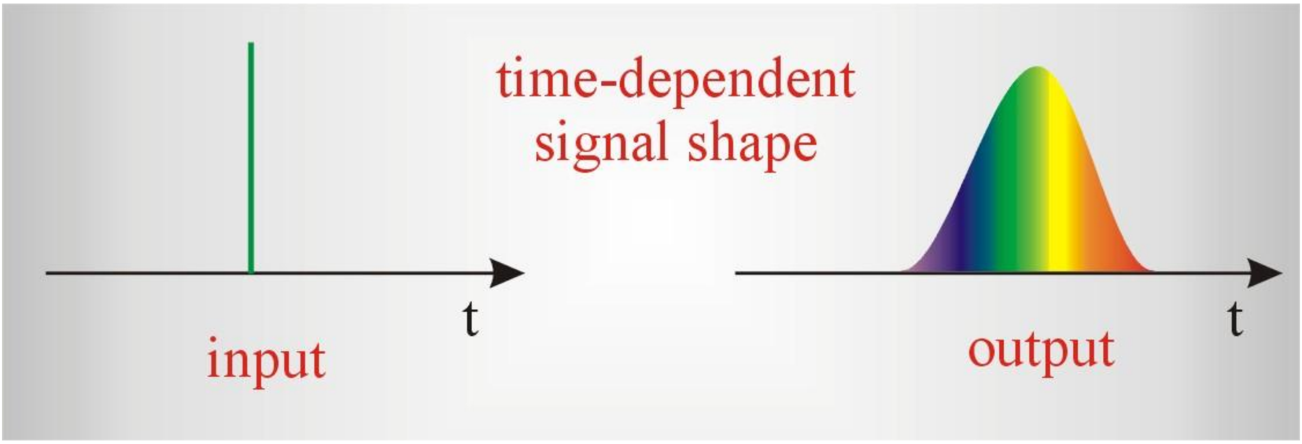



## 3. Dispersion types

Dispersion represents a broad class of phenomena related to the fact that the velocity of the electromagnetic wave depends on the wavelength. In telecommunication the term of *dispersion* is used to describe the processes which cause that the signal carried by the electromagnetic wave and propagating in an optical fiber is degradated as a result of the dispersion phenomena. This degradation occurs because the different components of radiation having different frequencies propagate with different velocities.
We distinguish various kinds of dispersion and all of them will be discussed in this chapter:

1. Chromatic dispersion
    - Waveguide dispersion (optical)
    - Material dispersion
    - Polarisation dispersion
2. Mode dispersion

These phenomena are particularly important in optical telecommunication. In different periods of the historical development of the optical telecommunication the different kinds of dispersion played a different role. In the first period, when the multimode fibers with the step profile of the refraction index were used and the light was transmitted only on small distances at low transimission speed, the chromatic dispersion played a negligible role in contrast to the mode dispersion. The development of the multimode optical fibers with the gradient profile of the refraction index had reduced the mode dispersion considerably. Employing the single-mode optical fibers eliminated entirely the phenomenon of the mode dispersion and allowed to propagate the signal over large distances. However, with the higher transmission speeds gigabites per second the chromatic dispersion became more and more essential on large distances. The technical problems related to the transmision in the second window, and particularly in the third transmission window became more and more dependent on the chromatic dispersion. Fig.3.1 illustrates the power losses caused by the chromatic dispersion on the distance in the III transmision window for the different transmission speeds, when we apply a single-mode laser DFB (distributed feedback Bragg) of spectral width of 0.1 nm as a light source to propagate in a single-mode fiber characterized by the dispersion coefficient of 17 ps/nm/km, typical for most glassy fibers.

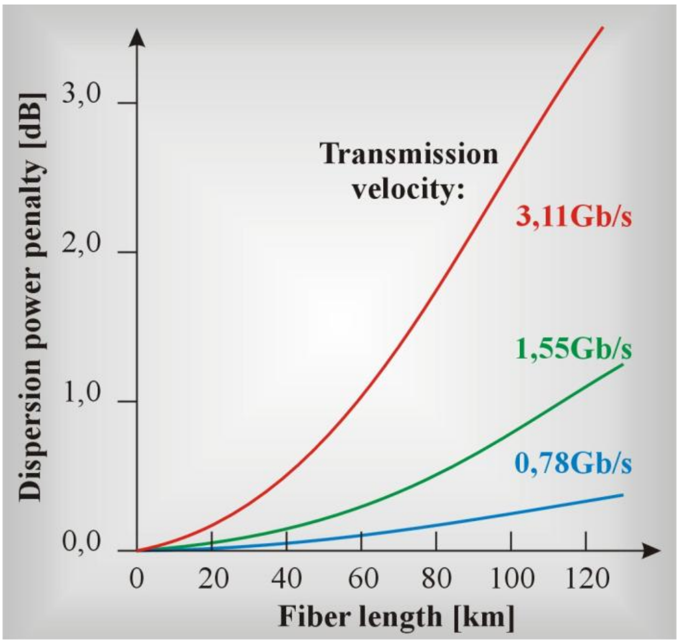



Fig.3.1. Attenuation caused by dispersion at transmission speed a) 0.78 Gb/s, b) 1.33 Gb/s, c) 3.11 Gb/s for the optical fiber characterized by the chromatic dispersion of 17 ps/nm/km and propagating the light from the single-mode laser DFB at spectral width of 0.1 nm

Usually, the maximum attenuation caused by dispersion can be tolerated up to the value of 2 dB, which means that at the transmission speed of 3.11 Gb/s we might apply the optical fiber with length up to 85 km without any regeneration. We can see that for the transmission speeds higher than 3 Gb/s dispersion plays an key role in case of larger distances and the transmission becomes dispersion-limited.

Simply speaking, chromatic dispersion means that the different wavelengths travel with different velocities even for the single-mode optical fibers. The chromatic dispersion is the characteristic feature of the material and it is imposible to avoid it, it can be only reduced. The dependence of the refraction index on the wavelegth for fused silica is shown in the Figure 3.2.

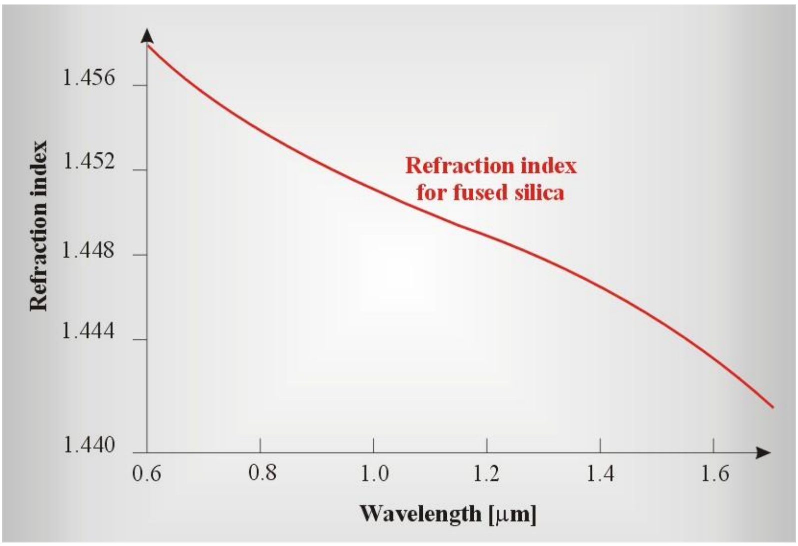



Fig.3.2. Dependence of refraction index and wavelength for fused silica.

The radiation of shorter wavelengths has larger refraction indices than that for the longer wavelengths, so the light at different wavelengths is traveling with different speeds. The more monochromatic light from the transmitter the larger the velocity difference between the longest and the shortest component propagating through the fiber. Every LED diode and a diode laser is characterized by a spectral width. For example, the multimode Fabry Perot (FP) laser has the spectral width of 2 nm, a single-mode laser – 0.1 nm, and Bragg laser DFB – 0.05 nm. Obviously, the smaller band width of the light source, the smaller dispersion effects. It indicates that DFB lasers results in the smallest material chromatic dispersion in a fiber. We will discuss in details the different light sources including LED diodes, diode lasers such as FP, DFB, DRB. Now we mentioned them because we needed only parameters, such us the spectral width to illustrate the problem of material chromatic dispersion.

## 3.1. Mode dispersion

So far, the discussion has been concerned on a single-mode optical fiber. In a multimode optical fiber there is an additional dispersion - the mode dispersion which occurs even, when the light introduced into a fiber is an ideal monochromatic source. Indeed, in a single –mode fiber we can assume with a good approximation that the optical path of the rays is directed along the optical axis of the fiber Fig.(1.10), because the radius of the core is very small (5-10 μm, see Chapter 1). In a multi-mode fiber the radius of the core is much larger (50-62.5 μm) and the rays can travel along different paths (Fig.1.11). In a multimode fiber with a step profile of the refraction index all rays travel with the same speed – the rays traveling along the fiber axis have the same speed as the rays traveling close to the core-cladding interface. As they cover the optical paths of different length at the same speed they reach the detector at different times. This leads to the temporal pulse broadening at the end of the fiber. This type of temporal broadening is called the mode dispersion

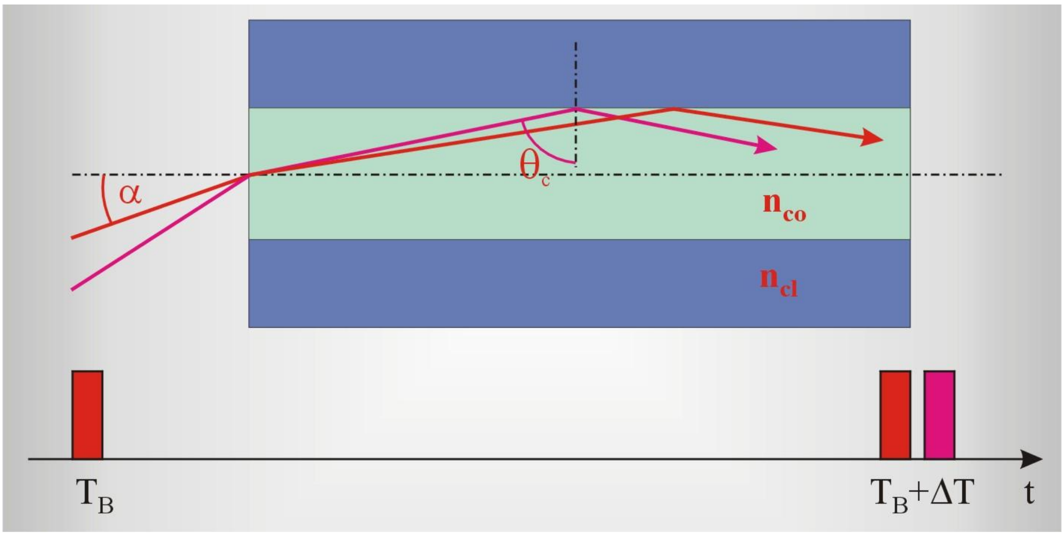



Fig.3.3. Illustration of temporal pulse broadening in a step fiber as a result of the mode dispersion

## 3.2. **Chromatic dispersion**

### 3.2.1. Waveguide dispersion (optical)

In chapter 1 we introduced the term of the normalized frequency $\upsilon$ and the cut-off frequency $\upsilon_0$

$$\upsilon = \frac{2\pi a}{\lambda_0}\sqrt{n_1^2 - n_2^2} \qquad (3.1)$$

as well as the effective refraction index

$$\boxed{\frac{\beta}{k_0} = n_{eff}} \qquad (3.2)$$

(3.2)

We have shown in chapter 1 that

1) for the frequency which exceeds the cut-off frequency $\upsilon_0$ only a small part of the electromagnetic field for a given mode propagates in the core. The main part of the optical power propagates in a cladding

$$n_{eff} = n_2 = \frac{\beta}{k_0} \qquad \text{when} \quad \upsilon \to \upsilon_0, \qquad (3.3)$$

2) as a frequency is growing up, the electromagnetic field of a given mode is contained partly in a core and partly in a cladding,
3) when the frequency grows up considerably above the cut-off frequency, the field of a given mode is contained nearly almost in a core

$$n_{eff} = n_1 = \frac{\beta}{k_0} \qquad \text{when} \quad \upsilon \to \infty. \qquad (3.4)$$

WAVEGUIDE DISPERSION describes the dependence of the effective refraction index $n_{eff} = f(\upsilon)$ on the normalized frequency of radiation propagating through the optical fiber. The waveguide dispersion results in distribution changes of a mode power between the core and the cladding. The waveguide dispersion is an important parameter resulting in the frequency dependence of the group delay $\tau_g = \frac{1}{\upsilon_g} = \frac{d\beta}{d\omega}$ , (3.5)

because β=f(υ) depends on the normalized frequency. In the next chapters we will explain why the goup delay plays an important role in modern optical fiber communication.

## 3.2.2. Material dispersion. Theoretical background: group velocity, group delay, dispersion index [1]

So far, in our considerations of waveguide dispersion we have assumed that there is no chromatic dispersion ( $n_1 \neq f(\omega)$ and $n_2 \neq f(\omega)$ where ω=2πν is the angulat frequency) which is of course a very rough approximation. Now we will take into account the chromatic, material dispersion of the core and the cladding. The typical dependence of the refraction index on wavelength is nonlinear as it is shown in Fig. 3.2 for the fused silica.

We will introduce some definitions, which will be needed to provide the theoretical description of the material dispersion phenomena, such as the group velocity, the group delay, the dispersion index.

**Phase velocity** - defines the velocity of the wave of the constant phase for a given mode

$$\upsilon = \frac{c}{n} = \frac{\omega}{k} \quad \text{for a planar wave,} \tag{3.6}$$

$$\upsilon = \frac{\omega}{\beta} \tag{3.7}$$

for a wave propagating in a fiber, where β is the propagation constant

**Group velocity** is expressed by a formula

$$\upsilon_g = \frac{d\omega}{d\beta} \tag{3.8}$$

*Phase velocity* is given by formula (3.6) and it describes the velocity of a monochromatic plane wave. Let us consider the case, when the wave is non-monochromatic. The wave function $\psi(z, t)$ for a such state is represented by the wave packet

$$\psi(z,t) = \int_{k_0-\Delta k}^{k_0+\Delta k} A(k)\exp(\mathrm{i}(kz - \omega t))\mathrm{d}k \, , \tag{3.9}$$

describing the set of planar waves characterized by the wave vectors **Δk** from the interval

$$k_0 - \Delta k \le k \le k_0 + \Delta k \tag{3.10}$$

and directed along z axis. Developing $\omega(k)$ into power series

$$\omega(k) = \omega_0 + \left(\frac{\mathrm{d}\omega}{\mathrm{d}k}\right)_0 (k - k_0) \tag{3.11}$$

and substituting into formula (3.9), one obtains

$$\psi(z,t) = 2A(k_0)\frac{\sin\left[\left(z - \left(\frac{d\omega}{dk}\right)_0 t\right)\Delta k\right]}{z - \left(\frac{d\omega}{dk}\right)_0 t}\exp\left(i\left(k_0 z - \omega_0 t\right)\right). \tag{3.12}$$

The term $2A(k_0)\frac{\sin\left\{\left[z - \left(\frac{d\omega}{dk}\right)_0 t\right]\Delta k\right\}}{z - \left(\frac{d\omega}{dk}\right)_0 t}$ can be treated as a amplitude $A$ of the wave packet $\psi(z, t)$.

This function is of $\frac{\sin x}{x}$ type, and at $t = 0$ it takes the form as presented in Fig. 3.4

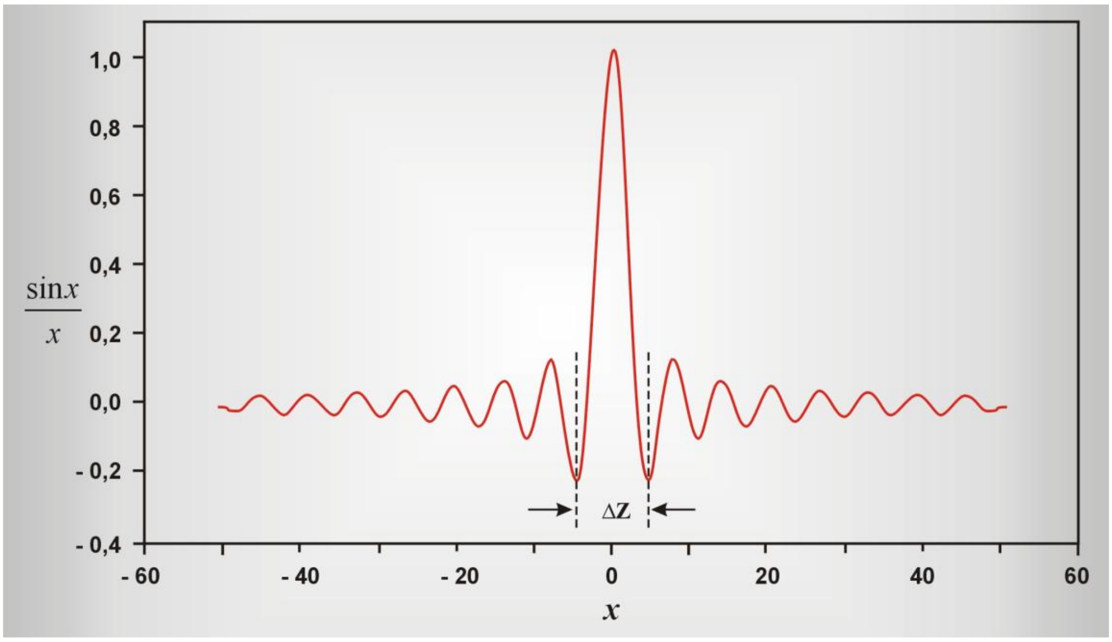



Fig. 3.4. Graph of the function of $\frac{\sin x}{x}$

The amplitude $A$ achieves maximum at z = 0, and $\Delta z$ can be considered as a measure of the wave packet broadening. Calculating the difference between the first minima $z_1$ and $z_2$ we can show that $\Delta z$ is equal to $\frac{2\pi}{\Delta k}$ . This means that the more monochromatic wave packet ($\Delta k \to 0$), the larger spatial broadening ($\Delta z \to \infty$, so the wave becomes planar). The expression (3.12) simply states that the central point of the wave packet corresponding to the maximum amplitude propagates in space at the velocity $\upsilon_g$

$$\upsilon_g = \left(\frac{d\omega}{dk}\right)_0, \tag{3.13}$$

which is called the *group velocity***.**

The formula (3.13) describes the set of planar waves freely propagating in the vacuum. In a real optical fiber, one should consider a limitation imposed on the propagation by the boundary conditions of the fiber as well as the refraction index of the core and the cladding described by so-called the effective refraction index, which we have already introduced in chapter 1 and which is related to the wave vector of the electromagnetic radiation $k_0$ by the formula

$$\frac{\beta}{k_0} = n_{eff} \qquad (3.14\ )$$

Therefore, in real conditions of light propagation in optical fiber the wave vector $k_0$ should be replaced by the propagation constant $\beta$ in the definition of the group velocity (3.13) and the expression for the group velocity in a fiber takes a form ( 3.15 )

$$\boxed{\upsilon_g = \frac{d\omega}{d\beta}} \qquad (3.15)$$

Considering the typical dependence of the refraction index on wavelength (Fig.3.2 ), we can suppose that more blue waves (shorter) move slower than the red ones, because they are characterized by a larger refraction index (3.6). However, we should remember that this statement is simplified, because it concerns the phase velocity of an ideally monochromatic planar wave. Very often this approximation works quite well and the conclusions derived from it are correct. However, we will show that it is not always true.

To analyse the question of the group velocity in details and the group velocity dispersion (GVD as well as the technically important parameter - the dispersion coefficient D, we should consider the phase $\Phi(\omega)$ of the wave propagating on the optical path $L$ through a fiber medium characterized by the refraction index $n(\omega)$. We have to remember that the light propagating in a optical fiber and used for the transmission of signals in telecommunication or computer networks has to be modulated in time. As a result of modulation a pulse of a given temporal duration is generated. The pulse can be treated as the wave packet of a given spectral width. The shorter the pulse in the time domain, the wider the spectrum in the frequency domain. We can assume that the optical pulse is a quasi - monochromatic when the condition $\frac{\Delta\omega}{\omega_0}$ << 1 is fulfilled, where $\Delta\omega$ is the spectral width of the pulse, and $\omega_0$ is the central frequency of the spectrum. Because $\omega_0 \approx 10^{-15} s^{-1}$, the quasi – monochromatic approximation is valid for pulses longer than 0.1 ps. It denotes that even for very fast modulations of the order of Tb/s, the approximation is fulfilled.

The electric field intensity $\boldsymbol{E}(\boldsymbol{r}, t)$ directed along polarisation direction $x$ can be expressed as

$$\boldsymbol{E}(\boldsymbol{r}, t) = \frac{1}{2}\hat{x}\left[E(\boldsymbol{r}, t)\, exp(-i\omega_0 t) + c.c.\right] \qquad (3.16)$$

where $\hat{x}$ is a vector in polarisation direction $x$, $E(\boldsymbol{r}, t)$ is a slowly changing function of time due to time modulation (slow in comparison with the period of optical modes $\omega_0$). This slowly changing function of time $E(\boldsymbol{r}, t)$ describing the temporal shape of the pulse is connected with a spectrum in the frequency domain $\hat{E}(\boldsymbol{r}, \omega - \omega_0)$ by the Fourier transform

$$\hat{E}(\boldsymbol{r}, \omega - \omega_0) = \int_{-\infty}^{\infty} E(\boldsymbol{r}, t)\, exp[i(\omega - \omega_0)t]dt \qquad (3.17)$$

The intensity of the electric field in the frequency domain $\hat{E}(\boldsymbol{r}, \omega - \omega_0)$ fullfils the Helmholtz equation

$$\nabla^2\hat{E} + n^2(\omega)k_0^2\hat{E} = 0 \qquad (3.18)$$

where the wave vector $k_0$ is expressed by formula

$$k_0 = \frac{\omega}{c} \qquad (3.19)$$

and $n(\omega)$ is the refraction index of medium in which the light propagates. For the wave propagating in a fiber $n(\omega)= n_{eff}$ , where $n_{eff}$ is the effective refraction index, which in extreme cases has a value of the refraction index of the core $n_1$, when the normalized frequency $\upsilon \to \infty$ or the refraction index of the cladding $n_2$, when normalized frequency is close to the cut-off frequency $\upsilon \to \upsilon_0$).
The intensity of the electric field in the frequency domain $\hat{E}(\boldsymbol{r}, \omega - \omega_0)$ can be expressed as

$$\tilde{E}(\boldsymbol{r}, \omega - \omega_0) = F(x, y)\tilde{A}(z, \omega - \omega_0)\, exp(i\beta z), \qquad (3.20)$$

where $F(x, y)$ describes the field distribution in the plane perpendicular to z axis, $\tilde{A}(z, \omega - \omega_0)$ describes the slowly changing electric field along $z$ axis – the direction of propagation of an optical signal, $\beta$ is the propagation constant corresponding to the wave vector for the planar wave in vacuum and expressed as

$$\upsilon = \frac{c}{n} = \frac{\omega}{\beta} \qquad (3.21)$$

After substitution (3.20 ) to Helmholtz equation (3.18) and using the method of variables separation we obtain the solution describing the propagation of an optical signal in a fiber. We used this method in chapter 1 to obtaining the electric field distribution for different modes – TE, TM, HE, EH.
In equation (3.20) the term $exp(i\beta z)$ describes the changes of phase $\Phi(\omega)$. If the optical signal moves on the length of $L$, the phase change is

$$\Phi(\omega) = \frac{\omega n(\omega)}{c} L \qquad (3.22)$$

In equation (3.22) we used (3.21)
Because, we do not know the detailed form of $\Phi(\omega)$ we expand the phase $\Phi(\omega)$ in Taylor series around the central frequency $\omega_0$

$$\Phi(\omega) = \Phi_0 + \left(\frac{d\Phi}{d\omega}\right)(\omega - \omega_0) + \frac{1}{2}\left(\frac{d^2\Phi}{d\omega^2}\right)(\omega - \omega_0)^2 + \frac{1}{6}\left(\frac{d^3\Phi}{d\omega^3}\right)(\omega - \omega_0)^3 + ... \qquad (3.23)$$

The temporal shape of the pulse $A(z, t)$ is related to the spectral shape $\tilde{A}_z(z, \omega - \omega_0)$ by the reverse Fourier transform

$$A(z, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A(z, \omega - \omega_0) e^{i\Phi(\omega)} e^{-i(\omega - \omega)t_0}\, d\omega \qquad (3.24)$$

Substituting (3.23) to (3.24) it becomes obvious, that the first term $\Phi_0$ doesn't have any influence on the temporal shape of the propagated pulses but only on the phase shift. The second term also does not influence the temporal shape of the pulse, it only generates the time delay of the pulse propagating through the medium. Indeed, comparing the first derivative of equation (3.22) with definition of the group velocity $\upsilon_g$ (3.15) we obtain

$$\frac{d\Phi}{d\omega} = \frac{n}{c}\left(1 + \frac{\omega}{n}\frac{dn}{d\omega}\right)L = \frac{d\beta}{d\omega}L = \frac{L}{\upsilon_g} = t_g, \qquad (3.25)$$

where $\frac{\beta}{k_0} = n_{eff}$. We can notice that $\frac{d\Phi}{d\omega}$ has a clear physical sense, it is the time $t_g$ needed to cover the distance $L$ by the spectral component moving with the group velocity $\upsilon_g$.

The third term in the expression (3.23)

$$\frac{1}{2}\frac{d^2\Phi}{d\omega^2}(\omega - \omega_0)^2 \qquad (3.26)$$

has influence on the temporal shape of the pulse. In fact, assuming that $\delta = \frac{d^2\Phi}{d\omega^2} = \mathit{const}$ and substituting to (3.24) we obtain

$$E'(t) = \frac{E_0}{\sqrt{\tau^2 + i\delta}}\, e^{-t^2/2\tau'^2}\, e^{i\frac{\delta}{\tau^2}\frac{t^2}{2\tau'^2}} \qquad (3.27)$$

where the modified temporal pulse duration is

$$\tau = \tau_0\sqrt{1 + \frac{\tau_c^4}{\tau_0^4}}, \qquad (3.27a)$$

where $\tau_c = \sqrt{|\delta|} = \sqrt{\left(\frac{d^2\Phi}{d\omega^2}\right)L} = \sqrt{\beta_2 L}$. (3.27b)

Derivation of the relation of (3.25 -3.27) we can find in the work [3] .

Thus, we have shown that the nonlinear term of dispersion $\frac{1}{2}\frac{d^2\Phi}{d\omega^2}(\omega - \omega_0)^2$ results in the temporal pulse broadening. The temporal pulse broadening is a result of different group velocities and it is called the GVD effect (Group Velocity Dispersion).

In some books [2] the equation (3.27 a) has a bit different form: the dispersion length $L_D$ ***is*** introduced as

$$L_D = \tau_0^2 / |\beta_2| \qquad (3.27c)$$

which defines the length of optical fiber $L_D$ and for which the effects of dispersion related with the nonlinearity of the refraction index $\beta_2$ are important, or they can be neglected. In fact, substituting (3.27 c) and (3.27 b) to (3.27 a) we obtain

$$\tau = \tau_0\sqrt{1 + \left(\frac{L}{L_D}\right)^2} \qquad (3.28)$$

For standard telecommunication fibers in the window 1550 nm $|\beta_2| \approx 20\,ps^2/km$ and pulses $\tau_0$ >100ps, $L_D \approx 500km$. It indicates that for a fiber of length 50-80 km, the effects related to GVD dispersion are negligible ($L << L_D$). However, for shorter pulses $\tau_0$ on the order of 1 ps , and for the fast modulation, $L_D \approx 50m$ , the GVD effect cannot be neglected for any length of a fiber,

| |
|---|
| because $L >> L_D$. |

To summarize, one can see from equation (3.27) that the nonlinear term of dispersion $\frac{1}{2}\frac{\mathrm{d}^2\Phi}{\mathrm{d}\omega^2}(\omega-\omega_0)^2$ causes the broadening of the temporal pulse. This term causes that the group velocities of different components become different. As a result the different components cover the optical path $L$ in different times, which causes the temporal pulse broadening.

In Fig. 3.5 the effect of broadening for the temporal pulse of the Gaussian shape during propagation in an optical fiber is shown.

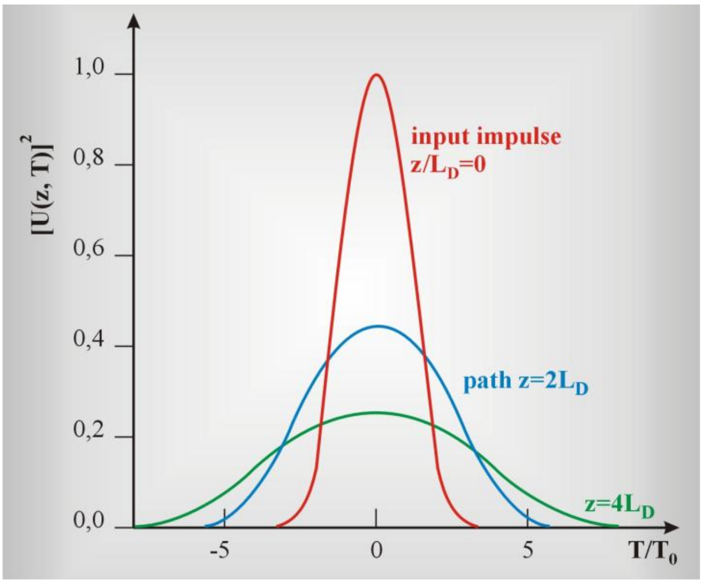



Fig.3.5. Broadening of the pulse of the Gaussian shape during the propagation in an optical fiber , input pulse for $z=0$, pulse after covering the path of $z=2L_D$, pulse after covering the path of $z=4L_D$ [2], where $L_D$ is the dispersion length of a fiber defined by the formula (3.27.c)

Let us assume that the temporal pulse of the Gaussian shape at $z = 0$ shows no group velocity dispersion (GVD = 0) ( Fig.3.6). After passing the path $z = 2L_D$ or $z = 4L_D$ the pulse has still the Gaussian shape, but it is significantly broadened due to the GVD effect. The GVD effect can be positive or negative. When the components of longer wavelengths moves faster than the shorter ones we say that GVD > 0 (Fig.3.7 a). When the components of longer wavelengths move slower than the shorter waves, we say that the fiber shows the negative dispersion (GVD < 0) (Fig.3.7 b).

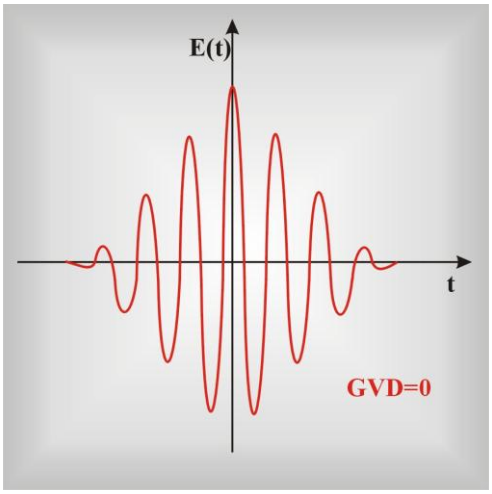



Fig.3.6. temporal pulse at Gausse shape at zero GVD=0 (ang. zero chirped)

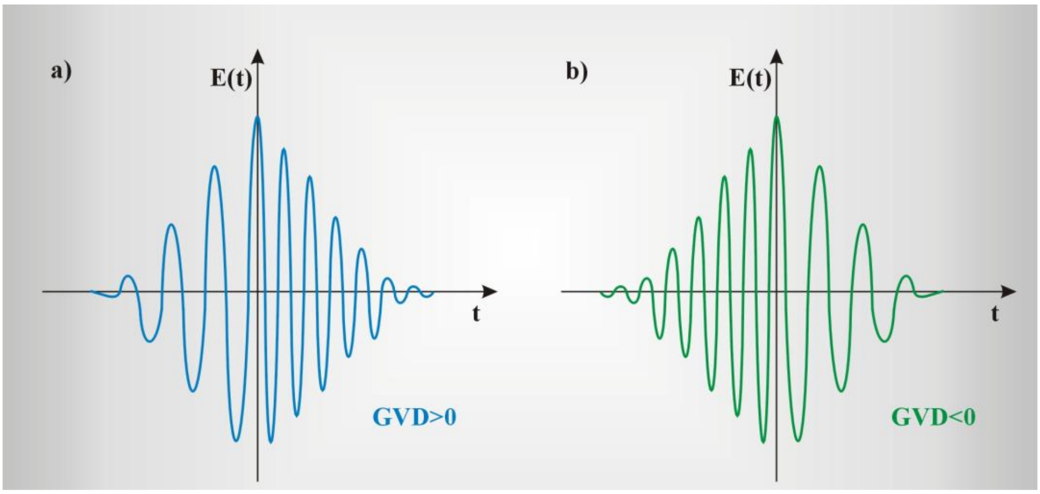



Fig.3.7. Gaussian shape characterised by the positive GVD (a) and the negative GVD (b)

No matter what is the sign of the GVD dispersion, the input pulse which shows zero chirp (Fig.3.6.) undergoes the temporal broadening. after moving through the optical fiber. The zero chirp means, that all spectral component of temporal pulse move with the same group velocity

$$\frac{d\Phi}{d\omega} = \frac{n}{c}\left(1 + \frac{\omega}{n}\frac{dn}{d\omega}\right)L = \frac{d\beta}{d\omega}L = \frac{L}{\upsilon_g} = t_g = const \qquad (3.29)$$

$$\frac{d}{dt}\left(\frac{d\Phi}{d\omega}\right) = 0 \qquad (3.30)$$

However, the situation changes when the input pulse just shows negative dispersion (GVD < 0) (Fig. 3.7 b). Then the positive GVD effect causes firstly shortening of the pulse up to the moment when GVD=0 is reached, and later its broadening (Fig.3.8 )

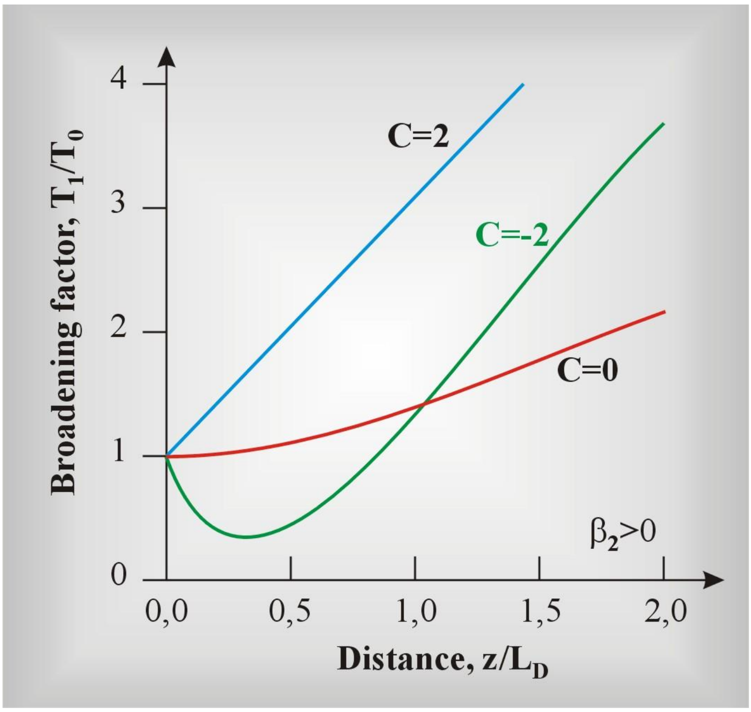



Fig.3.8. Broadening of the Gaussian pulse with input chirp characterized by parameter C during propagation in optical fiber of positive dispersion GVD > 0, ----- pulse at $C = 0$, C = 2 - pulse for input positive GVD dispersion, C =-2 - pulse for input negative GVD dispersion [2]. When the optical fiber shows dispersion GVD < 0, the same curves describe bands broadening, when we exchange signs of C

It is easy to understand this initial effect of pulse narrowing in Fig. 3.8 for C=-2. The input pulse has GVD opposite to GVD of the optical fiber in which it begins to propagate. This means that the shorter waves which at entrance moved faster, in the optical fiber begin to show the effect of delay, whereas the longer waves begin to move faster. It leads to the compensation of GVD effect. The pulse achieves the minimum for a such distance in the optical fiber for which both effects are compensated and the GVD = 0. For this distance the the pulse is the shortest and the most broadened in the frequency domain (transform – limited spectral band width). Further propagation in a fiber leads to the pulse broadening as the the effect of GVD becomes positive.

One can see from (3.22) that the second order dispersion term $\frac{d^2\Phi}{d\omega^2}$ can be expressed by the following formula

$$\frac{d^2\Phi}{d\omega^2} = \left(\frac{2}{c}\frac{dn}{d\omega} + \frac{\omega}{c}\frac{d^2n}{d\omega^2}\right)L = \frac{d^2\beta}{d\omega^2}L. \qquad (3.31)$$

If $\frac{d^2\Phi}{d\omega^2}$ is different from zero, the group velocities $\upsilon_g$ related to different frequencies are different and therefore we say, that the medium is characterized by the the group velocity dispersion - GVD. The typical values $\boldsymbol{\frac{d^2\Phi}{d\omega^2}}$ for wavelength of 800 nm are the following: sapphire crystal -580 $fs^2$/cm; fused silica - 360 $fs^2$/cm, glass SF10 - 1500 $fs^2$/cm.

To summarize the influence of GVD on the shape and the temporal duration of the modulated pulses propagating in the optical fiber we can state that the GVD effect becomes essential for pulses of picosecond time scale or shorter. Therefore, the nonlinear effects of the refraction index begin to play an important role in fast optical transmission. Practically, it becomes important when the transmission speeds exceeds 100Gb/s. Fast transmisions requires fast modulation. Fast modulation generates pulses in the form of quasi - monochromatic wave packet in a given spectral range. Because the refraction index $n(\omega)$ of every material depends on the radiation frequency, every frequency component in the pulse propagate with a bit different group velocity $\upsilon_g$. The wider spectral range, the larger differences (dispersion) of the group velocity (GVD).
So far, we have considered only the influence of the first three terms in Taylor series (3.23). In some cases we also have to consider the forth term $\frac{1}{6}(\frac{d^3\Phi}{d\omega^3})(\omega - \omega_0)^3$. Including this term results in change of the Gaussian pulse shape. The pulse is not only broadened, but also the pulse in the fiber does not keep the Gaussian shape any more and depends on the value of $\beta_3 = \frac{d^3\Phi}{d\omega^3}$ (Fig. 3.9).

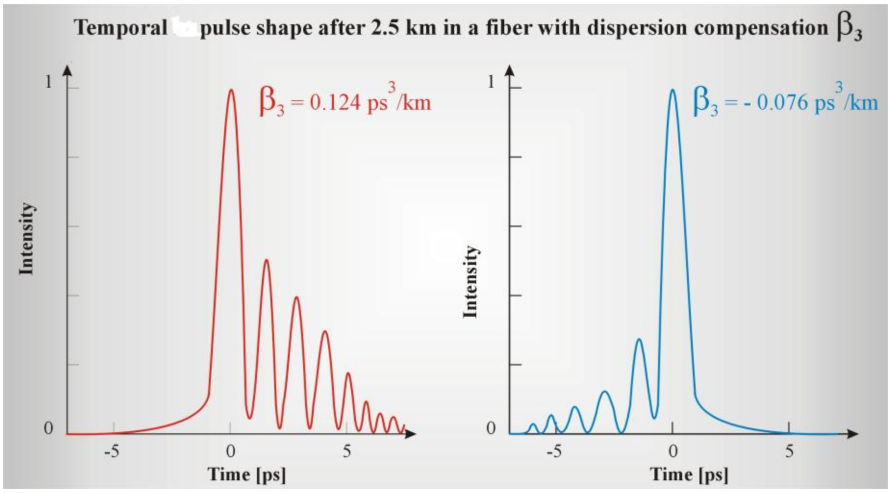



Fig.3.9. The shape of the temporal pulse 0.5 ps after passage of the path of 2.5 km in a fiber with the dispersion $\beta_3$=0.124 (a) and $\beta_3$=-0.076 $ps^3 / km$ [4]

Introduction of the third order nonlinearity (TOD, third order dispersion) characterized by $\beta_3$ permitts to introduce the next important parameter S defined as

$$S = \frac{dD}{d\lambda} = \left(\frac{2\pi c}{\lambda^2}\right)^2 \beta_3 + \left(\frac{4\pi c}{\lambda^3}\right)\beta_2 \qquad (3.23)$$

In Figure 3.10, the typical dependence of the refraction index $n(\lambda)$ versus wavelength λ is shown.

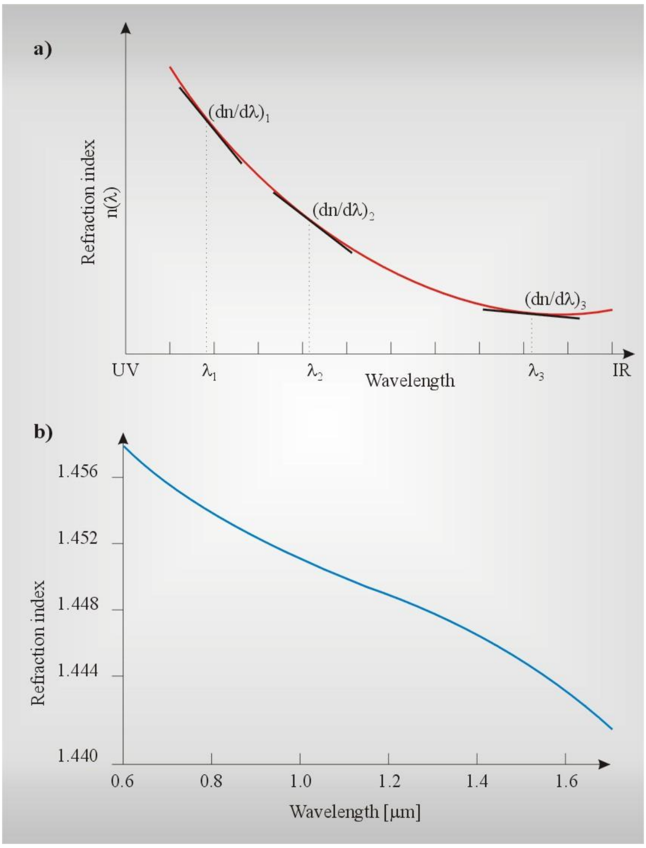



Fig.3.10. Schematic dependence of the refraction index $n(\lambda)$ as a function of wavelength $\lambda$.(a), the refraction index for fused silica (b)

We should stress that Fig.3.10. represents only a small range of the dependence on the wavelength corresponding to the non-resonance area (where the glass does not absorb). The full range represents Fig.3.11.

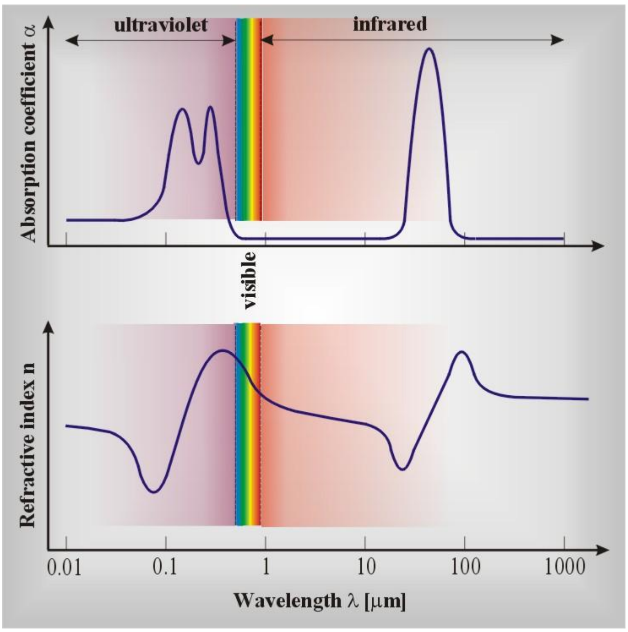



Fig.3.11. The dependence of the absorption coefficient $\alpha(\lambda)$ and the refraction index $n(\lambda)$ as a function of wavelength for fused silica

For a given wavelength the refraction index $n(\lambda)$ defines the phase velocity by the relation $\upsilon = \frac{c}{n} = \frac{\omega}{\beta}$. The slope of the curve, $\frac{dn(\lambda)}{d\lambda}$, determines the group velocity

$$\upsilon_g = \frac{c}{(n(\lambda) - \lambda \frac{dn}{d\lambda})} \tag{3.33}$$

for the wave packet at wavelength λ. The formula (3.33) for the group velocity can be easily derived from definition of the group velocity $\upsilon_g = \frac{d\omega}{d\beta}$ and the relation $\upsilon = \frac{c}{n} = \frac{\omega}{\beta}$

The expression (3.33) can be written as

$$\upsilon_g = \frac{c}{N} \tag{3.34}$$

where the group refraction index $N$ is introduced

$$N = n(\lambda) + \omega \frac{dn}{d\omega} = n(\lambda) - \lambda \frac{dn}{d\lambda} \tag{3.35}$$

One can be sen from eq. (3.33) that the first derivative $\frac{dn(\lambda)}{d\lambda}$ defines the group velocity. When $\frac{dn(\lambda)}{d\lambda} = const, \frac{d^2n}{d\lambda^2} = 0$ the all the spectral components and the pulse moves with the same group velocity $\upsilon_g$. This condition means that the dependence of the refraction index on wavelength should be linear, which happens in practice very rarely. Thus, the second derivative $\frac{\mathrm{d}^2 n}{\mathrm{d}\lambda^2}$ which is a measure of nonlinearity for the refraction index decides about the group velocity dispersion (GVD) of material. In another words, using the expression (3.25) we can say that GVD=0 when the group delay $t_g$ is independent on frequency, that is $t_g = \frac{d\Phi}{d\omega}$ = const.

Another expression that also describes the group velocity dispersion GVD is the second derivative of propagation constant

$$\beta_2 = (\frac{d^2\beta}{d\omega^2})_{\omega=\omega_0} \tag{3.36}$$

expressed by equation

$$\beta_2 = \frac{d^2\beta}{d\omega^2} = \frac{1}{c}(2\frac{dn}{d\omega} + \omega\frac{d^2n}{d\omega^2}) \tag{3.37}$$

which results from (3.31)

| |
|---|
| When $\beta_2 > 0$, it is said, that the fiber shows normal (or positive ) GVD, when $\beta_2 < 0$, it is said, that the fiber shows abnormal (or negative) GVD. |

In material showing positive GVD effect, the components at longer wavelengths moves faster than components at shorter wavelengths, in material showing the negative GVD effect the situation is the opposite. The larger group velocity dispersion GVD, the larger broadening of the temporal pulse. It is said, that the pulse is modulated positively (*positively chirped*), when the longer waves travel in medium faster than the short ones.

Often, instead of $\beta_2$, the group velocity dispersion is describedwith the **dispersion coeffcient D** defined as

$$D = \frac{dt_g}{d\lambda} \; [\frac{ps}{nm \bullet km}] \text{, gdzie } t_g = \frac{1}{\upsilon_g} = \frac{d\beta}{d\omega} \quad (3.38)$$

The dispersion coefficient D determines the temporal broadening of the pulse in ps (picoseconds) after passage of 1 km of optical fiber, if the width of the spectral line of ligth source is 1 nm

We should notice , that the dispersion coefficient D defined above

$$D = \frac{dt_g}{d\lambda} \quad (3.39)$$

where $t_g = \frac{1}{\upsilon_g} = \frac{d\beta}{d\omega} = \beta_1$ (3.40)

has the sign opposite to $\beta_2$, as

$$D = \frac{d\beta_1}{d\lambda} = -\frac{2\pi c}{\lambda^2}\beta_2 \approx \frac{\lambda}{c}\frac{d^2 n}{d\lambda^2} \quad (3.41)$$

so

When $D < 0$ it is said, that the fiber shows the normal (positive) GVD,
when $D > 0$ it is said, that the fiber shows the anomal (negative) GVD.

The another parameter which characterizes chromatic dispersion is the coefficient $d_{12}$ defined as the following:

$$d_{12} = \beta_1(\lambda_1) - \beta_1(\lambda_2) = \upsilon_g^{-1}(\lambda_1) - \upsilon_g^{-1}(\lambda_2) \quad (3.43)$$

where $\lambda_1$ and $\lambda_2$ are the central wavelengths of two temporal pulses, $\beta_1$ is calculated from formula obtained from (3.25)

$$\beta_1 = \frac{d\beta}{d\omega} == \frac{1}{\upsilon_g} = t_g = \frac{n}{c}\left(1 + \frac{\omega}{n}\frac{dn}{d\omega}\right) \quad (3.44)$$

In Fig. 3.12. the dependence of the parameters characterizing the group velocity dispersion GVD, $\beta_2$ and $d_{12}$, as *a* function of wavelength for fused silica $SiO_2$ is presented. One can seen from Fig. 3.12 that in standard optical fibers $\beta_2$ reaches zero value for wavelength of 1.31 $\mu$m which denotes that GVD dispersion is equal to zero in the second transmission window. In practice we can shift the zero dispersion towards longer waves in the third window. It can be achievied by several methods:

- doping with e.g. $GeO_2$ or $P_2O_5$,
- modification of the effective refraction index $n_{eff}$ of a core through the waveguide dispersion.

Fig.3.12. shows the dependence of $d_{12}$ on $\lambda_2$, when $\lambda_1$=0.532 $\mu$m for silica glass $SiO_2$ with the normal dispersion (when $\beta_2$ >0), where the longer wavelength components of the the temporal pulse moves faster than that of the shorter one. The figure shows that, for example, the pulse $\lambda_2$=1.064 $\mu$m

(for which also $\beta_2$ >0, as it results from Fig.3.12.) propagating along the same optical path as the pulse $\lambda_1$=0.532 $\mu$m will preceede it of about 80 ps/m. The parameter, which describes the divergence effect caused by the different group velocities is defined as $L_W$ (walk-off length)

$$L_W = \tau_0 / |d_{12}| \quad (3,45)$$

where $\tau_0$ is a duration time of the pulse. For an example given above, the walk-off length is 25 cm for pulse of 20 ps.

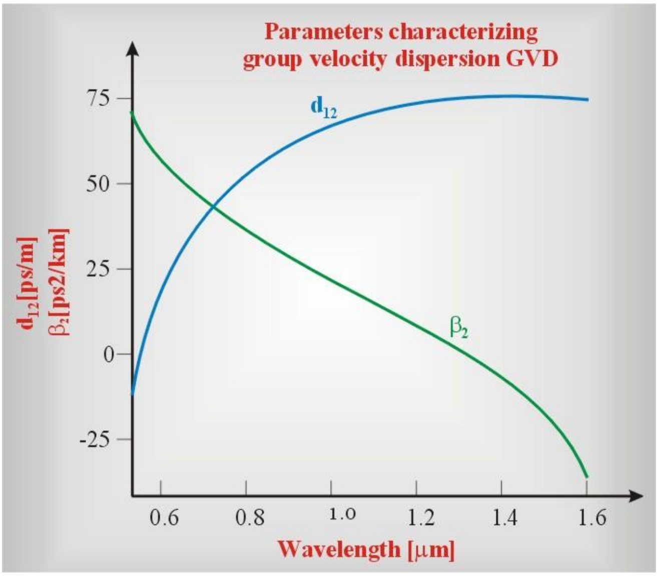



Fig.3.12. Parameters characterizing the group velocity dispersion GVD, $\beta_2$ and $d_{12}$ as a function of wavelength for fused silica $SiO_2$ [4]

Fig.3.13. shows the dependence of the dispersion coefficint D as a function of wavelength for a standard single mode fiber [5]. One can see that D is equal to zero at about 1.31 $\mu$m.

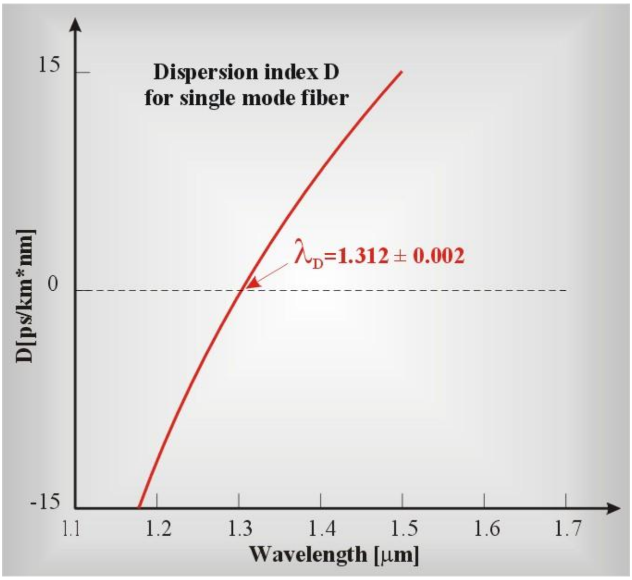



Fig.3.13. Dependence of dispersion index D and wavelength for a single mode fiber [5]

Using (3.41) and the dependence of the refraction index on wavelength, one can show that for a typical single mode fiber made of fused silica the dispersion coefficient D is expressed with the empirical relation

$$D = \frac{S_0}{4}(\lambda - \frac{\lambda_0^4}{\lambda^3}) \qquad (3.42)$$

where

$S_0$=0.092 ps/(nm$^2$ km), $\lambda_D$ = 1311 nm

Fig. 3.14 illustrates the dispersion coefficient D as a function of wavelength in the range from 1250 nm to 1650 nm. The zero of GVD occurs at 1311 nm, in contrast to the III window at 1550 nm where D is at about 17 ps/nm/km.

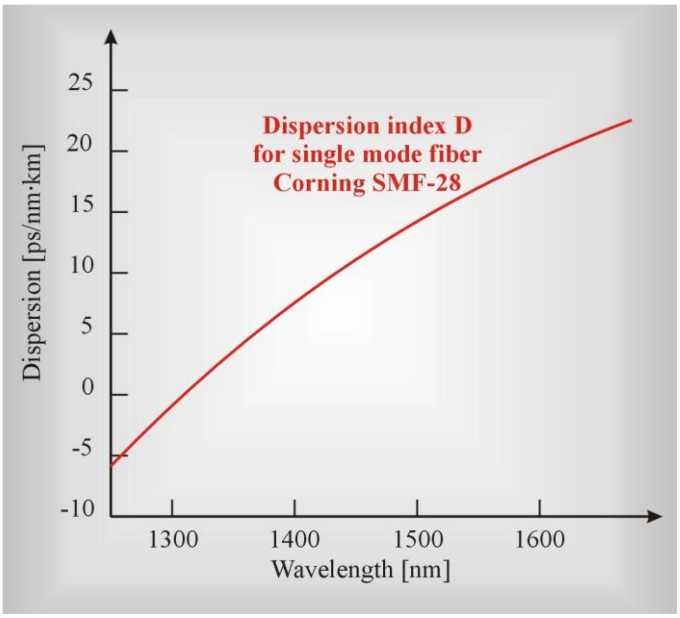



Fig.3.14. Dependence of dispersion index D and wavelength for a typical single mode fiber [Corning SMF-28, www. Fiberoptic.com ]

Now we will consider in details the dispersion shifted fibers. After substituting the group velocity $\upsilon_g$ (3.33) into the definition of the dispersion coefficient

$$D = \frac{dt_g}{d\lambda}\left[\frac{ps}{nm \bullet km}\right] \qquad (3.46)$$

We obtain

$$D = \frac{1}{c}\frac{dN}{d\lambda} \qquad (3.47)$$

So, the dispersion coefficient $D=0$, when $\frac{dN}{d\lambda} = 0$, which happens when $N$ has an extremum. Fig.3.15. shows the dependence of the group refraction index on wavelength for pure silica $SiO_2$. It is clear from the figure that the group refraction index has a minimum at 1.31 $\mu$m which indicates that at this wavelength a fiber shows zero GVD.

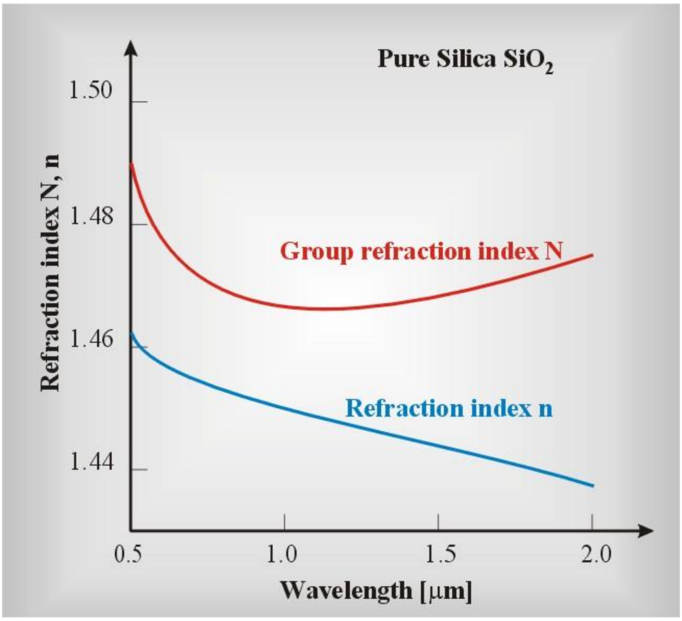



Fig.3.15. The group refraction index *N* (a) and the refraction index *n* (b) as a function of wavelength for pure silica $SiO_2$

Therefore, in a standard fiber a zero dispersion occurs in the second transmission window (1.31 μm). It would be better if the zero dispersion is accompanied by a low attenuation. However, the attenuation in the second transmission window is higher than in the third window at 1.55 μm.

Therefore, the effort has been concentrated on inventing such a fiber, in which the dispersion minimum (D = 0) overlapps with the attenuation minimum in the third transmission window.

The fibers, in which it has been successfully done are called the dispersion shifted fibers. They were discussed partly in chapter I. Here we have already a sufficient supply of theoretical knowledge to understand the methods of achieving the shift of dispersion towards the third window. The Figure 3.16 illustrates the idea of obtaining the dispersion shifted fiber.

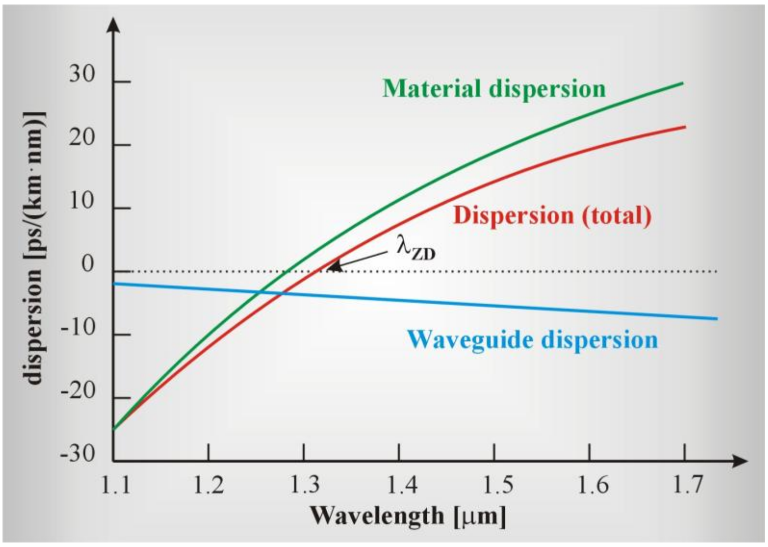



Fig.3.16. Dependence of material, waveguide and total dispersion on wavelength

The total dispersion of a single mode fiber is the sum of contributions coming from the material dispersion and the waveguide dispersion. One can see from Fig. 3.16 that the increase of the negative slope of waveguide dispersion shifts the zero of dispersion towards longer wavelengths. Thus, the obvious way of dispersion minimum shifting to the III window of transmission is enlarging the influence of the waveguide dispersion. It can be done by enlargement of difference between the core and the cladding refraction indices. However, we should notice, that by enlarging this difference, we also increase the cut-off frequency $\upsilon_0 = \frac{2\pi a}{\lambda_0} \sqrt{n_1^2 - n_2^2}$ , what creates the conditions for propagating more than one mode. To remove this undesirable effect we have to reduce the diameter of core $a$ (e.g. from 8 $\mu$m in a standard fiber to 5 μm in a dispersion shifted fiber). When GVD is be shifted far over 1.6 $\mu$m, fibers show large positive value of $\beta_2 > 0$ and they are called dispersion - compensating fibers, DCFs.

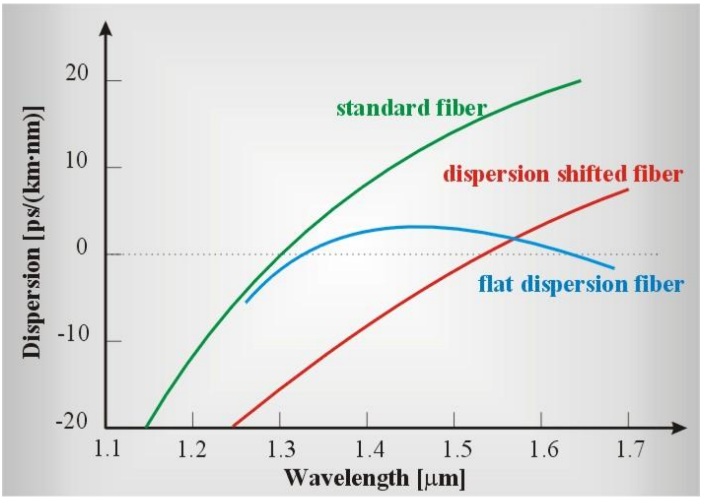



Fig.3.17. Dependance of dispersion index D and wavelength in differend kinds of fibers,
a) standard fiber, b) dispersion shifted fiber, c) flat dispersion fiber

The slope of the dispersion coefficient D ( Fig. 3.17) depends on nonlinearities of the third order $\beta_3 = \frac{d^3\beta}{d\omega^3}$ ( 3.23 ) (TOD- third order dispersion). The strong dependence of the dispersion effect on wavelength creates many problems in WDM (wave division multiplexing) techniques. Therefore, in recent years there have been created fibers with the reduced slope. The most profitable case is a flat profile shown in Fig. 3.18. The low slope can be achieved by using multiple cladding layers. Fig.3.18 shows the slope of D versus wavelength for SC – single clad, DC – double- clad, QC - quadruple – clad fibers.

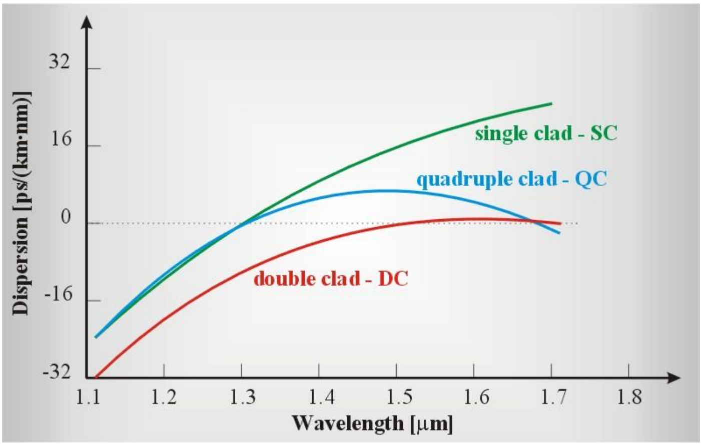



Fig.3.18. Dispersion coefficient D as a function of wavelength for three kinds of fibers, for SC- single clad, DC –double-clad, QC- quadruple-clad.

Using this technology we can get almost flat profile in the range of 1.3-1.6 $\mu$m.
We should stress that the zero dispersion removes all problems related to GVD, but creates new nonlinear phenomena such as four waves mixing (FWM) or cross phase modulation (CMP) which can be very disadvantageous for the transmission, particularly for the WDM transmission. In many cases the total lack of dispersion generates cross-talks between the channels as a result of the nonlinear phenomena of four waves mixing. This phenomenon will be discussed later in this chapter. The GVD disperion should be small (typically D≈1-3 ps / km-nm), but non - zero in the whole range of the optical amplifiers EDFA (1530-1565 nm) reducing the effects of non-linear FWM and CMP. The single mode fibers with the non - zero shifted dispersion are nowadays the best medium for DWDM broadcasting at high speeds in the III the window on large distances. Combining the low dispersion with the small slope profile helps to reach larger frequency range of broadcasting without the necessity to compensate the dispersion parameters in nets working with velocities of 2.5 - 10Gbit/s. These parameters becomes more and more significant with the optical transmission up to 40Gbit /s.
To summarize, the modern optical fibers are characterized by the small dispersion coefficient D of flat profile. Low dispersion both for the band C and L (Fig.3.19) creates many new possibilities including

- long distant transmission without dispersion compensation,
- high speeds up to 40Gbit/s,
- Transmission in the L band.

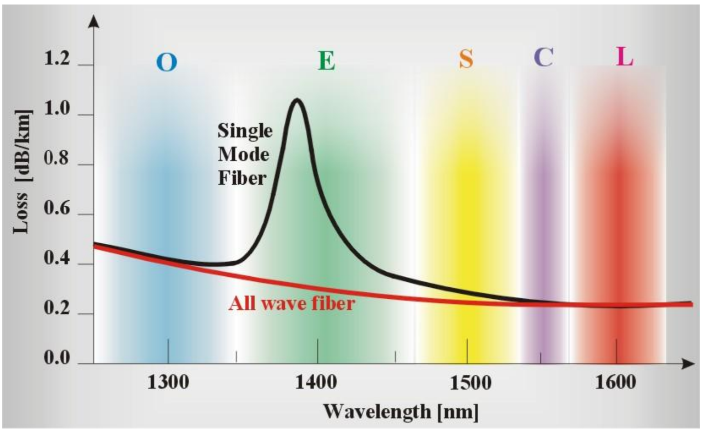


Fig. 3.19. Transmission bands

Although the present fiber technology is based on positive GVD materials, more and more often fibers showing anomal (or negative) GVD (when $\beta_2 < 0$) become in the centre of interest related to the new generation of fibers for soliton transmission. We will discuss this issue in further chapters.

| Kinds of dispersion shifted fibers | Description |
|---|---|
| Dispersion shifted single mode fibers | DS- SMF – Dispersion Shifted-Single Mode Fiber, recommendation of G.653, gradient refraction index, strong negative dispersion in II window (below 20 ps/nm*km, zero dispersion at 1550 nm in III window, applications in TDM single channel long-distant transmission in III window, less useful in multichanneled WDM transmission in III window, as the lack of dispersion leads to cross-talks as a result of non - linear four wave mixing phenomenon (FWM) |
| Non zero dispersion shifted –single mode fiber | NZDS- SMF – Non Zero Dispersion Shifted-Single Mode Fiber, recommendation of G.655, small, but not zero dispersion in whole range of transmission of optical amplifiers EDFA (1530-1565 nm) reduces the nonlinear effects of FWM and a cross phase modulation CPM, so far, the best medium for DWDM transmission in III window over long distances |

ITU-T International Telecommunication Union – Telecommunication Standarization Sector

Fig.3.20 illustrates the different types of dispersion shifted single mode fibers applied in the III optical window at1550 nm. Blue region marks the EDFA window (erbium doped fiber amplifier) and it represents the wavelengths used at present in DWDM multiplexing.

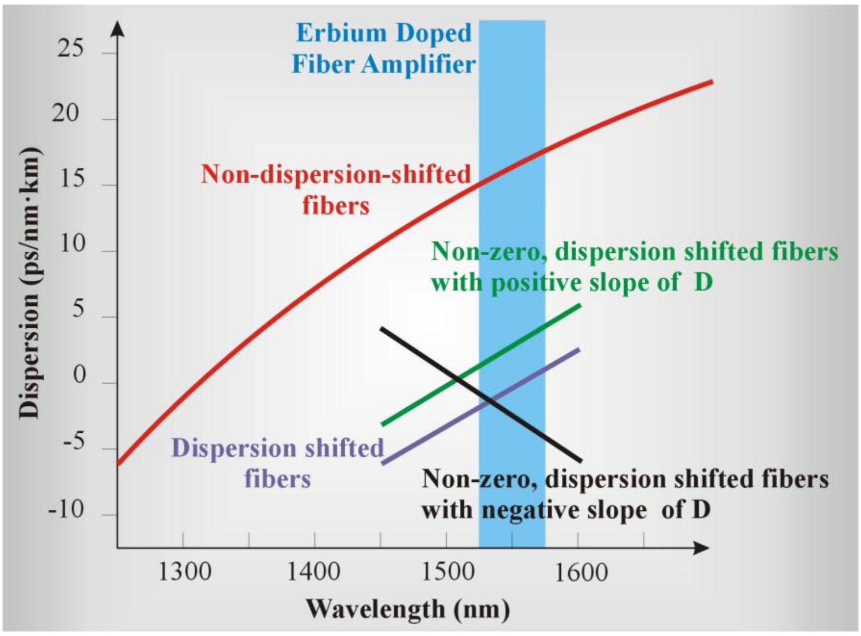


Fig.3.20 The different types of single mode fibers with shifted dispersion

- for non-dispersion shifted single mode fibers (Non-DSF) zero GVD occurs at 1310 nm,
- for single mode dispersion shifted fibers (DSF) zero GVD occurs at 1550 nm, employed in single channel TDM ransmission, nonlinear effects cause problems for multichannel DWDM broadband transmission,
- single mode non-zero, dispersion shifted fibers with positive slope of the dispersion coefficint D ((+D) NZ-DSF) are similar to DSF, but zero-dispersion is shifted out of window 1550 nm. At 1550 nm a fiber has small, but non-zero GVD, slope of D is positive
- single mode non-zero, dispersion shifted fibers with negative slope of the dispersion coefficient D (-D) NZ-DSF are similar to DSF, but the zero-dispersion is shifted out of window 1550 nm. At 1550 nm fiber has a small, but non-zero GVD, slope of D is negative.

The optical fibers of type (+D) NZ-DSF) and (-D) NZ-DSF having the opposite GVD are used to compensate the GVD. The sequence of equal distances of fibers (+D) NZ-DSF) and (-D) NZ-DSF causes that the total GVD is negligibly small. Such solutions can be employed in multiplexing DWDM techniques. Fig. 3.21 illustrates this method of GVD compensation.

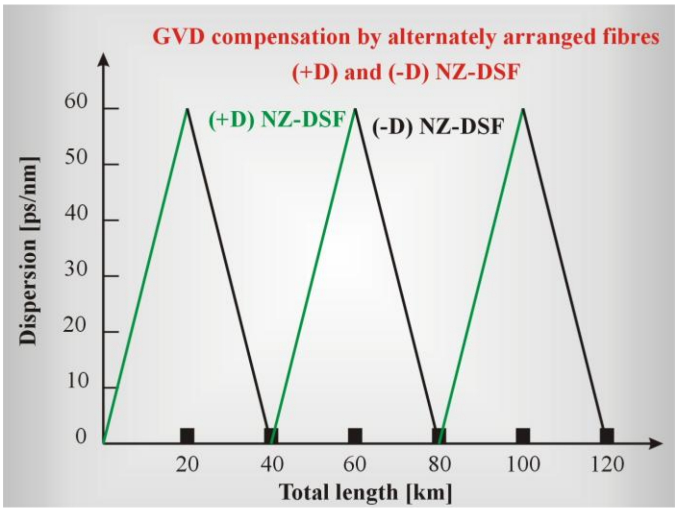
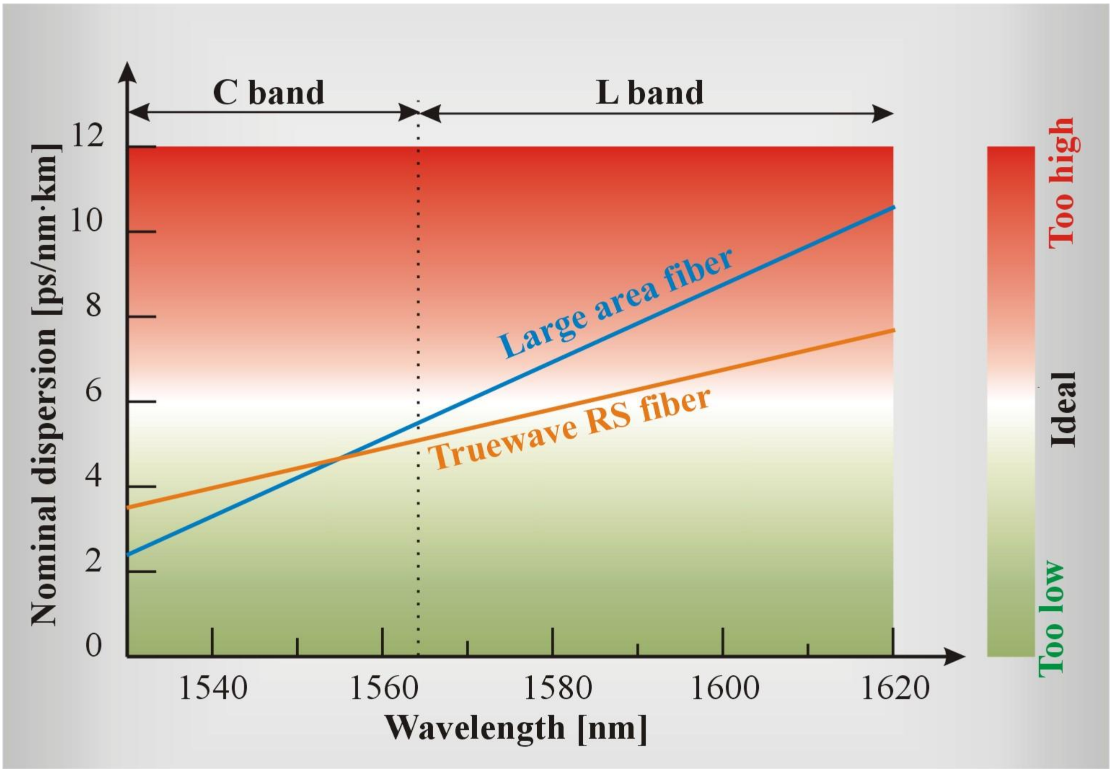


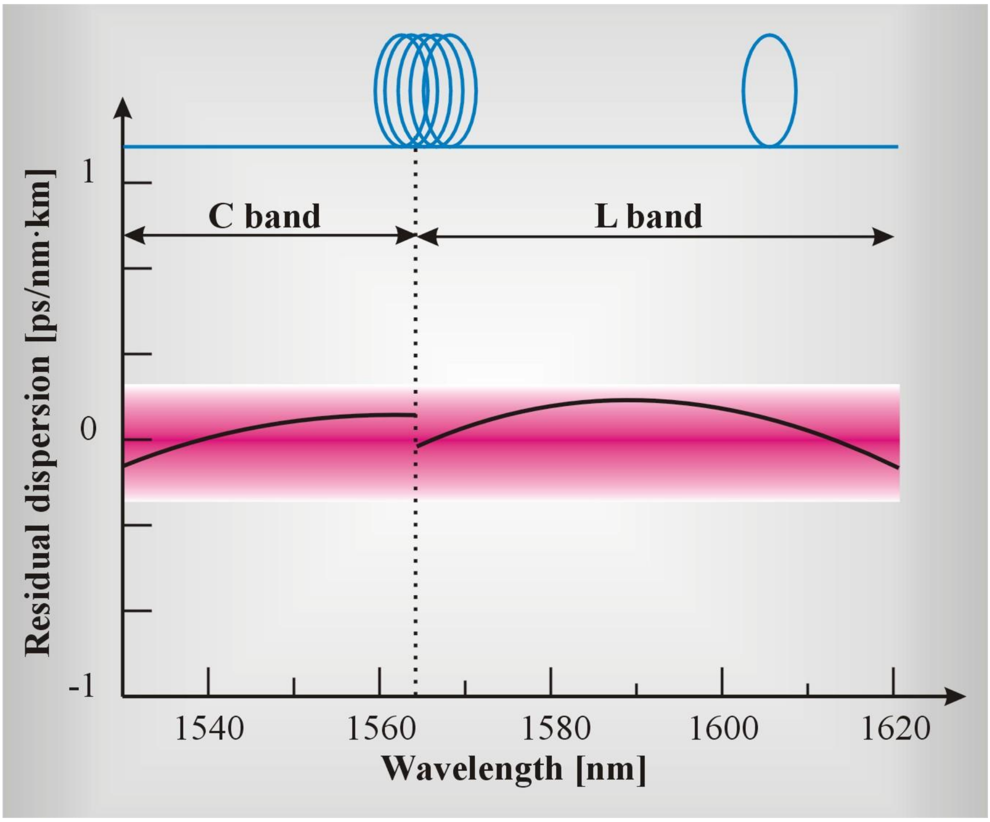

Fig.3.21. Method of GVD compensation by the sequence of fibers (+D) NZ-DSF) and (-D) NZ-DSF

Examples of non-zero dispersion shifted single mode fiber - NZDS- SMF

| | |
|---|---|
| True Wave (Fig.3.22) | 1994, Lucent Technologies, version True Wave + as well as True Wave – (alternately dispersion) permit to transmit without up to 1000 km with speed of 2.5 Gb/s or 300 km with speed of 10 Gb/s, dispersion D in the range 1460-1625nm is from 2 to 14ps/nm*km), TrueWave® is produced according to the standard of ITU-T G.655 (NZDF). The fiber of TrueWave RS offers the smallest slope of dispersion profile |
| All Wave | Bell Laboratory (Lucent), transmission in all four transmission windows II,III, IV, V. So far, transmission in V window has been inaccessible due to high attenuation caused by absorption of $OH^-$ ions. It may be used in the whole band from 1280 to 1625 nm. A fiber has a very low attenuation in water peak - at 1383 nm attenuation does not exceed 0.31dB/km. The fiber is in accordance with the newest standard of ITU-T G.652.D |
| LEAF | Large Effective Area, 1998, Corning , lower noises, it permitts to enlarge a distance between EDFA amplifiers up to 100 km |
| TERALIGHT | 1999, Alcatel, zero dispersion at 1440 nm, small positive slope of dispersion in the whole range of EDFA amplifiers, perfect fiber to long-distance multichannel broadband UWDM transmissions |




a)




b)

Fig.3.22 Comparison of the dispersion properties for the NZDS fiber and the large area fiber (a), residual dispersion for the NZDS fiber

### 3.2.3. Polarization Dispersion

We discussed the phenomena of polarization in optical fibers in chapter 1. In an ideal optical fiber there is no distinguished optical axis, the material of the core and of the cladding are isotropic, which means that a phenomenon of birefringerence does not exist. In real optical fibers the tensions, change of thickness, the accidental changes of shape, core diameter cause an accidental formation of distinguished optical axises and local birefringerence. As a consequence, two ortogonal components traveling in a fiber as ordinary and extraordinary ray move in a fiber with different velocities. The different velocities of the two ortogonal components generate the phase difference changing in time of propagation along a fiber and change of polarization. Beside the change of polarization with time of propagation, the different velocities of ordinary ray (polarization vector is perpendicular to the plane of the optical axis) and extraordinary (polarization in the same plane as the optical axis) cause that the rays reach the end of a fiber in different time. The changes of polarization are not essential, as long as a continuous light in a fiber is propagated (continuous wave, CW) because the majority of detectors are not sensitives to polarization state changes. However, in many applications the maintenance of a constant polarization is essential, e.g. in optical interferometer, optical lasers, sensors, optoelectrical modulators, in coherent transmission as well as in the coupling of integrated optical circuits. The question of dispersion caused by the different velocity of the ordinary and the extraordinary ray become particularly important in systems of optical communication at large speeds on the order of Gb/s, in which the short pulses travel large distances through the optical fiber. The different velocities of the ortogonal rays cause the group velocity dispersion GVD, which results in temporal broadening of the pulse. The polarization mode dispersion (PMD), combined with the GVD causes the distortion of CSO image (composite second order) in the amplitude modulated video systems, manifesting itself in appearance of a diagonal or circulating lines on TV screen. For digital systems at large speeds, the effect of PMD causes the increase of the bit error rate.

As it was discussed in chapter 1 describing fibers providing the PM polarization , the measure of a birefringerence is a parameter called the mode birefringerence $B_m$

$$B_m = \frac{|\beta_y - \beta_x|}{k_0} = n_{ef}^x - n_{ef}^y , \qquad (3.48)$$

where $\beta_y$ as well as $\beta_x$ are the ortogonal mode propagation constants, $n_{ef}^x$ and $n_{ef}^y$ are effective refraction indices along $x$ and $y$ direction, $k_0$ is the wave vector.
The another parameter defining a fiber birefringerence is the beat length

$$L_B = \frac{2\pi}{|\beta_y - \beta_x|} = \frac{\lambda}{B_m} , \qquad (3.49)$$

where $L_B$ is a path, on which the phase difference of the ortogonal modes increases by $\frac{\pi}{2}$. This phenomenon this repeats periodically. The parameter which characterizes the phenomenon of dispersion caused by polarization PMD is the time delay $\Delta T$ between the two ortogonal components. This parameter is a measure of temporal pulsebroadening on the distance of $L$ for a fiber characterized by the mode birefringerence $B_m$ and it is expressed by formula

$$\Delta T = \left| \frac{L}{\upsilon_{gx}} - \frac{L}{\upsilon_{gy}} \right| = L|\beta_{1x} - \beta_{1y}| = L\delta\beta_1 \qquad (3.50)$$

where

$\delta\beta_1 = k_0 (dB_m / d\omega)$

However, the parameter $\Delta T$ is not a good parameter to characterize the standard fibers, that are unable to keep the constnt polarization, because $\Delta T$ is the stochastic variable, and its average approaches zero. Therefore, for standard fibers without maintaining polarization the better parameter is the root-mean-square, RMS for $\Delta T$ [6]

$$\sigma_T^2 = \langle (\Delta T)^2 \rangle = 2(\Delta' l_c)^2 [\exp(-L / l_c) + L / l_c - 1] \qquad (3.51)$$

where $\Delta'$ is an internal mode dispersion of a fiber, $l_c$ is the length of correlation defined as distance on which the correlation keeps between the two ortogonal components of polarization. Typical length of correlation is $10^2$ m, that is at L>0.1 km one can assume, that $l_c << L$,
and the formula obtains the form

$$\sigma_T \approx \Delta' \sqrt{2 l_c L} \equiv D_p \sqrt{L} \qquad (3.52)$$

where $D_p$ is the PMD parameter, expressed in $ps / \sqrt{km}$. Typical values of PMD are on the order of 0.1-1 $ps / \sqrt{km}$

## SUMMARY

There are the following kinds of dispersion:

1. Chromatic dispersion

Waveguide dispersion – the effective refraction index depends on the normalized frequency

Material dispersion - light ray is not monochromatic and the different wavelength components propagate through a fiber with different velocities. The temporal pulse broadening is due to the non - zero second derivative of the refraction index

Polarisation dispersion - the tensions, change of thickness, the accidental changes of shape, core diameter cause an accidental formation of distinguished optical axises and local birefringerence. As a consequence, two ortogonal components traveling in a fiber as ordinary and extraordinary ray move in a fiber with different velocities. The different velocities of the two ortogonal components generate the phase difference changing in time of propagation along a fiber and change of polarization. Beside the change of polarization with time of propagation, the different velocities of ordinary ray and extraordinary cause that the rays reach the end of a fiber in different time.

2. Mode dispersion – in a multimode fiber with a step profile of the refraction index all rays travel with the same speed – the rays traveling along the fiber axis have the same speed as the rays traveling close to the core-cladding interface. As they cover the optical paths of different length at the same speed they reach the detector at different times. This leads to the temporal pulse broadening at the end of the fiber. This type of temporal broadening is called the mode dispersion

Parameters characterizing chromatic dispersion:

$$\beta_1 = \frac{d\beta}{d\omega} == \frac{1}{\upsilon_g} = t_g = \frac{n}{c}\left(1 + \frac{\omega}{n}\frac{dn}{d\omega}\right)$$

$$\beta_2 = \frac{d^2\beta}{d\omega^2} = \frac{1}{c}(2\frac{dn}{d\omega} + \omega\frac{d^2n}{d\omega^2})$$

$$\beta_3 = \frac{d^3\beta}{d\beta^3}$$

when $\beta_2 > 0$ it is said, that the fiber shows normal (or positive ) GVD,
when $\beta_2 < 0$ it is said, that the fiber shows anomal (or negative ) GVD
Often, instead of $\beta_2$, to express the group velocity dispersion we use the dispersion coefficient D defined as

$$D = \frac{dt_g}{d\lambda}\ [\frac{ps}{nm \bullet km}],\ \text{gdzie}\ t_g = \frac{1}{\upsilon_g} = \frac{d\beta}{d\omega}$$

The dispersion coefficient D determines the temporal pulse broadening in ps (picoseconds) after passage of 1 km of a fiber, if the width of spectral line of light source is 1 nm. D has the opposite sign as $\beta_2$, because

$$D = \frac{d\beta_1}{d\lambda} = -\frac{2\pi c}{\lambda^2}\beta_2 \approx \frac{\lambda}{c}\frac{d^2n}{d\lambda^2}$$

The other parameter which characterizes the chromatic dispersion is the coefficient $d_{12}$ defined as

$$d_{12} = \beta_1(\lambda_1) - \beta_1(\lambda_2) = \upsilon_g^{-1}(\lambda_1) - \upsilon_g^{-1}(\lambda_2)$$

The parameter which describes this effect of divergence caused by different propagation velocities is $L_W$ (walk-off length)

$$L_W = \tau_0 / |d_{12}|$$

The important parameter which characterizes the slope of dispersion coefficient D is

$$S = \frac{dD}{d\lambda} = \left(\frac{2\pi c}{\lambda^2}\right)^2\beta_3 + \left(\frac{4\pi c}{\lambda^3}\right)\beta_2$$

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