Generation of high-energy, few-cycle optical pulses
PART I : Foundations of ultrafast pulse compression

Günter Steinmeyer
Max-Born-Institut, Berlin, Germany
steinmey@mbi-berlin.de

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MBI
Max-Born-Institute

Overview

First Lecture:

Foundations of Ultrashort Pulse Compression

- Description of short laser pulses (duration, chirp, spectrum)
- Group Delay Dispersion and its compensation
- Gain, loss, and nonlinear optical effects (SPM and SAM)
- Soliton and solitonlike pulse shaping

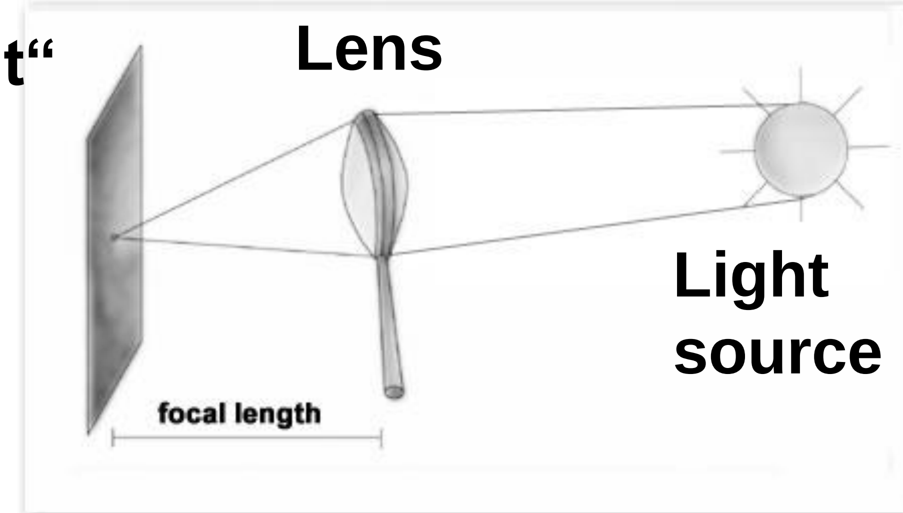
Second lecture:

Practical methods for ultrafast pulse compression

- Laser oscillators
- Amplification of short pulses, CPA
- Active pulse compression (fiber-grating, hollow fiber, filament)

Piling up photons in as small a volume as possible

„Experiment“



What is the maximum excitation density we can get?

Sun solar constant ≈ 1000 W/m^2

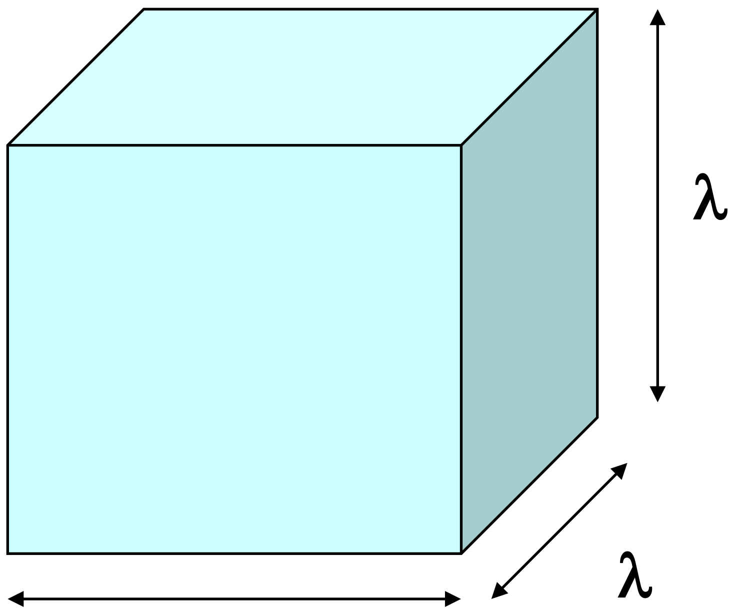
Lens area, maybe 0.1m^2

Assuming 10 µm spot size: 1 W/µm^2

But we are only focusing in two dimensions…

The lambda cube

Focusing along z means temporal compression !!



One cycle

$= \lambda/c$

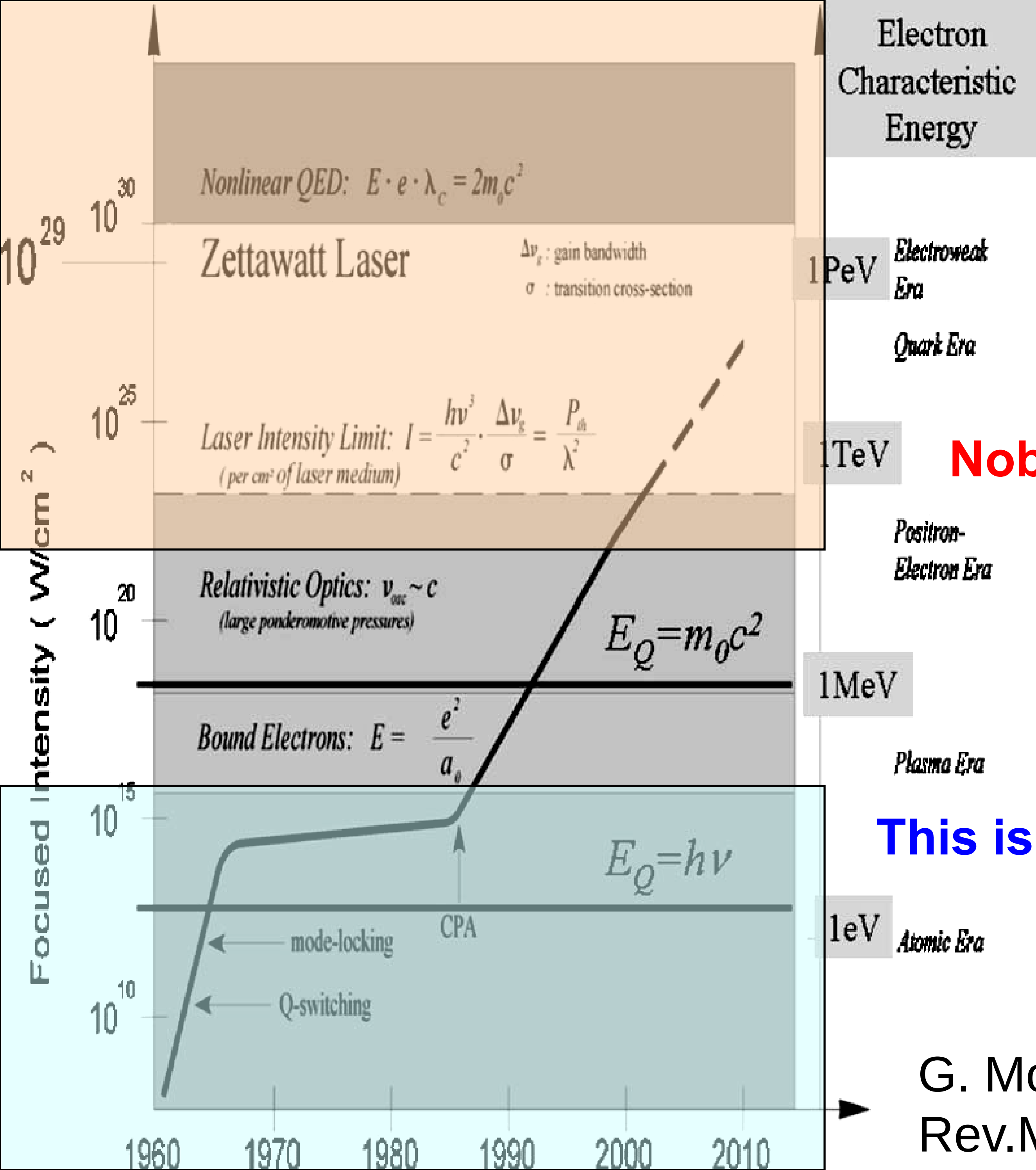
Solar focus (uncompressed):

<u>10000 photons/λ^3</u>

Amplified compressed laser pulse

1 mJ, focused to 1 µm

<u>10^{16} photons/λ^3 !!</u>

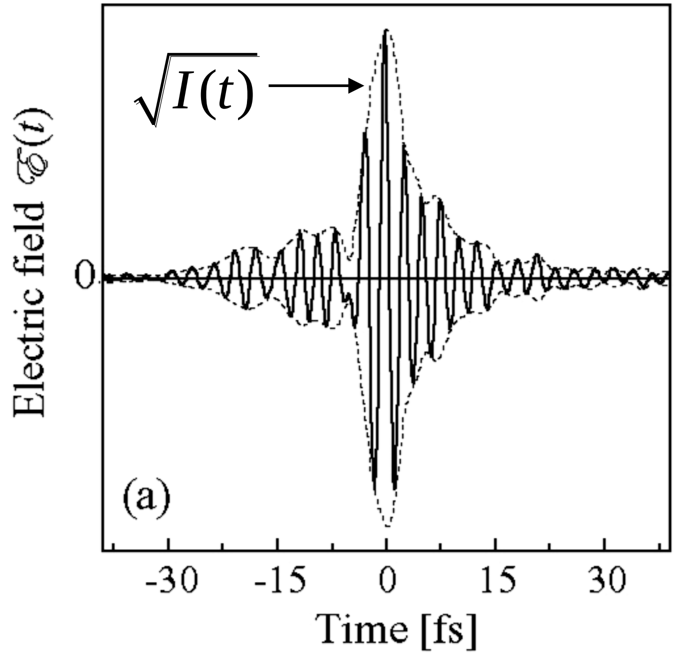


Nobody's ever been there…

This is as far as I will take you…

G. Mourou et al.
Rev.Mod.Phys. **78**, 309 (2006).

A light wave has intensity and phase vs. time.



Neglecting the spatial dependence for now, the pulse electric field is given by:

$$E(t) = \mathrm{Re}\left\{ \sqrt{I(t)}\, \exp\{i[\omega_0 t - \phi(t)]\} \right\}$$

Intensity

Carrier frequency

Phase

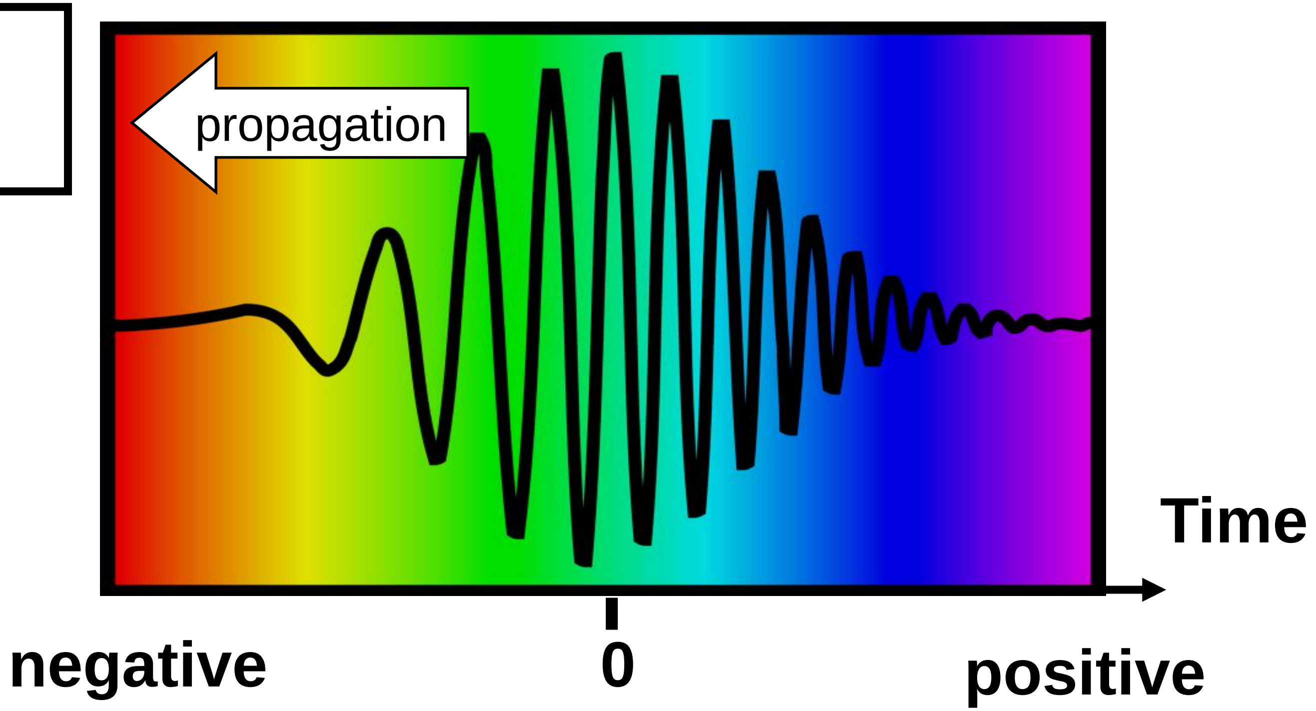
Slowly-varying envelope approximation, can be used down to about two optical cycles with some care.

The Chirp (Instantaneous frequency)

The temporal phase, $\phi(t)$, contains frequency-vs.-time information.

The pulse ***instantaneous angular frequency***, $\omega_{inst}(t)$, is defined as:

$$\omega_{inst}(t) \equiv \omega_0 - \frac{d\phi}{dt}$$

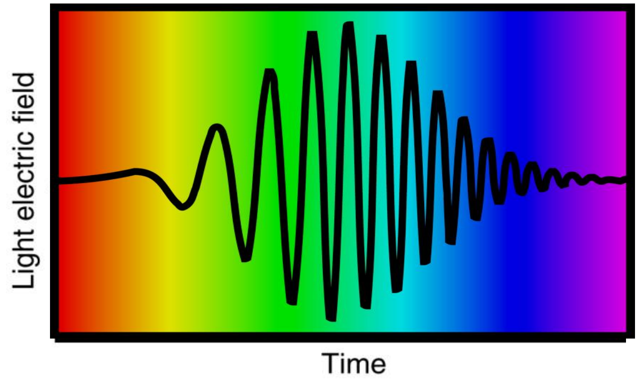


This pulse increases its frequency linearly in time (from red to blue).

In analogy to bird sounds, this pulse is called a "chirped" pulse.

This pulse is **positively chirped**, i.e., red leading blue, as from material dispersion !

The Chirped Pulse (continued)



We can write a linearly chirped Gaussian pulse mathematically as:

$$E(t) = \operatorname{Re} E_0 \underbrace{\exp\left[-(t/\tau_G)^2\right]}_{\text{Gaussian amplitude}} \exp\left[i\left(\underbrace{\omega_0 t}_{\text{Carrier wave}} + \underbrace{\beta t^2}_{\text{Chirp}}\right)\right]$$

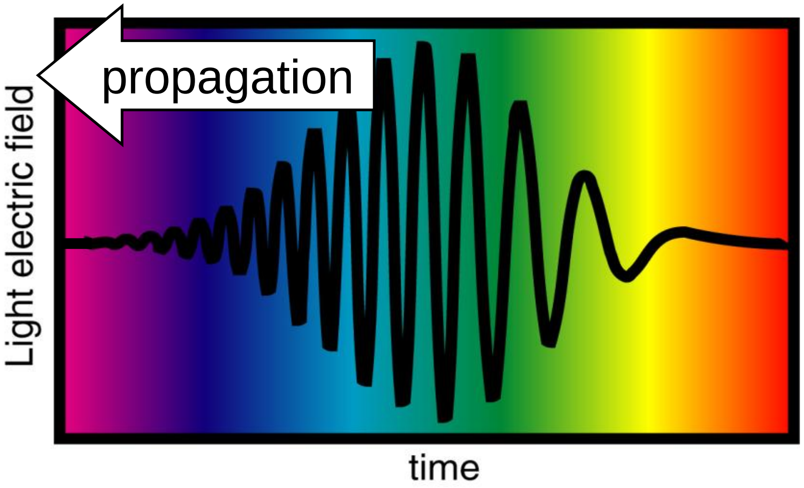
Gaussian amplitude **Carrier wave** **Chirp**

Note that for β > 0, when t < 0, the two terms partially cancel, so the phase changes slowly with time (so the frequency is low). And when t > 0, the terms add, and the phase changes more rapidly (so the frequency is larger)

The Negatively Chirped Pulse

We have been considering a pulse whose frequency *increases* **linearly with time: a** *positively* **chirped pulse.**

One can also have a *negatively* **chirped (Gaussian) pulse, whose instantaneous frequency** *decreases* **with time.**



We simply allow β to be *negative* **in the expression for the pulse:**

$$E(t) = \operatorname{Re} E_0 \exp\left[-(t/\tau_G)^2\right] \exp\left[i\left(\omega_0 t + \beta t^2\right)\right]$$

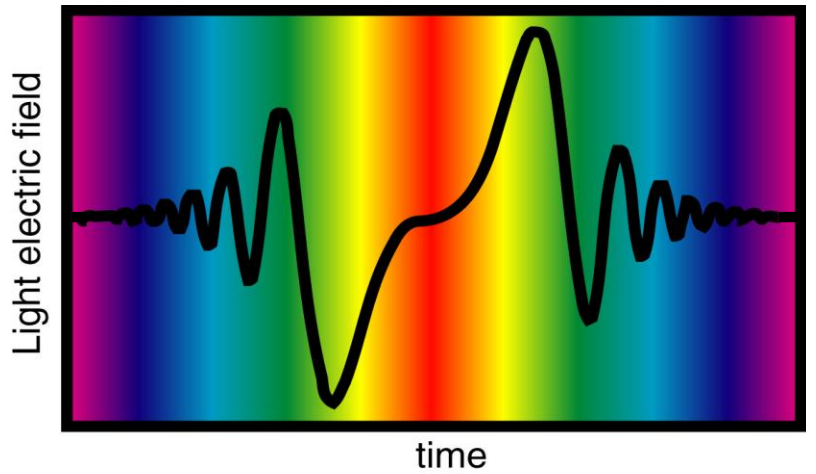
And the instantaneous frequency will decrease with time:

$$\omega_{inst}(t) = \omega_0 + 2\beta t = \omega_0 - 2|\beta| t$$

Nonlinearly Chirped Pulses

The frequency of a light wave can also vary nonlinearly with time.

This is the electric field of a Gaussian pulse whose frequency varies quadratically with time:



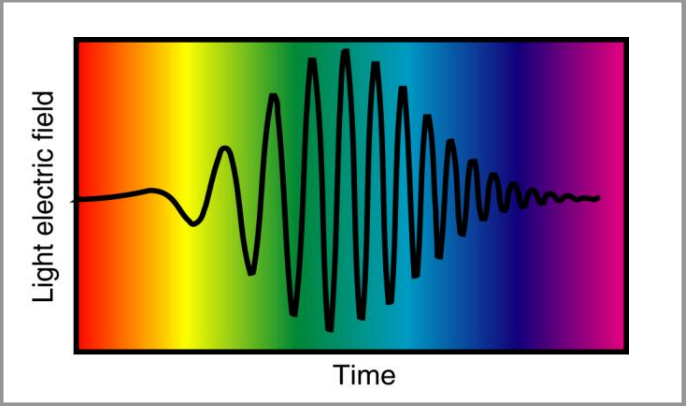
$$\omega_{inst}(t) = \omega_0 + 3\gamma t^2$$

This light wave has the expression:

$$E(t) = \operatorname{Re} E_0 \exp\left[-\left(t/\tau_G\right)^2\right]\exp\left[i\left(\omega_0 t + \gamma t^3\right)\right]$$

Arbitrarily complex frequency-vs.-time behavior is possible.

The Fourier Transform of a Chirped Pulse



Writing a linearly chirped Gaussian pulse as:

$$E(t) = E_0 \exp\left[-\alpha t^2\right] \exp\left[i\left(\omega_0 t + \beta t^2\right)\right]$$

or:

$$E(t) = E_0 \exp\left[-(\alpha - i\beta) t^2\right] \exp\left[i(\omega_0 t)\right]$$

A Gaussian with a complex width!

Fourier-Transforming yields:

$$\tilde{E}(\omega) \propto E_0 \exp\left[-\frac{1/4}{\alpha - i\beta}(\omega - \omega_0)^2\right]$$

A chirped Gaussian pulse Fourier-Transforms to itself!!!

Rationalizing the denominator and separating the real and imag parts:

$$\tilde{E}(\omega) \propto E_0 \exp\left[-\frac{\alpha/4}{\alpha^2 + \beta^2}(\omega - \omega_0)^2\right] \exp\left[-i\frac{\beta/4}{\alpha^2 + \beta^2}(\omega - \omega_0)^2\right]$$

The chirped Gaussian

$$E(t) = E_0 \exp\left[-\alpha t^2\right] \exp\left[i\left(\omega_0 t + \beta t^2\right)\right]$$

FT

$$\tilde{E}(\omega) \propto E_0 \exp\left[-\frac{\alpha/4}{\alpha^2+\beta^2}\left(\omega-\omega_0\right)^2\right] \exp\left[-i\frac{\beta/4}{\alpha^2+\beta^2}\left(\omega-\omega_0\right)^2\right]$$

Increasing β at constant α: Increasing the chirp at constant pulse duration

$\Rightarrow$ Wider spectrum with increasing parabolic phase

Linear chirp yields parabolic phase

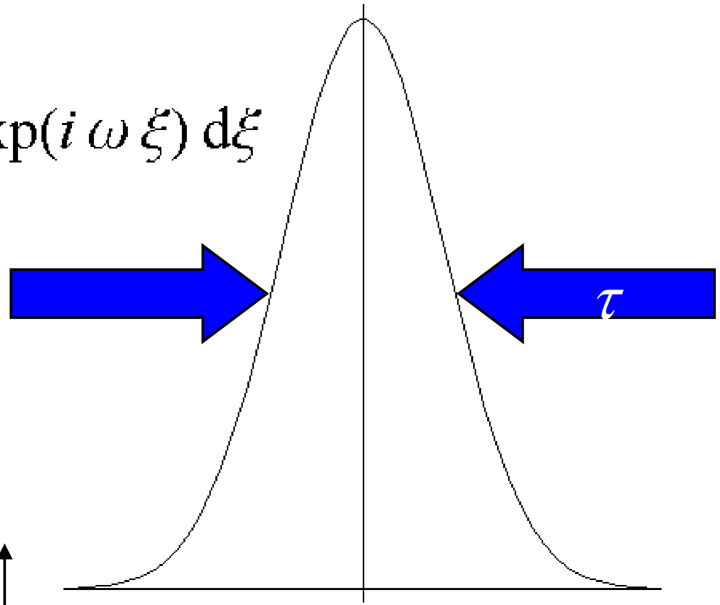
($\Rightarrow$ Group Velocity Dispersion, GVD)

Sign of curvature of phase corresponds to sign of chirp
positive chirp = normal GVD
negative chirp = anomalous GVD

Spectrogram

$$S(\omega, t) = \int f^*(\xi - \tfrac{1}{2}\, t)\, f(\xi + \tfrac{1}{2}\, t) \exp(i\, \omega\, \xi)\, \mathrm{d}\xi$$

$$f(t) = \exp\left(-\frac{t^2}{\tau_0{}^2}\right)$$



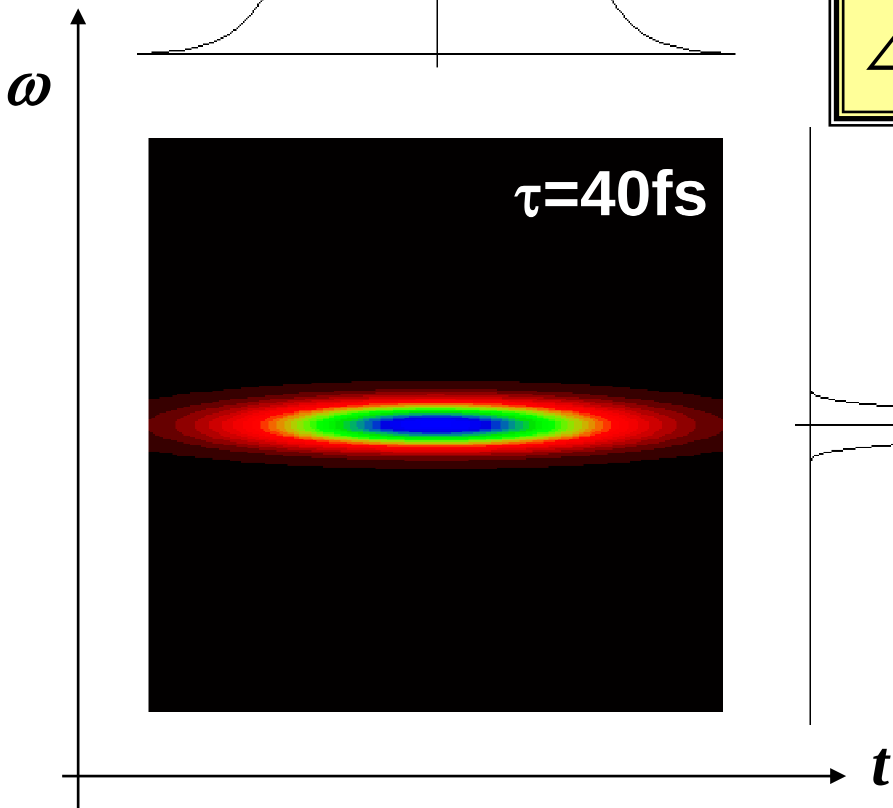
Heisenberg‘s uncertainty relationship b/t $\Delta\omega$ and τ:

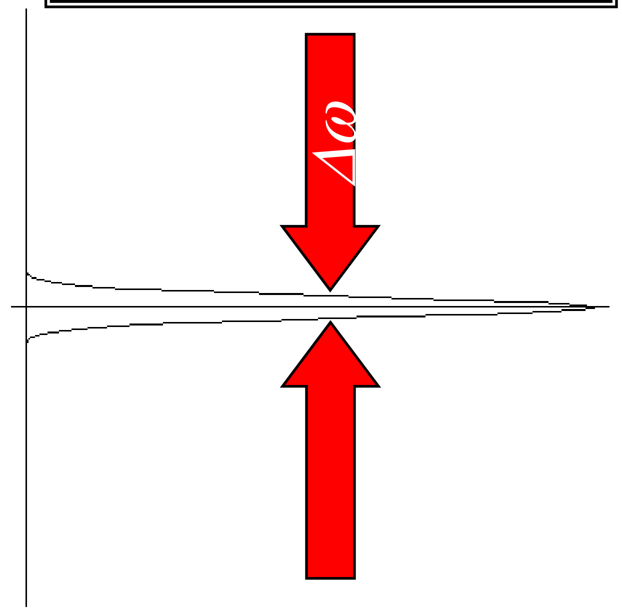
short pulse requires broad spectrum

$$\Delta\omega \cdot \Delta\tau \geq 2\pi$$

but:

broad spectrum does not automatically yield short pulse...

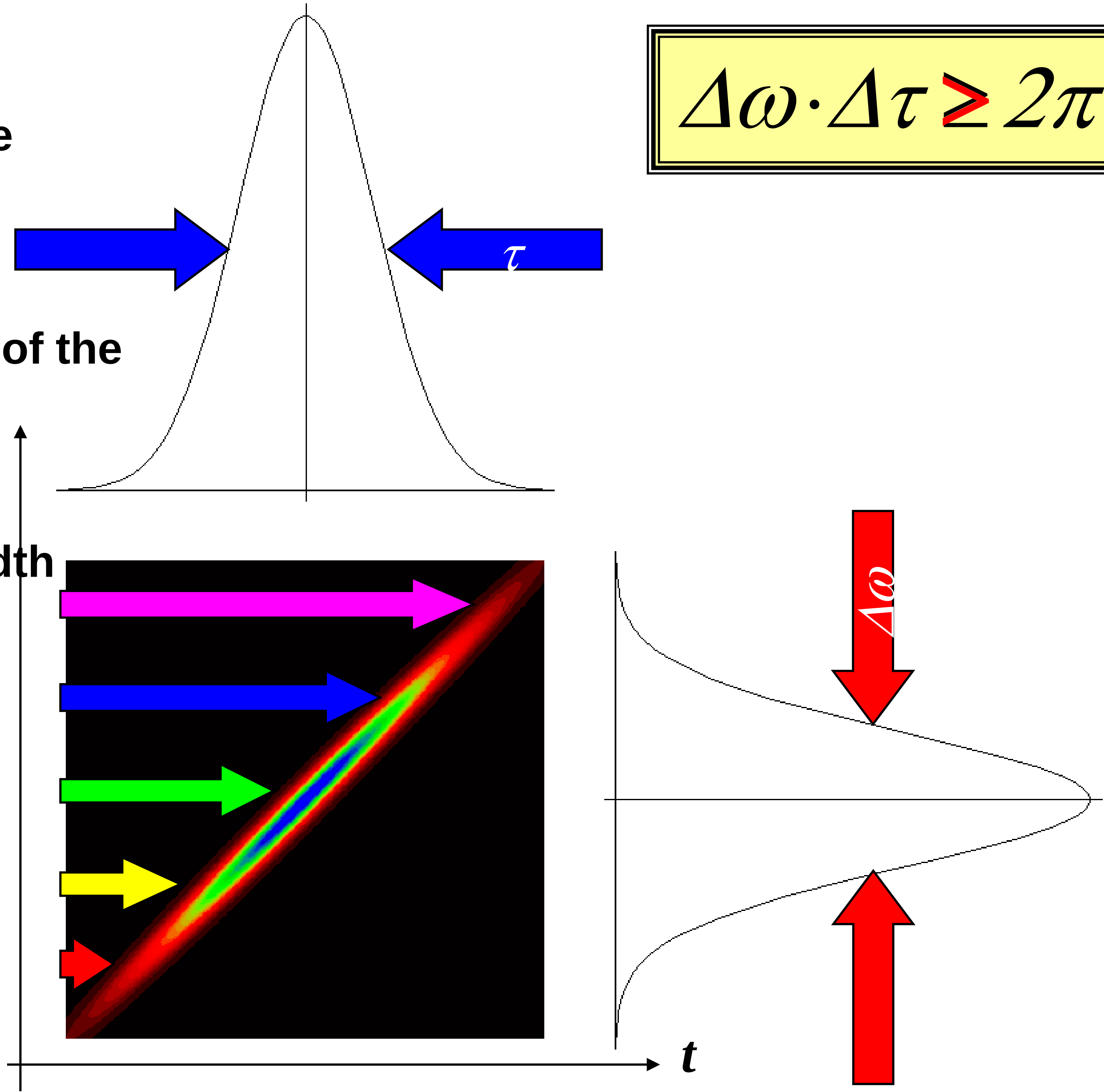




Chirped pulses in the spectrogram

$$\Delta\omega \cdot \Delta\tau \geq 2\pi$$

- different Fourier components experience different group delay
- Temporal smearing of the pulse
- Despite spectral width no short pulse
- „not bandwidth limited“



Group Delay vs. Frequency GD(ω)

The frequency-domain quantity that is analogous to the instantaneous frequency vs. t is the "group delay" vs. ω.

If the wave in the frequency domain is:

$$E(\omega) = \sqrt{S(\omega)}\exp\left[-i\varphi(\omega)\right]$$

then the group delay is the derivative of the spectral phase:

$$\boxed{\tau_g(\omega) = d\varphi / d\omega}$$

The Group Delay vs. ω for a Chirped Pulse

The group delay of a wave is the derivative of the spectral phase:

$$\tau_g(\omega) \equiv d\varphi / d\omega$$

For a linearly chirped Gaussian pulse, the spectral phase is:

$$\varphi(\omega) = \frac{\beta/4}{\alpha^2+\beta^2}\left(\omega-\omega_0\right)^2$$

So:

$$\tau_g = \frac{\beta/2}{\alpha^2+\beta^2}\left(\omega-\omega_0\right)$$

And the delay vs. frequency is also linear.

When the pulse is long ($\alpha \to 0$), then:

$$\tau_g = \frac{1}{2\beta}\left(\omega-\omega_0\right)$$

which is just the inverse of the instantaneous frequency vs. time.

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Chirp vs. Spectral phase curvature

- An unchirped pulse exhibits a flat spectral phase
- Spectral phase slope is unimportant (const. group delay) for the pulse shape
- Positively chirped pulses (red leading blue) exhibit positive (or normal) phase curvature or Group Delay Dispersion (GDD)
- Negatively chirped pulses (red trailing blue) exhibit negative (or anomalous) phase curvature
- The unchirped pulse is the shortest pulse possible for a given spectrum (rms def., not FWHM !)

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Spectral-Phase Taylor Series

It's common practice to expand the spectral phase in a Taylor Series:

$$\varphi(\omega) \quad = \quad \varphi_0 + \varphi_1\,[\omega - \omega_0] + \varphi_2\,[\omega - \omega_0]^2 / 2! + \ldots$$

What do these terms mean?

φ_0: Absolute phase $\qquad E(t)\exp(i\varphi_0) \to \tilde{E}(\omega)\exp(i\varphi_0)$

φ_1: Group Delay $\qquad E(t+\varphi_1) \to \tilde{E}(\omega)\exp(i\omega\varphi_1)$

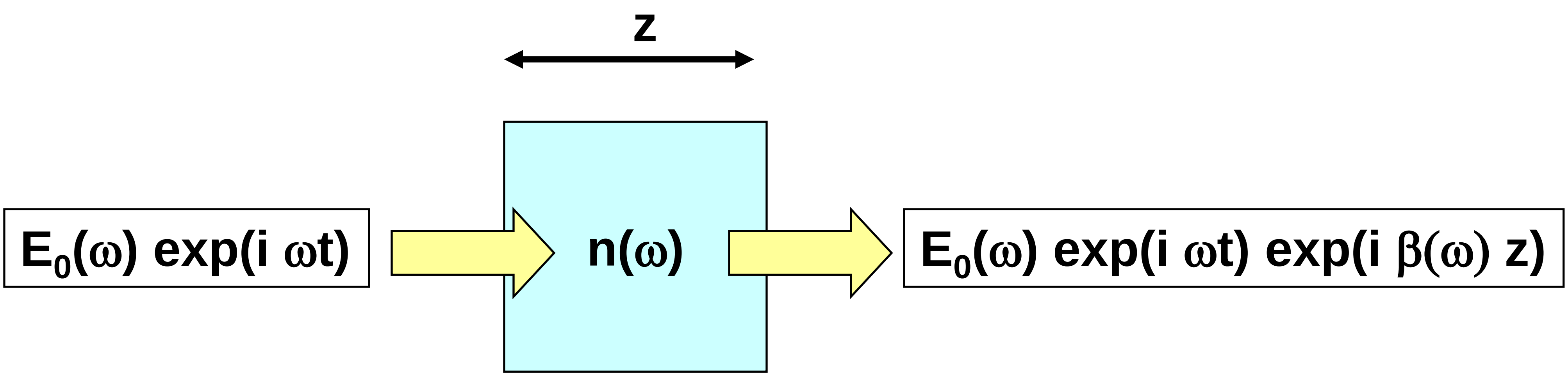
Fourier Shift Theorem

φ_2: Group velocity dispersion (linear chirp)

Leading Order Term to cause pulse broadening

φ_3: Third-order dispersion

Propagation through dispersive materials



$$\beta(\omega) = n(\omega)\ \omega/c = \beta_0 + \beta_1\ (\omega-\omega_0) + \beta_2\ (\omega-\omega_0)^2/2 + + \beta_3\ (\omega-\omega_0)^3/6 + \ldots$$

$\beta_1 = n_g/c = (n+\omega\ dn/d\omega)/c = 1/v_g$ **Group delay, inverse group velocity**
units: fs/mm

$\beta_2 = (2\ dn/d\omega + \omega\ d^2\nu/d\omega^2)/c$ **Group velocity dispersion**
units: fs²/mm

GDD / GVD

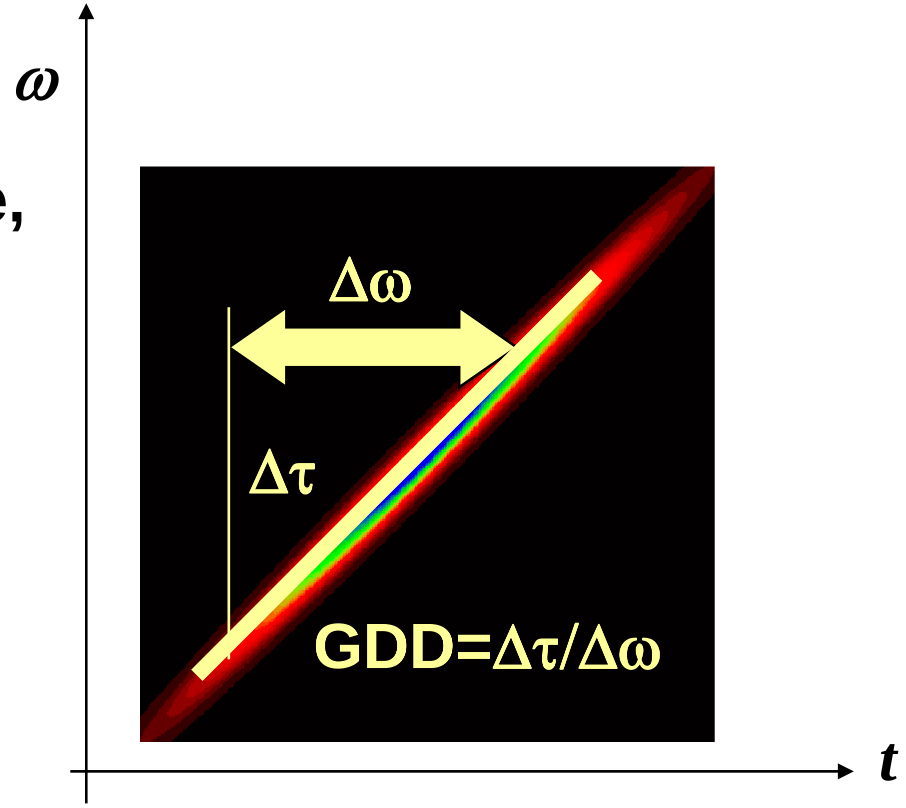
Group Delay Dispersion, units [fs^2]

More correctly: [fs/(Prad/s)], i.e. delay over ang. freq.

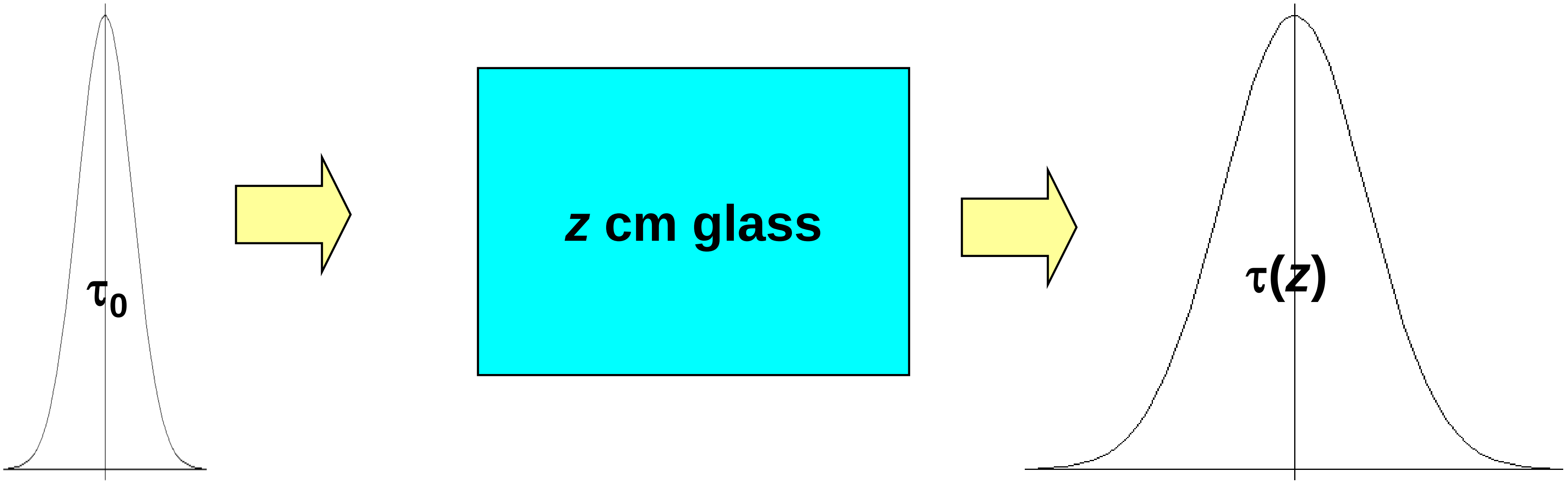
describes how much a particular Fourier component appears to be delayed vs. another one at distance $\Delta\omega$

GDD is property of the pulse, not the material !

GVD, units [fs^2/mm] is specific property of an optical material



Pulse broadening due to propagation



$I(t)=\exp(-t^2/\tau_0^2)$

$t_{FWHM}=1.665\ \tau_0$

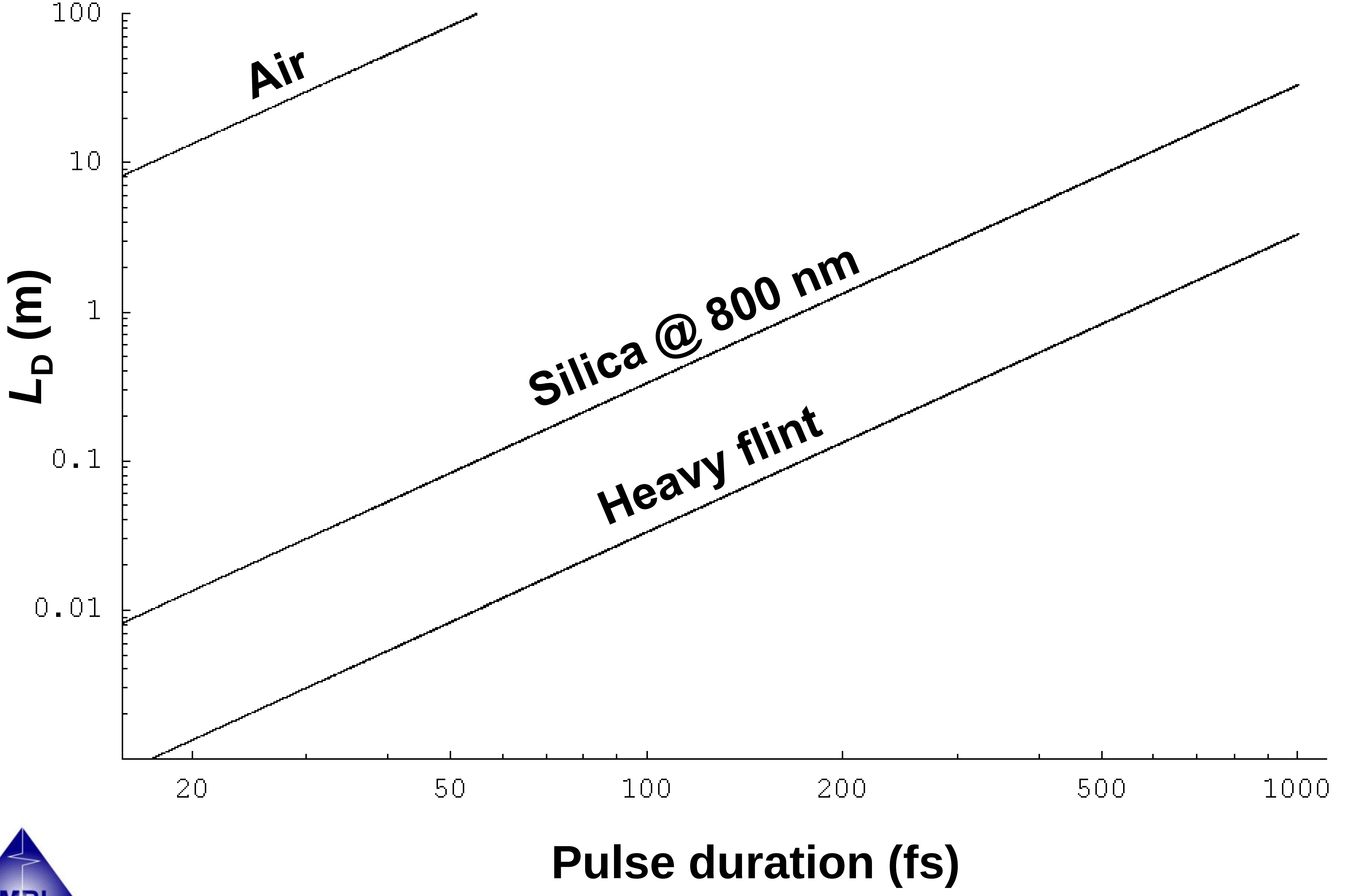
$$\tau(z)=\tau_0\left[1+(z/L_D)^2\right]^{1/2}$$

$$L_D=\tau_0^2/\beta_2$$

Rule of thumb:

$|N^2\ \mathrm{fs}^2|$ become important for a pulse with N fs duration

The dispersion length



Types of dispersion

material dispersion
(origin: atomic and vibrational resonances)

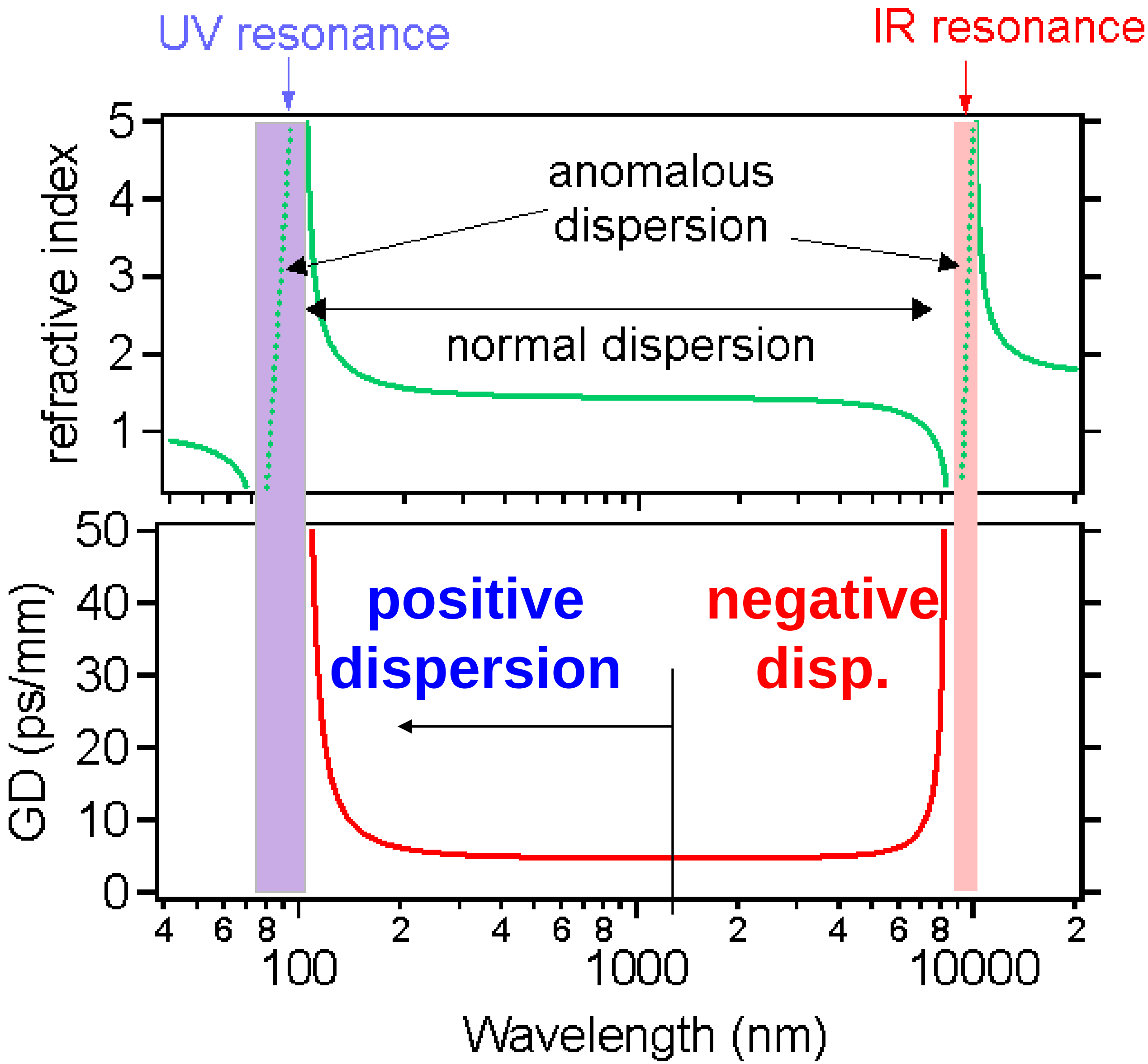
geometric dispersion
(origin: angular dispersion)

interferometric dispersion
(resonances due to cavity/multi pass interferometer)

chirped mirrors
(photonic structure, designed to provide particular dispersion)

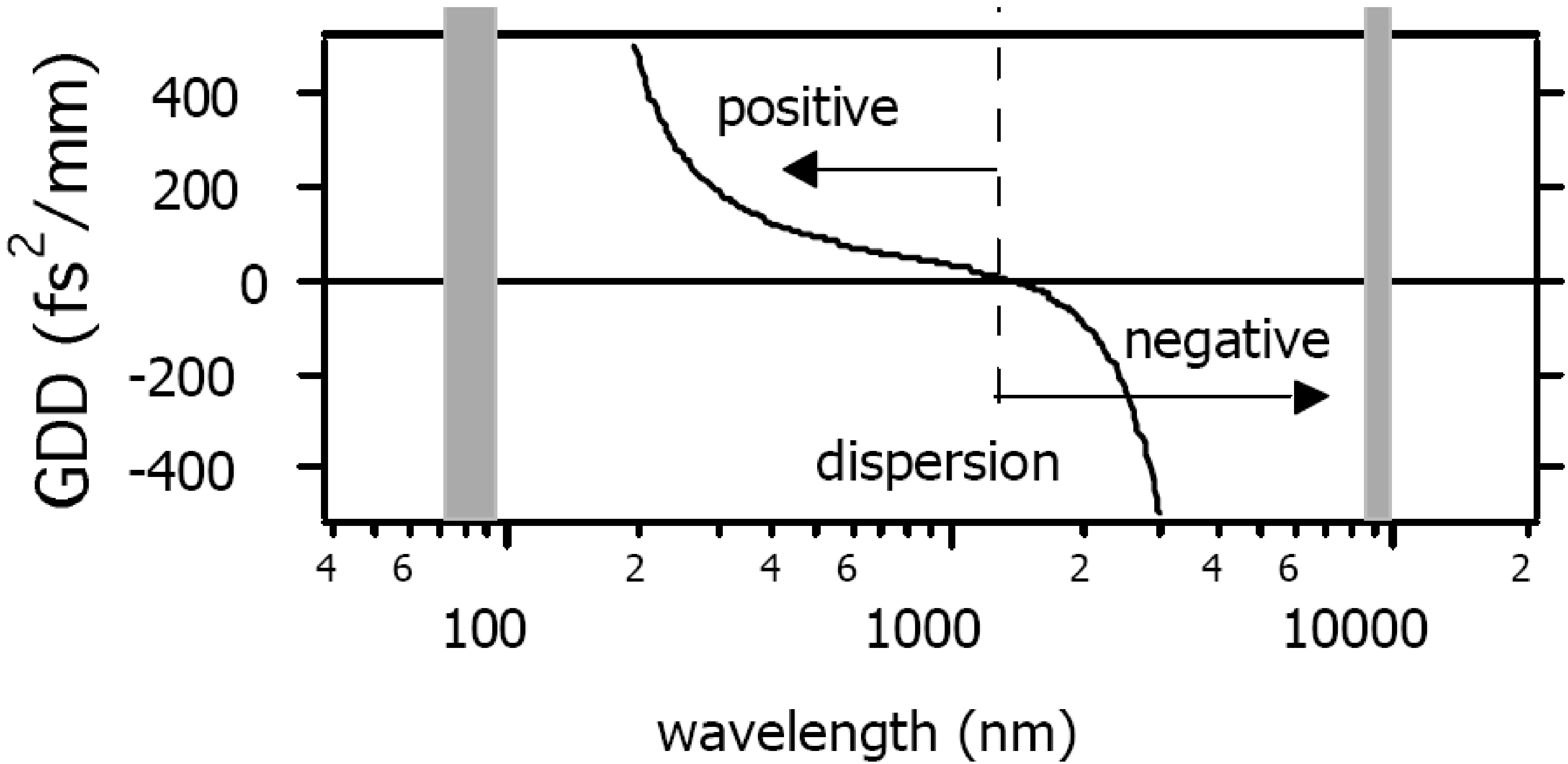
Origin of material dispersion

- **UV- and IR-Resonances cause characteristic „phase“ of an optical medium**
- **Resonances „store“ energy, causing a group delay close to resonance**
- **Below ~1000nm, only positive slope of GD(ω)**
  positive dispersion



Reviews: G.Steinmeyer, *J. Opt. A* **5**, R1 (2003)
I.A. Walmsley, *Rev. Sci. Instrum.* **72**, 1 (2001)

Resulting GDD



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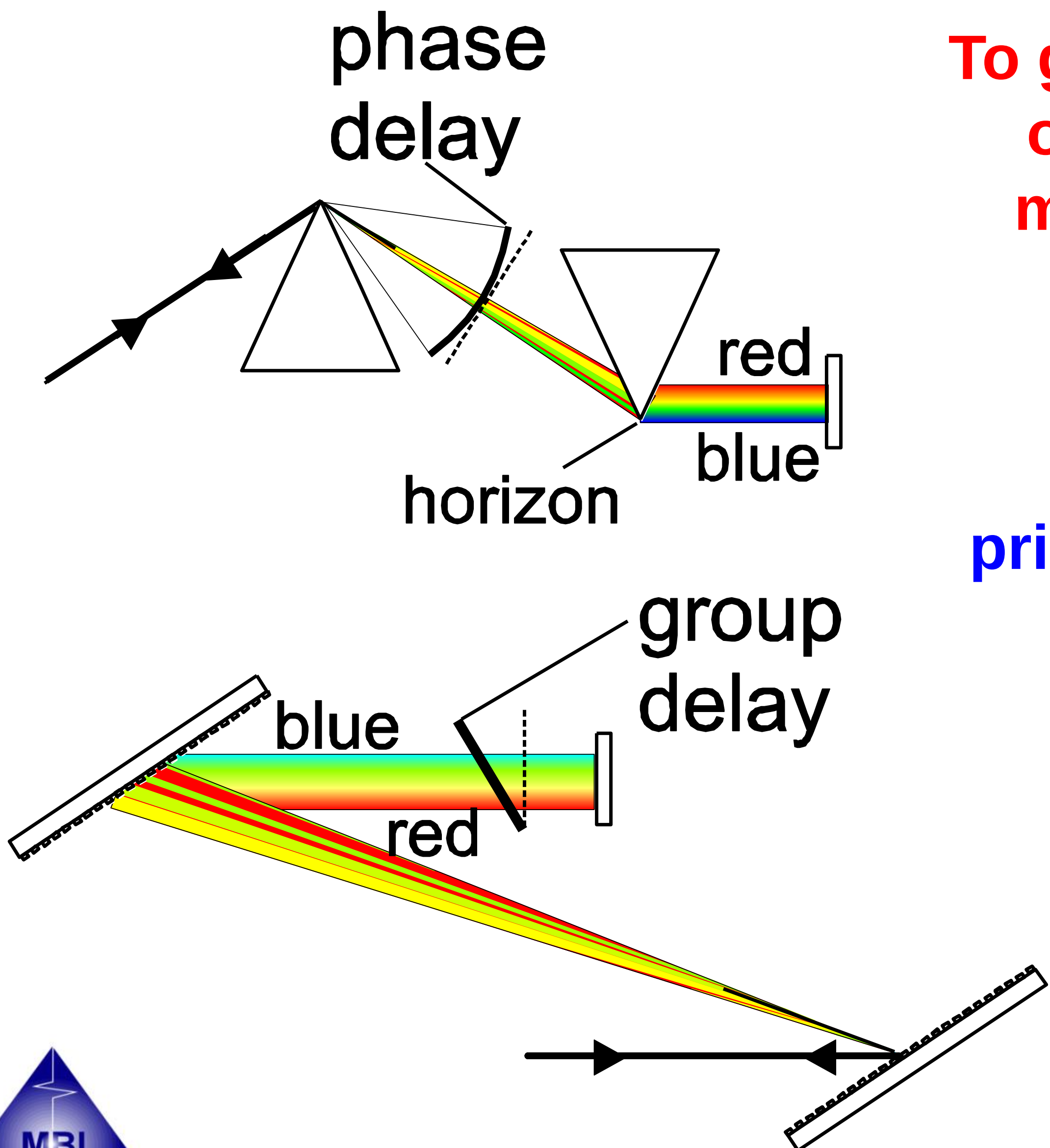
dispersion compensation



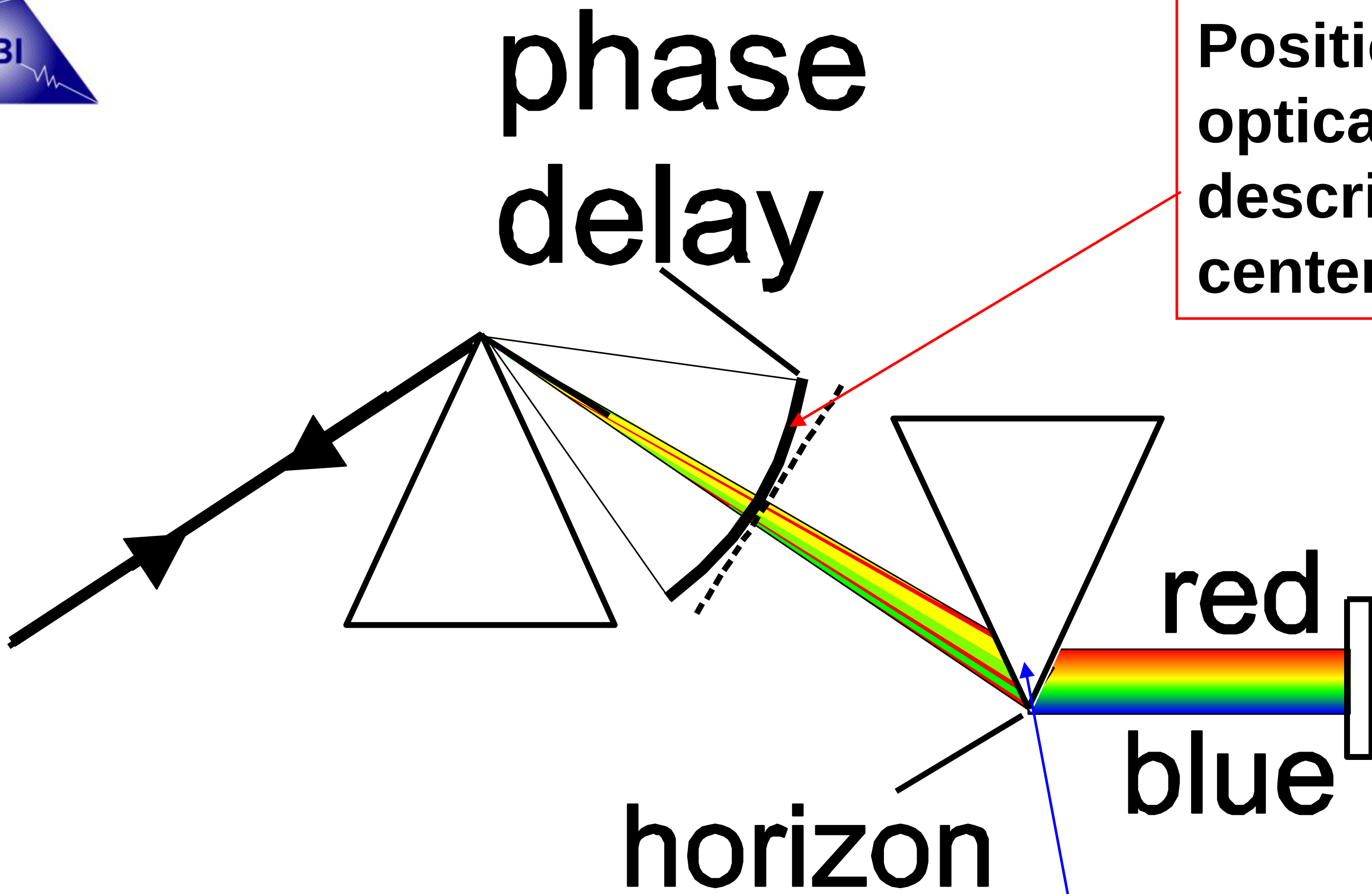
To generate the shortest pulse, one needs to compensate material dispersion effects

Traditional way:

prism and grating assemblies

Refs.:
E.B.Treacy, *IEEE JQE* **5**, 454 (1969)
Fork et al., *Opt. Lett.* **9**, 150 (1984)

Geometric dispersion – the prism compressor



Positions of equal optical path length describe circle with center at prism tip

Second prism only parallelize beam paths
No effect on dispersion to leading order

Refs.: Fork et al., *Opt. Lett.* **9**, 150 (1984)
Sherriff, JOSA B 15, 1224 (1998)

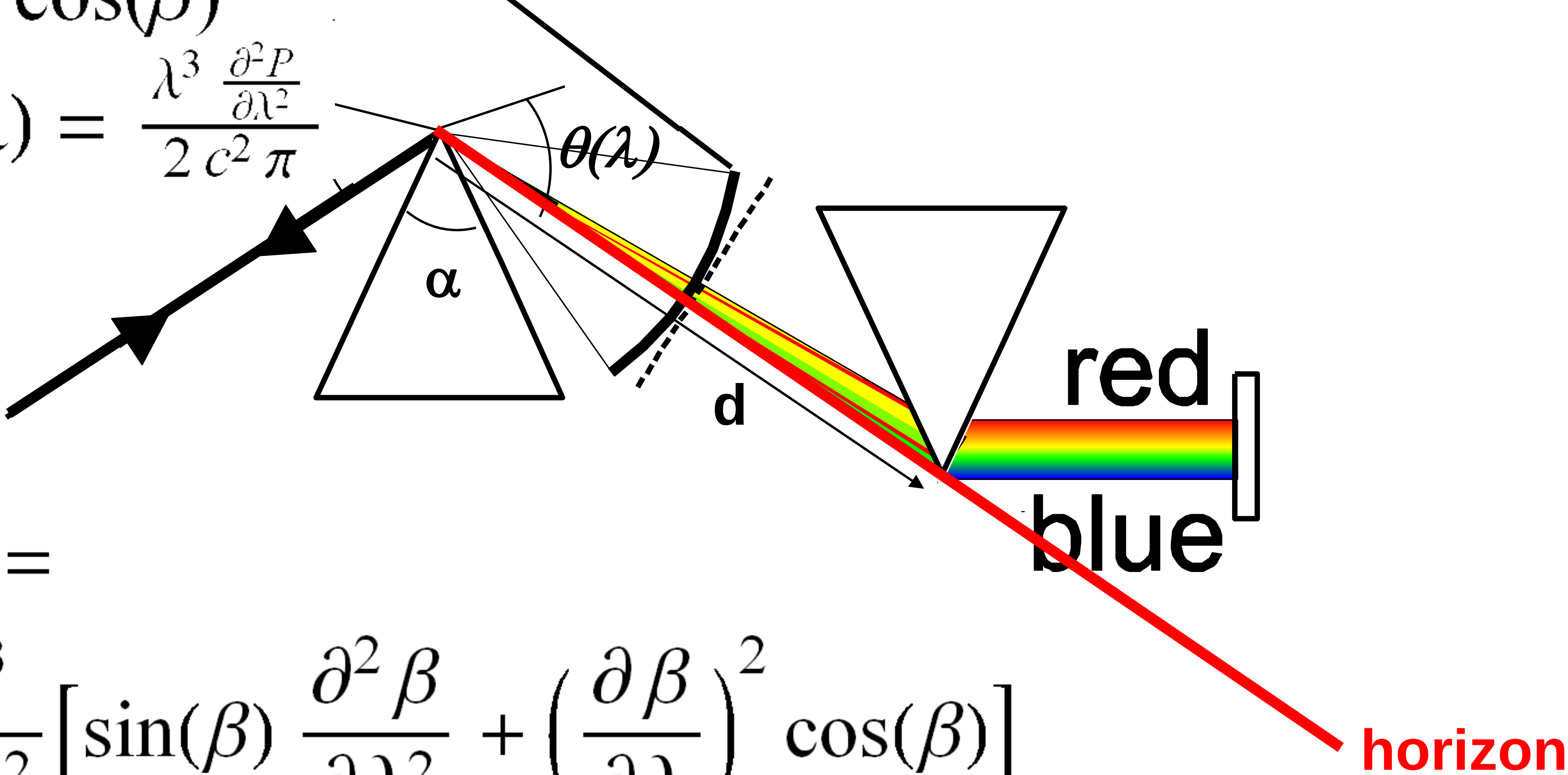
The prism compressor

$$\theta(\lambda)=\arcsin\left(n(\lambda)\,\sin\left[\alpha-\arcsin\left(\frac{\sin(\theta_{\text{in}})}{n(\lambda)}\right)\right]\right)$$

$$\beta(\lambda) := \theta(\lambda)-\theta(\lambda_{\text{hor}})$$

$$P(\lambda)=d\cos(\beta)$$

$$GDD(\lambda)=\frac{\lambda^3\,\frac{\partial^2 P}{\partial\lambda^2}}{2\,c^2\,\pi}$$



$$GDD(\lambda)=$$

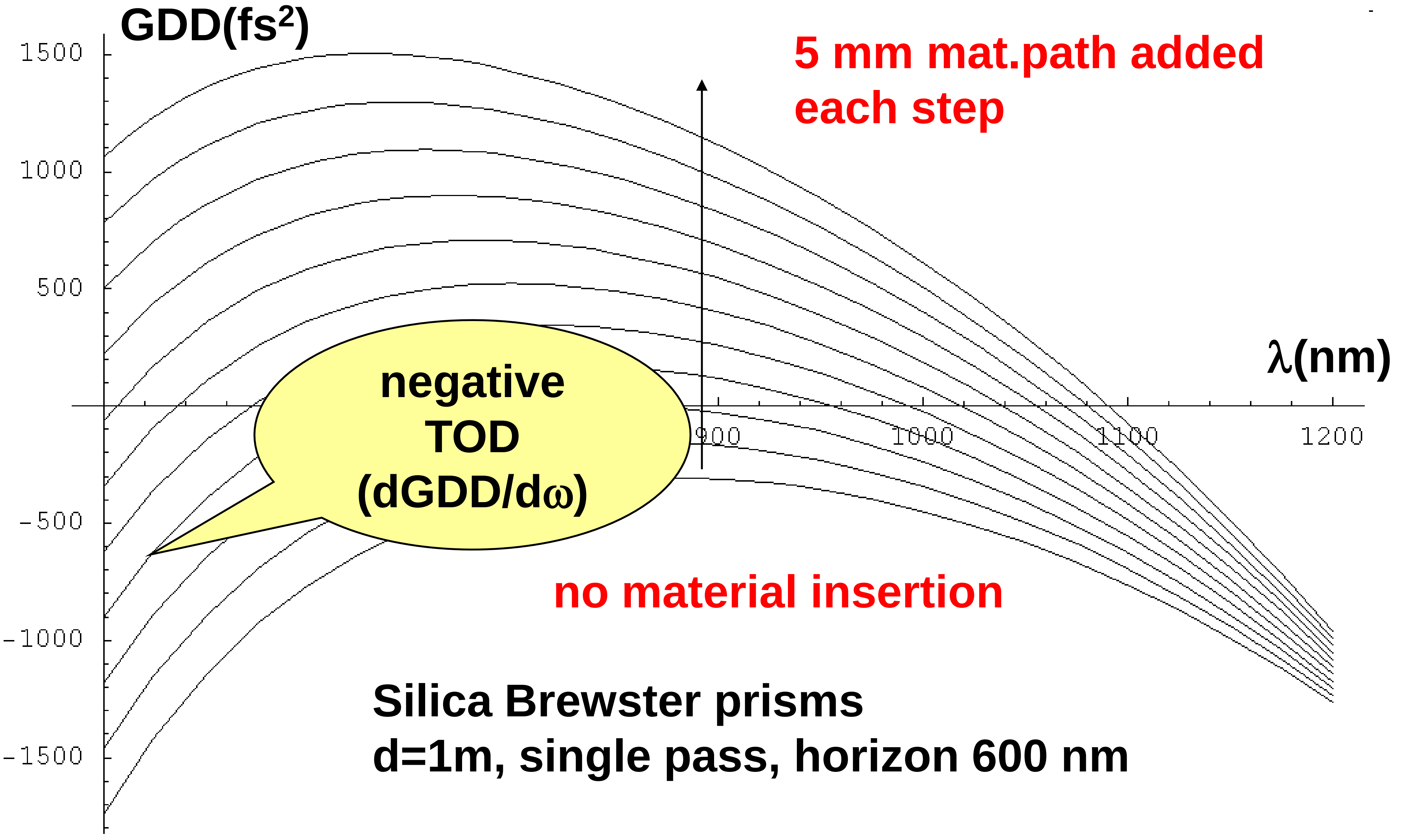
$$-\frac{d\,\lambda^3}{2\,\pi\,c^2}\left[\sin(\beta)\,\frac{\partial^2\beta}{\partial\lambda^2}+\left(\frac{\partial\beta}{\partial\lambda}\right)^2\cos(\beta)\right]$$

The prism compressor

$$\mathrm{GDD}(\lambda) = -\frac{d\,\lambda^3}{2\,\pi\,c^2}\left[\sin(\beta)\,\frac{\partial^2 \beta}{\partial \lambda^2} + \left(\frac{\partial \beta}{\partial \lambda}\right)^2 \cos(\beta)\right]$$

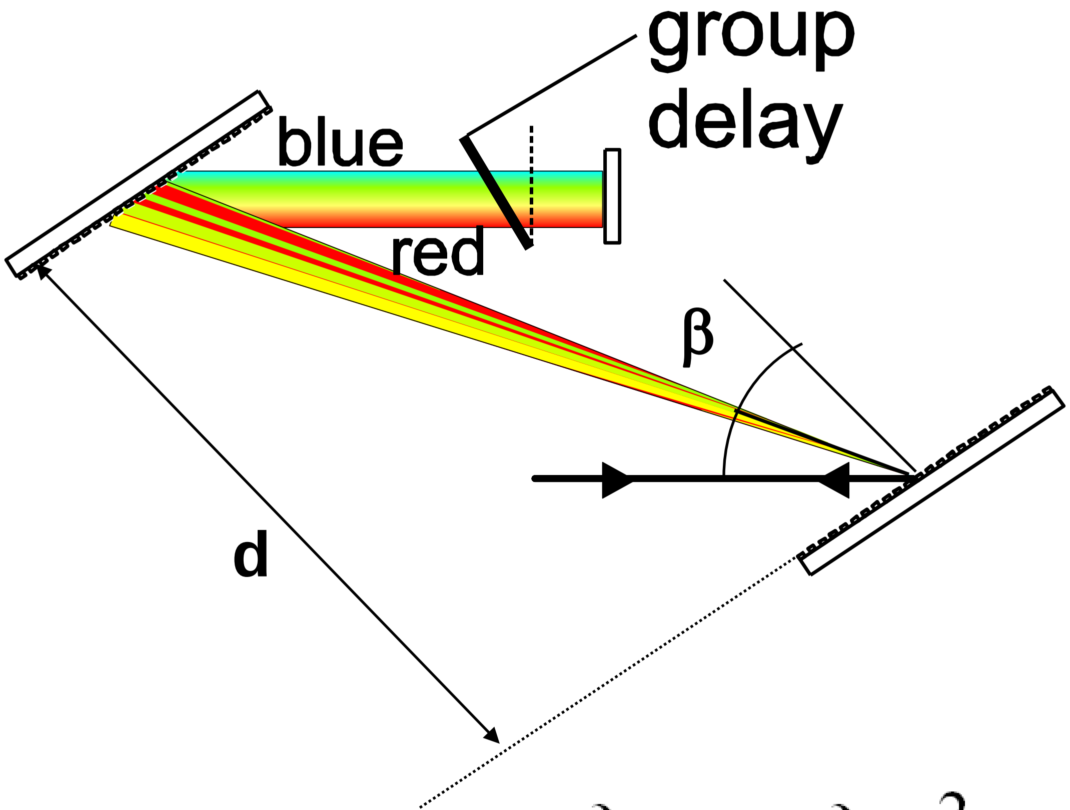
- **angular dispersion is converted into GDD**
- **cosine term dominant for small β**
- **geometric GDD <u>always</u> negative**
- **has to be adjusted for material path through prism 1**

The prism compressor



A 1m prism compressor can compensate 5cm of mat. disp.

The grating compressor

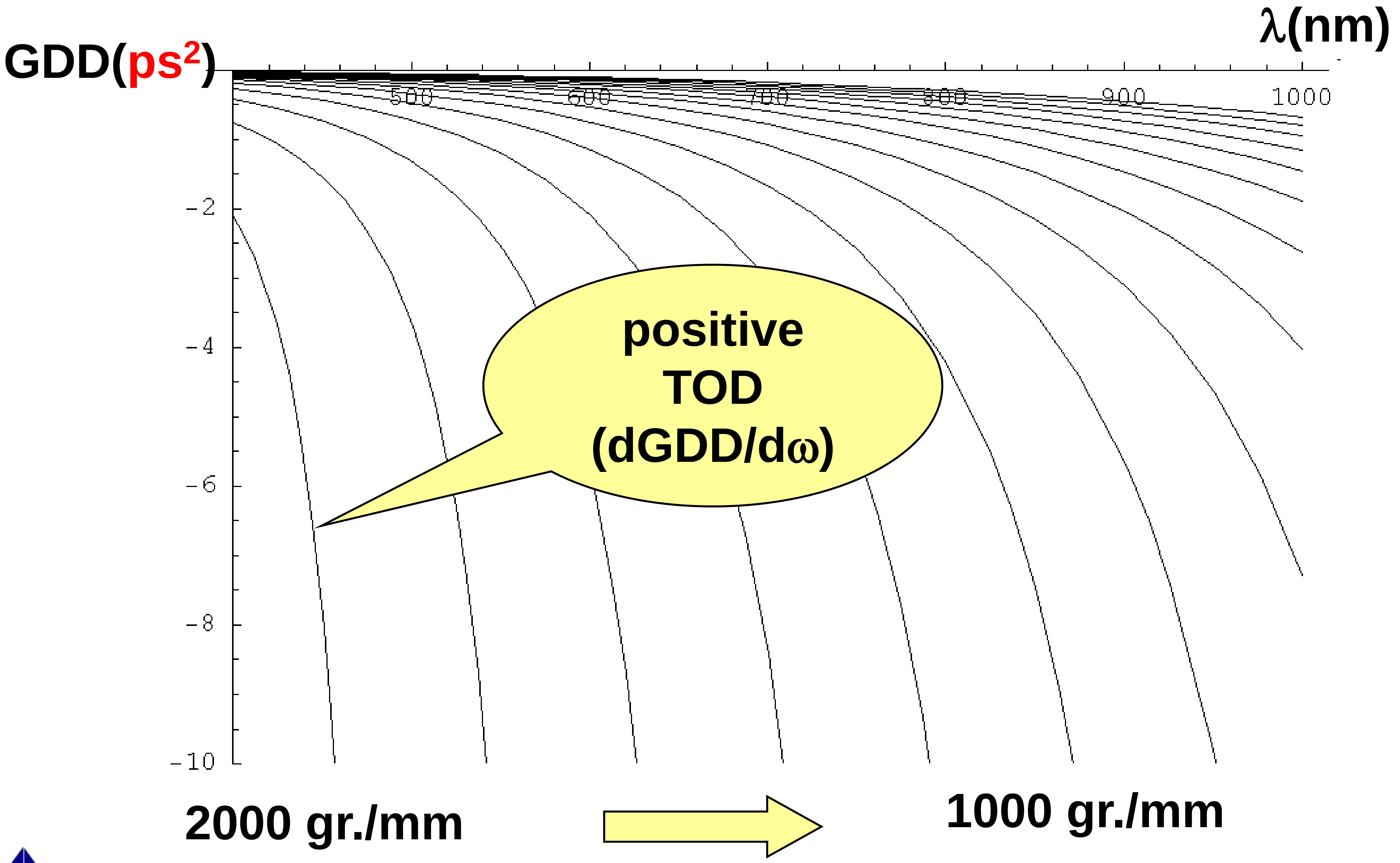


Λ : grating period

$$\mathrm{GDD}(\lambda) = -\frac{\lambda}{2\pi c^2}\left(\frac{\lambda}{\Lambda}\right)^2 d\left[1-\left(\frac{\lambda}{\Lambda}-\sin\beta\right)^2\right]^{-3/2}$$

Ref.: E.B.Treacy, *IEEE JQE* **5**, 454 (1969)

The grating compressor



normal incidence, diffraction into 1st negative order, d=1m

Prism vs. Grating compressor

Prism	Grating
1000 fs^2	ps^2
Near lossless	Loss=15-50%
Negative TOD	Positive TOD

Only negative geometric dispersion

Translates angular dispersion into GDD

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(origin: atomic and vibrational resonances)

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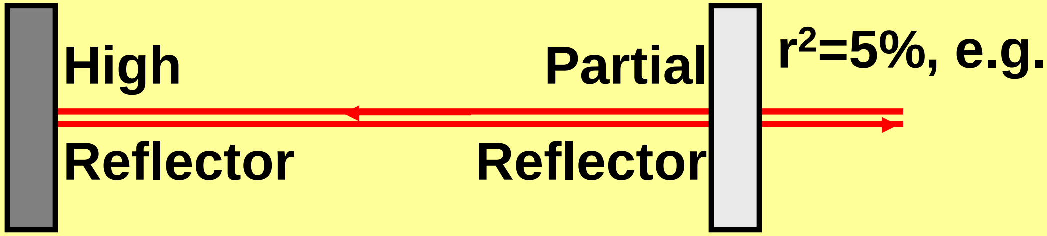
The GTI

Gires-Tournois Interferometer



$$\Phi(\omega) = \arctan\left(\frac{(1-r^2)\sin\psi}{2r-(r^2+1)\cos\psi}\right)$$

$$\psi = -2\omega n L / c$$

$$\mathrm{GD}(\omega) = \frac{\mathrm{d}\Phi}{\mathrm{d}\omega} = \frac{(r^2-1)\,\frac{\mathrm{d}\psi}{\mathrm{d}\omega}}{r^2+1-2\,r\,\cos\psi}$$

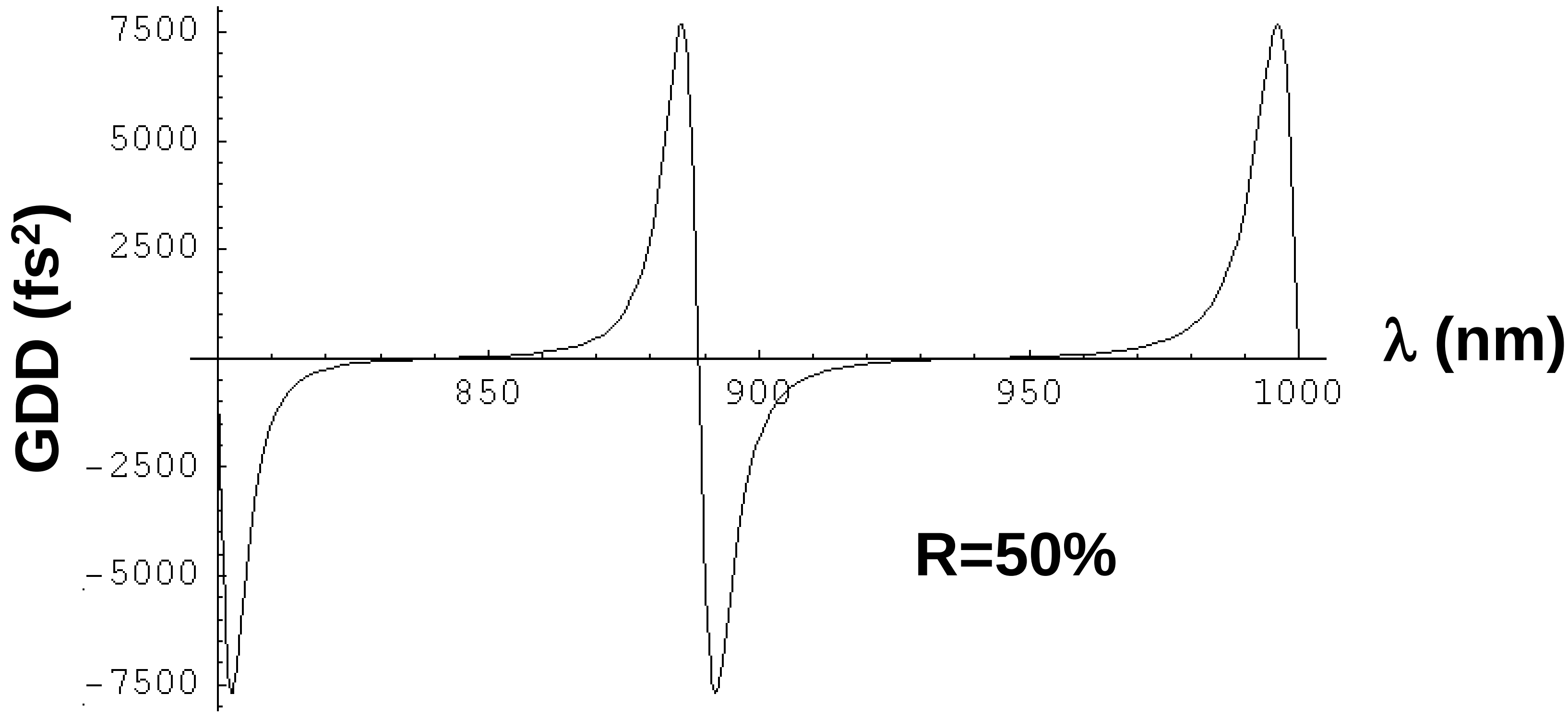
The GTI

$$\mathrm{GD}(\omega) = \frac{\mathrm{d}\Phi}{\mathrm{d}\omega} = \frac{\left(r^2 - 1\right)\frac{\mathrm{d}\psi}{\mathrm{d}\omega}}{r^2 + 1 - 2\,r\,\cos\psi}$$

GTI has constant 100% reflectivity but periodic phase, group delay, and GDD

Period: $\Delta\nu = c/2nL$ (just like a Fabry-Perot)

The GTI



Small values of *r*: GTI has sinusoidal shape
Large values of *r*: Dispersion develops into a third-order pole

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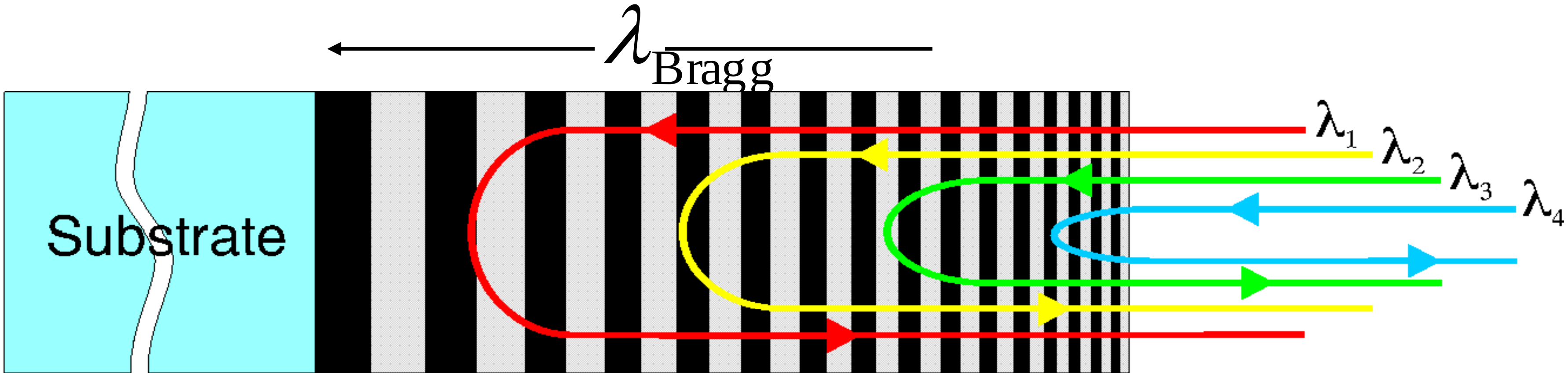
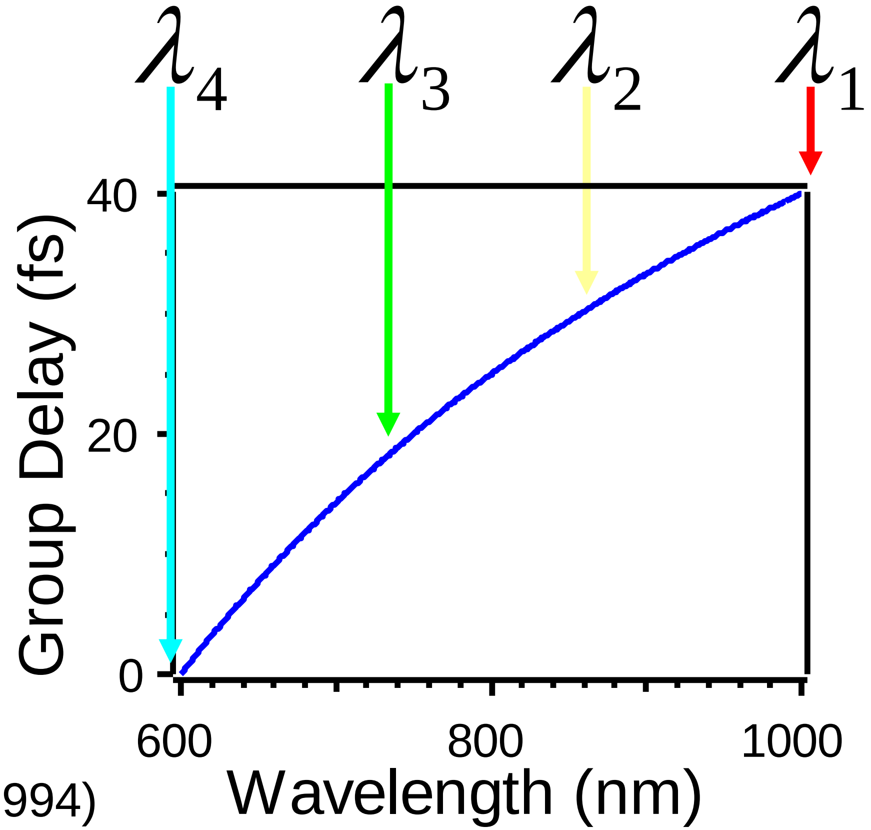
Chirped mirrors



$$\varphi(\omega) = \frac{\omega}{c} n(\omega) L(\omega)$$

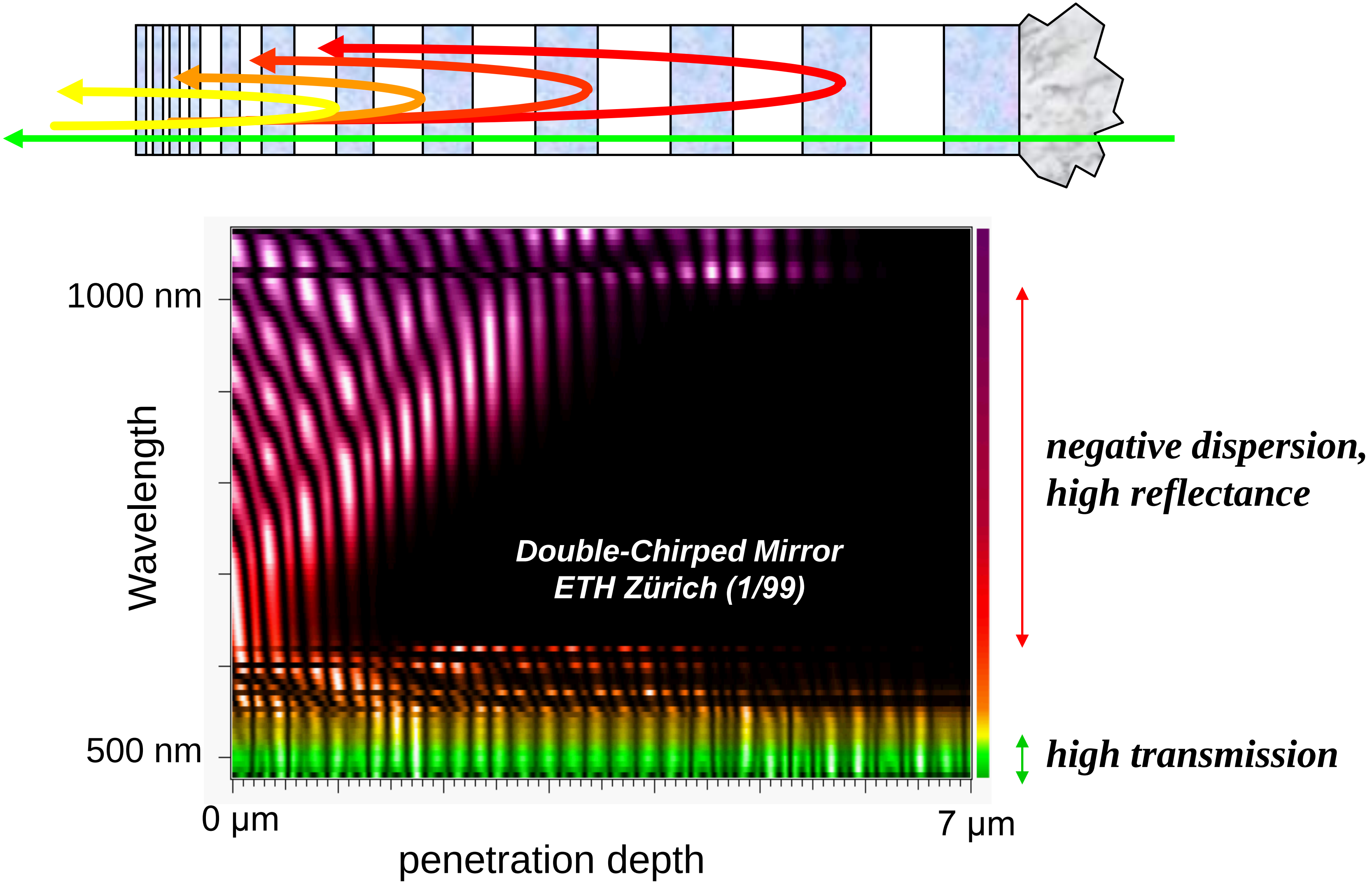


⇒ **arbitrary monotonous GD can be compensated**

⇒ **Compensation of arbitrary material dispersion**

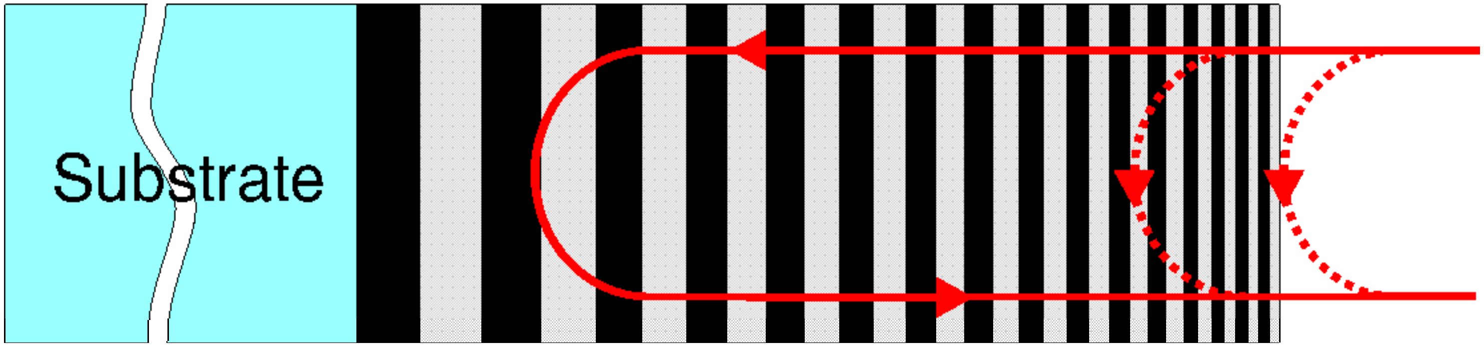
R. Szipöcs et al., *Opt. Lett.* **19**, 201 (1994)

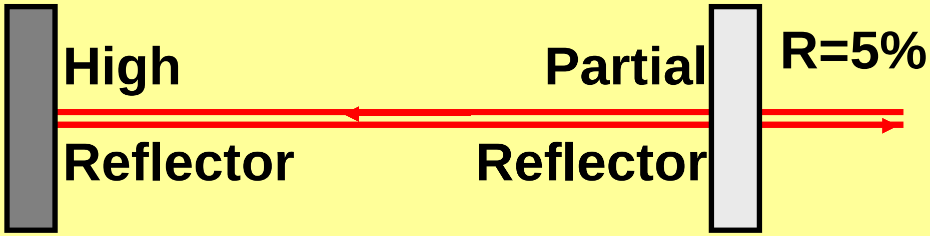
Chirped Mirrors



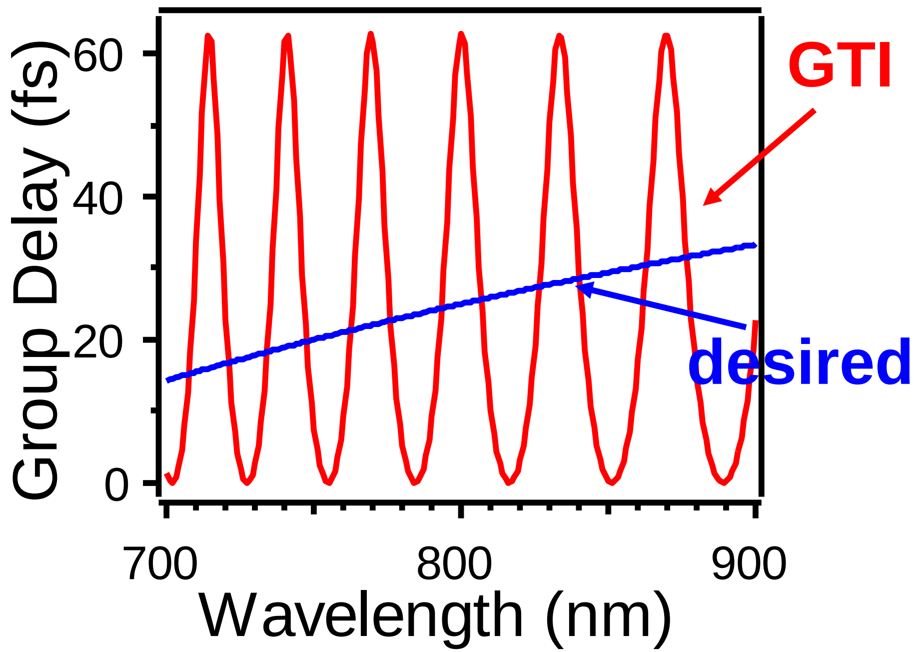
Ref.: G.Steinmeyer, *Science* **286**, 1507 (1999)

Dispersion oscillations





- Front face + highly reflecting mirror form *GTI*
- *Dispersion oscillations*
- Magnitude comparable with desired disp.

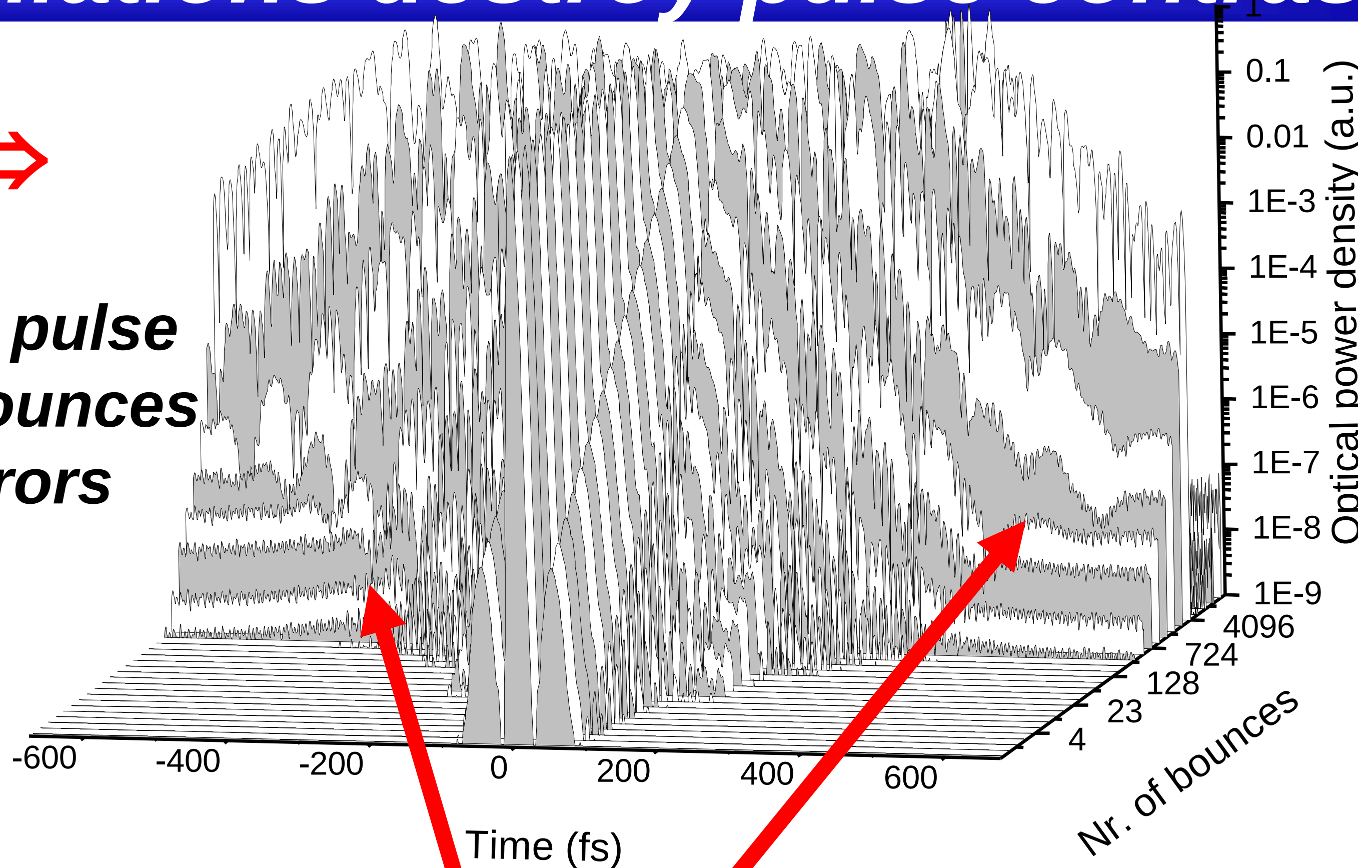


Ref.: Gires et Tournois,
C.R. Acad. Sc. Paris, **258**, 6112 **(1964)**

Disp.oscillations destroy pulse contrast

Simulation ⇒

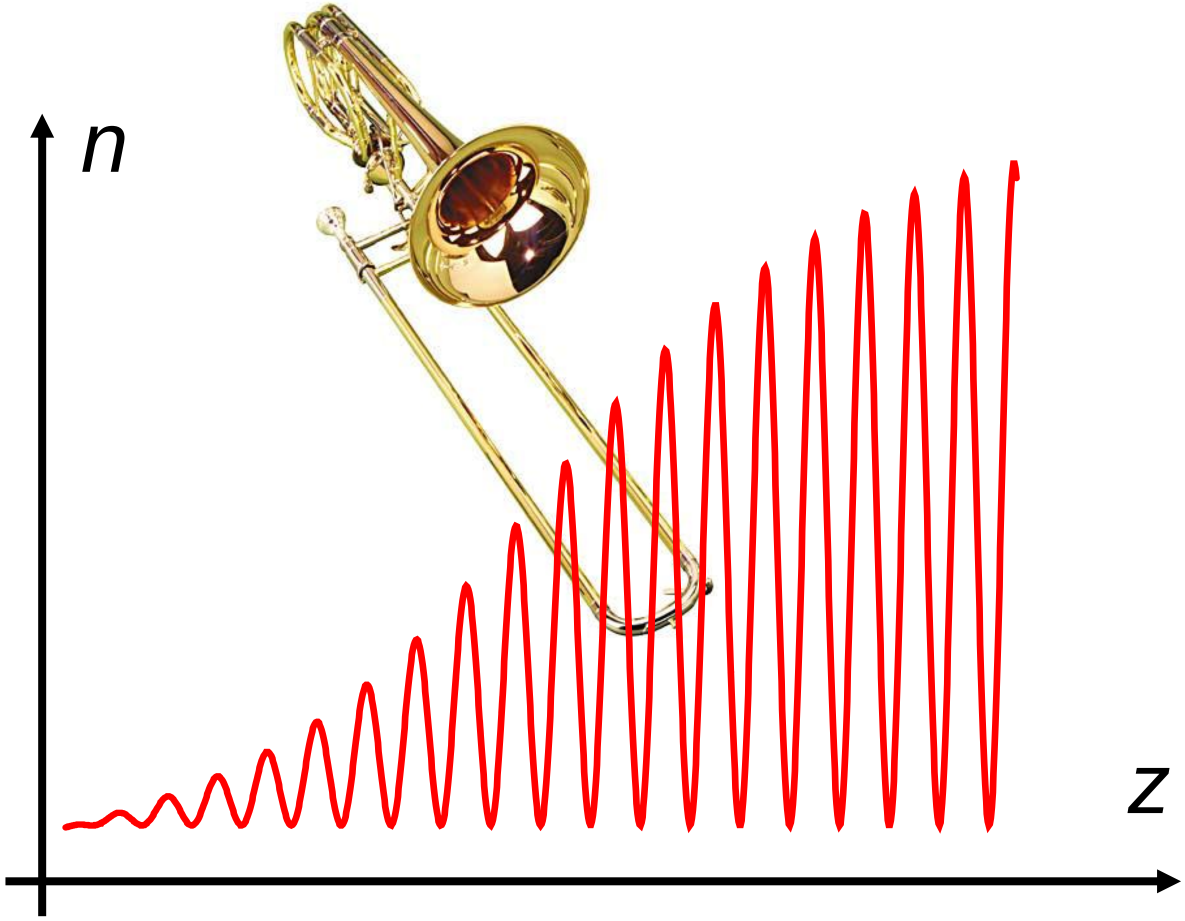
Decay of a 5-fs pulse after several bounces off chirped mirrors



pulse energy diffuses into temporal continuum

Ref.: G. Steinmeyer, *IEEE J. QE.* **39,** 1027 (2003)

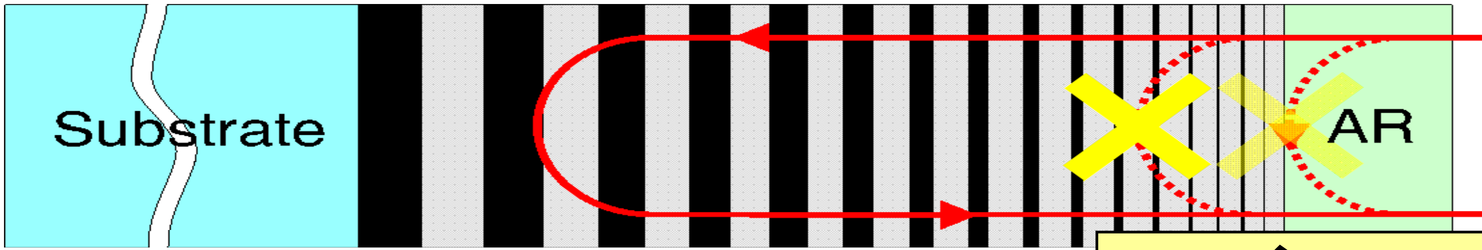
Fight ripple of Bragg gratings: Apodization

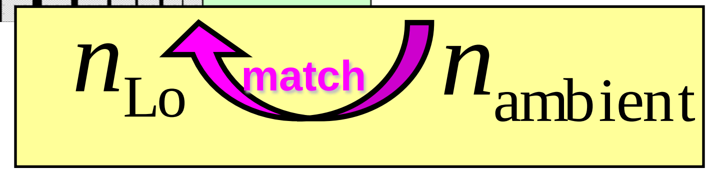


apodize Bragg grating to remove impedance discontinuities !

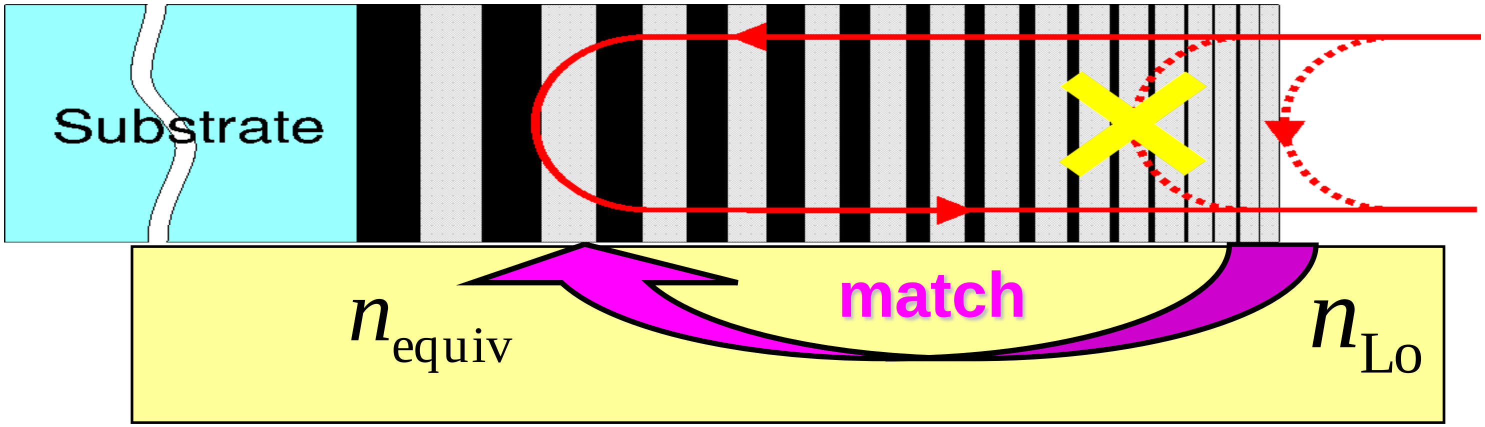
Ref.: J.Albert et al., *Electron. Lett.* **31** (1995)

A remedy: double-chirped mirrors





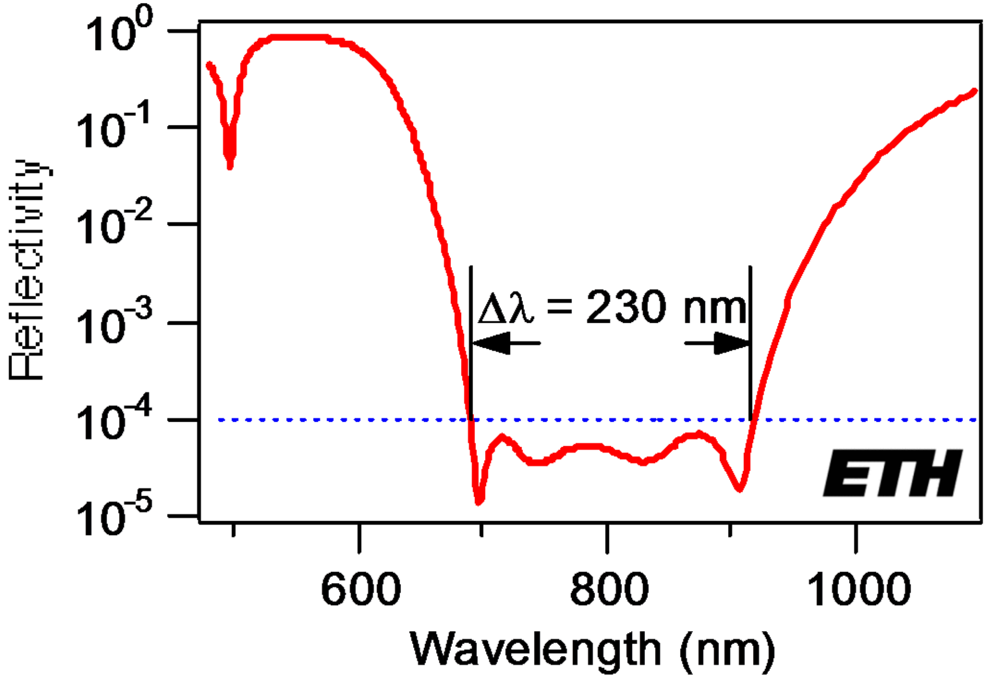
1 • **AR layer for impedance matching to air**

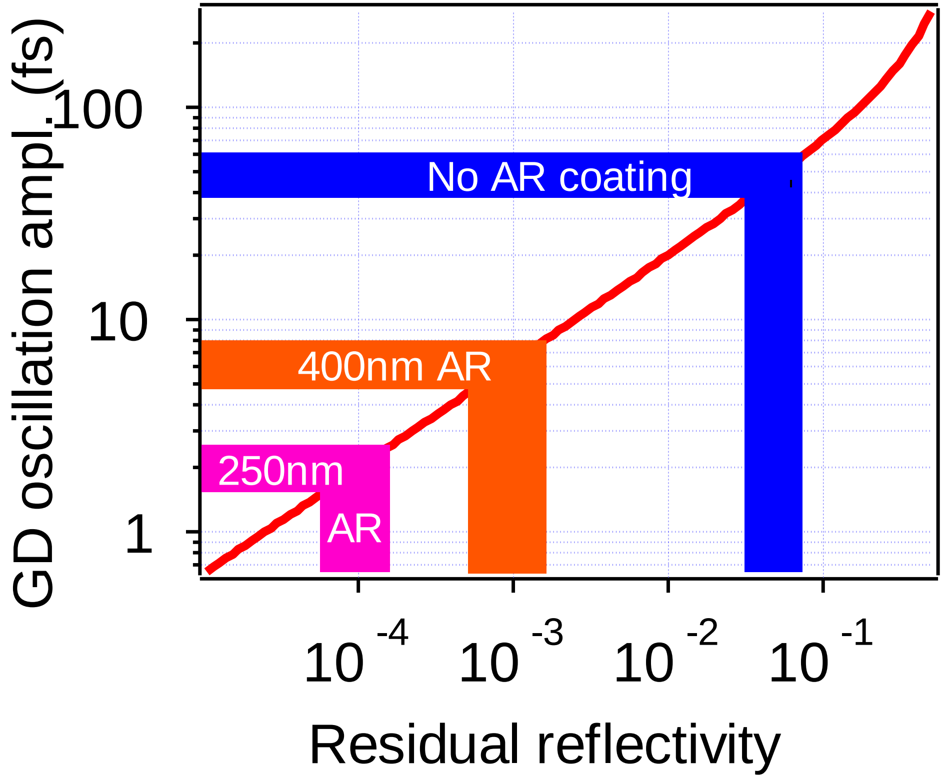


2 • **Chirping the duty cycle b/t high and low index materials for adiabatic matching inside the stack**

Kärtner et al., Opt. Lett. **22**, 831 (1997)
Matuschek et al., IEEE J. Sel. Top. Quantum Electron. **4**, 197 (1998)

Limits of double chirped mirrors





It‘s simply impossible to design an AR coating with *arbitrarily small reflectivity* and *arbitrarily large bandwidth* !

J.A. Dobrowolski et al., *Appl. Opt.* **35**, 644 (1996)

Summary dispersion

Dispersion control is of utmost importance for obtaining the shortest possible pulse for a given spectrum

Material dispersion can often only be compensated by „engineered“ dispersion such as
- resulting from angularly dispersive assemblies
- interferometers
- chirped mirrors

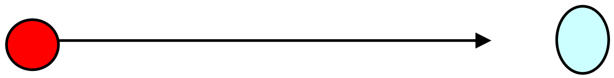
Dispersion control over a wide bandwidth becomes exponentially more challenging as higher orders start to play an increasing role

Some words about laser gain

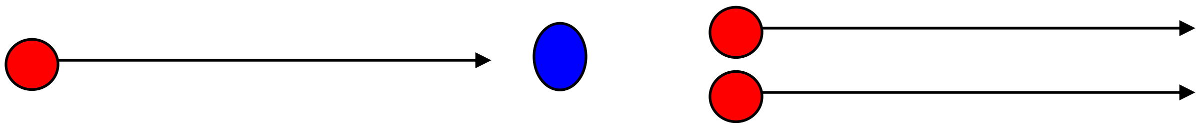
- **Saturation fluence and cross section (most important parameter to model gain)**
- **Frantz-Nodvik equation**
- **Saturation fluence in absorbers**

Elementary light matter interaction

Absorption



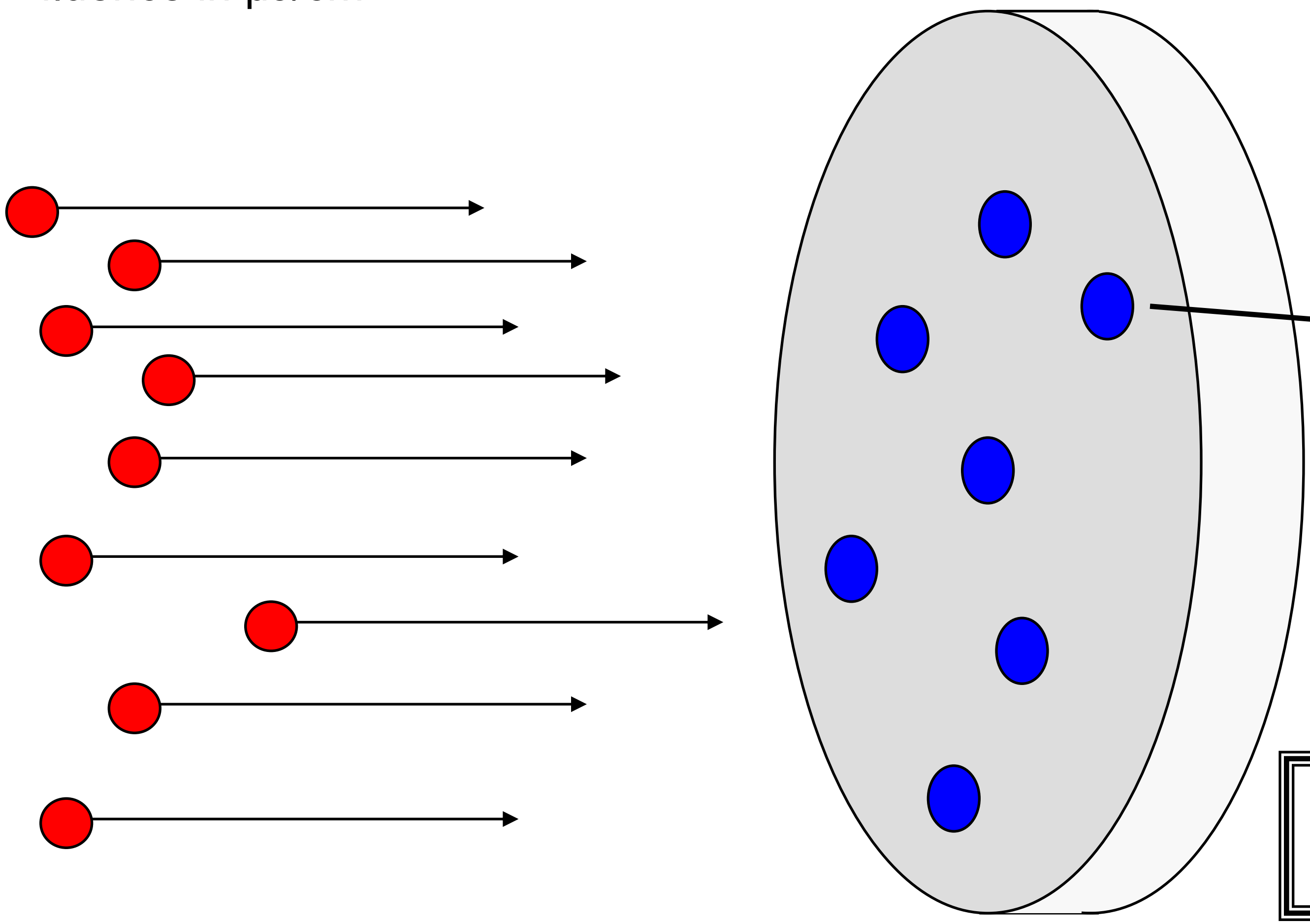
stimulated Emission



Considering Abs. and stimulated emission in the following

saturation fluence - microscopic picture

photons,
fluence in $\mu J/cm^2$



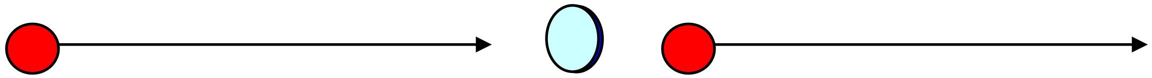
atomic absorbers,
cross section =
equivalent area

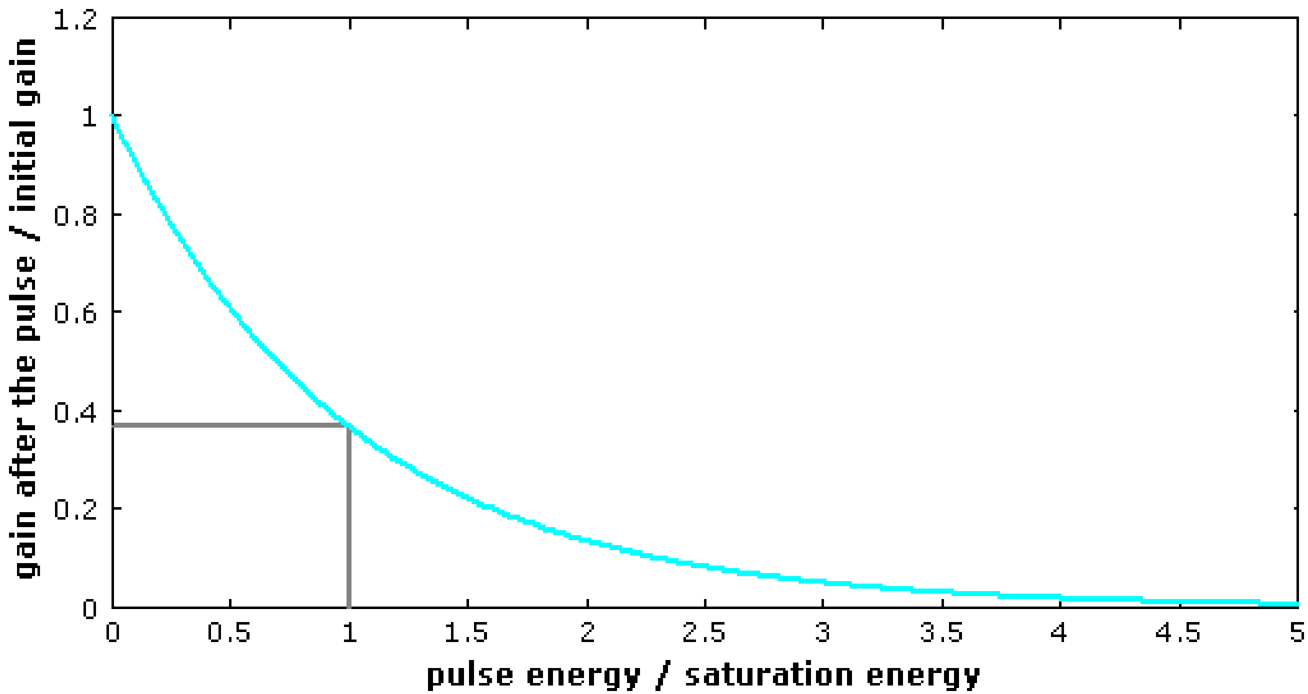
$$\sigma = E_{\text{photon}} / F_{\text{sat}}$$

Saturation fluence if on the average, one photon impinges on every atom

Macroscopic picture for absorber

stimulated Emission





Dynamical gain saturation

Frantz Nodvik equation

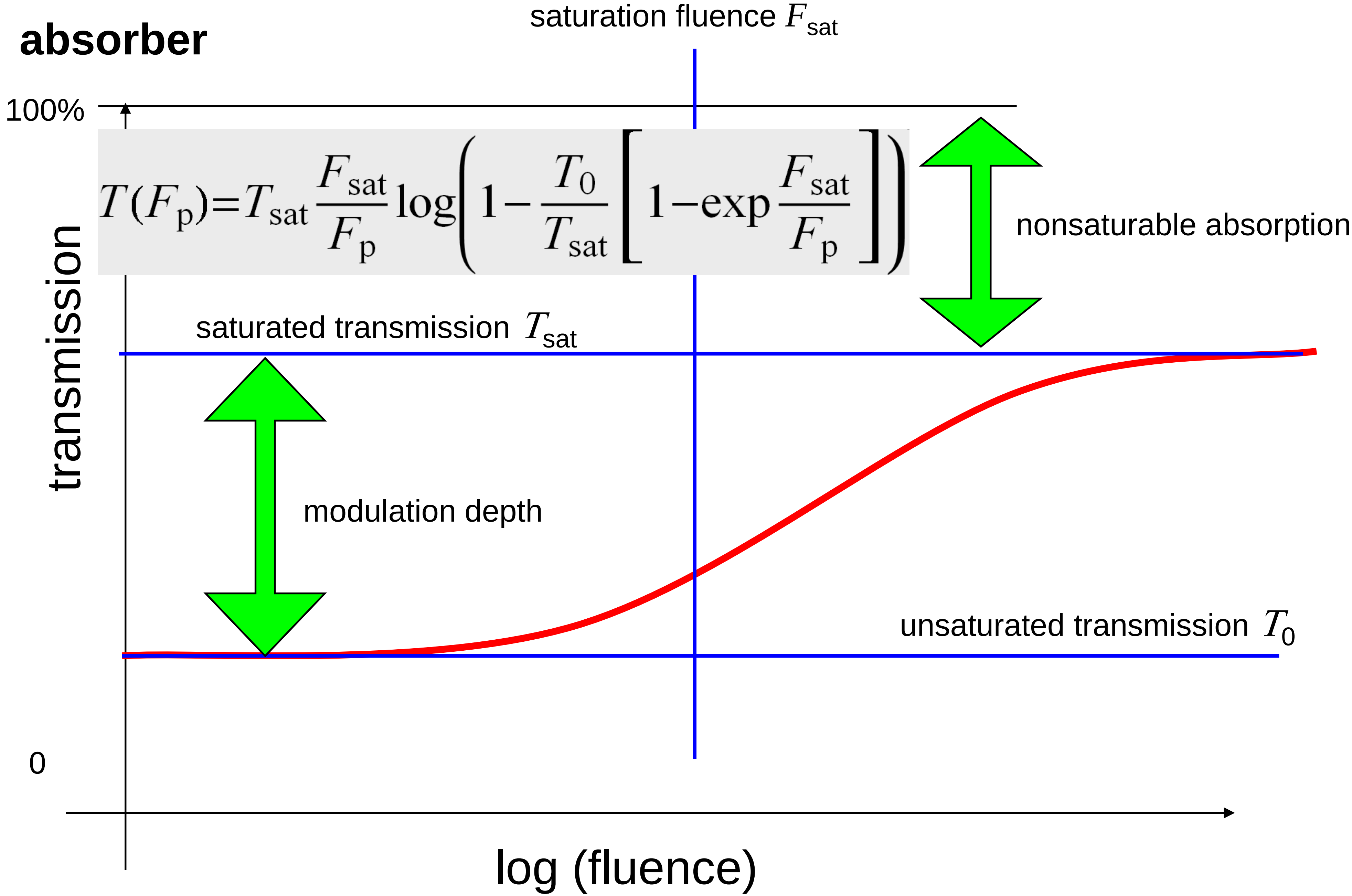
small-signal gain: $$g_0 = \exp\left(\frac{F_{pump}}{F_{sat}}\right)$$

$$F_{out} = F_{sat} \ln\left[1 + \left(\exp\frac{F_{in}}{F_{sat}} - 1\right)\exp g_0\right]$$

L. M. Frantz and J. S. Nodvik, *J. Appl. Phys.*, **34**, pp. 2346-2349, 1963.

macroscopic picture for absorber

absorber

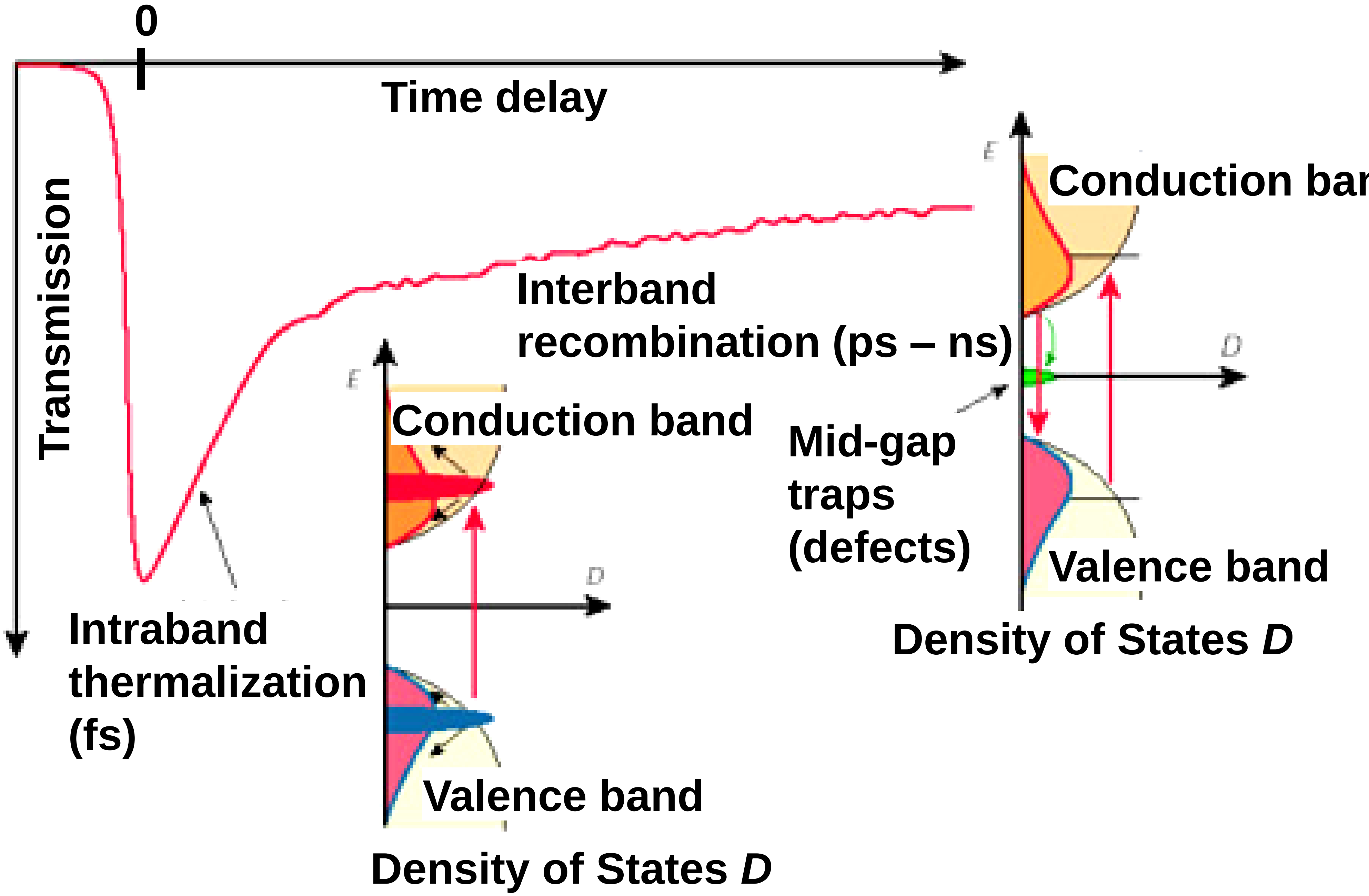


$$T(F_p) = T_{sat} \frac{F_{sat}}{F_p} \log\left(1 - \frac{T_0}{T_{sat}} \left[1 - \exp \frac{F_{sat}}{F_p} \right] \right)$$

Nonlinear optical effects

- All known nonlinear optical effects (SHG etc.) also play a role in ultrafast optics
- Two classes of special importance
  - nonlinear absorber (clean pulses, provide higher transmission for high peak powers)
  - instanteneous effects (fs response time)
- Both effects impossible at the same time!
- But: we have instantaneous phase nonlinearities
- And: we can translate phase nonlinearities into amplitude nonlinearities

Saturable absorption (in a SESAM)



Real saturable absorption

Relies on band-filling effects

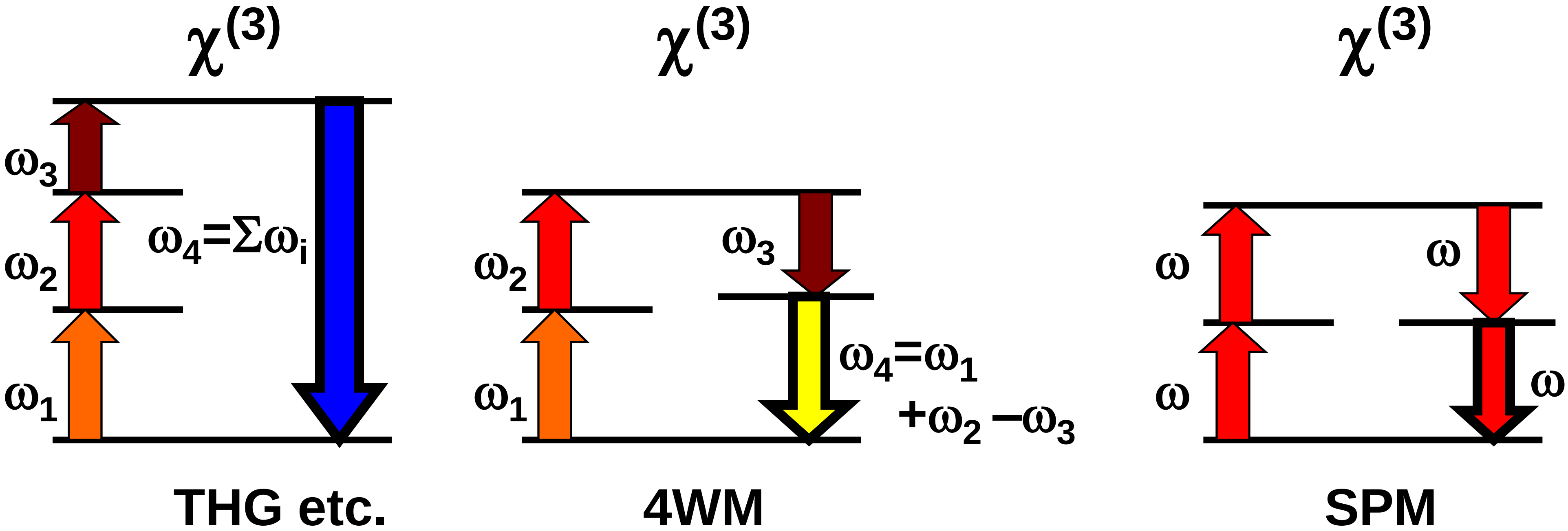
Relaxation not instantaneous

Acceleration possible (but side effects may occur)

Are there ways to build „artificial“ absorbers that are arbitrarily fast?

Solution: exploit phase nonlinearity
(reactive nonlinearities)

Self-phase modulation (I)



SPM is the totally degenerate case of 4WM

Three photons/waves at freq. ω combined convert into one new photon/wave at the same freq.

This photon/wave is phase-shifted.

$$E_{out}=\chi^{(3)} E^2 E^* = [\chi^{(3)} I] E$$

Phase shift prop. to input intensity.

Self-phase modulation (II)

$$I(t) = I_0 \exp\left(-\frac{t^2}{\tau^2}\right)$$

$$n(I) = n_0 + n_2 \cdot I$$

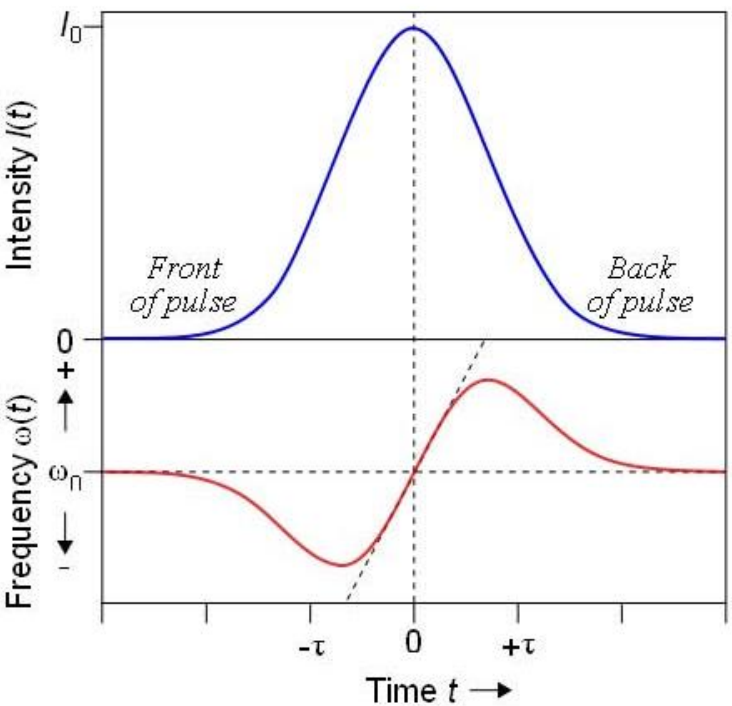
nonlinear phase prop. to intensity:

$$\phi(t) = \omega_0 t - \frac{2\pi}{\lambda_0} \cdot n(I) L$$

Carrier freq. proportional to derivative:

$$\omega(t) = \frac{d\phi(t)}{dt} = \omega_0 - \frac{2\pi L}{\lambda_0} \frac{dn(I)}{dt},$$

$$\omega(t) = \omega_0 + \frac{4\pi L n_2 I_0}{\lambda_0 \tau^2} \boxed{t \cdot \exp\left(\frac{-t^2}{\tau^2}\right)}.$$



Linear chirp in the center of the pulse

Types of self-phase modulation

Types of self-phase modulation:

1. **electronic polarization type**
   (non-resonant, bound electrons, quasi instantaneous)
   positive in dielectrica, values on the order of a few 10^{-20} m^2/W

   The Stolen „constant“: $n_2=3.2\ 10^{-20}$ m^2/W for silica

   Ref.: Stolen & Lin, Phys. Rev. A **17**, 1448 (1978).

   response time ≈ inverse band gap (i.e., ≈1 fs for dielectrica)

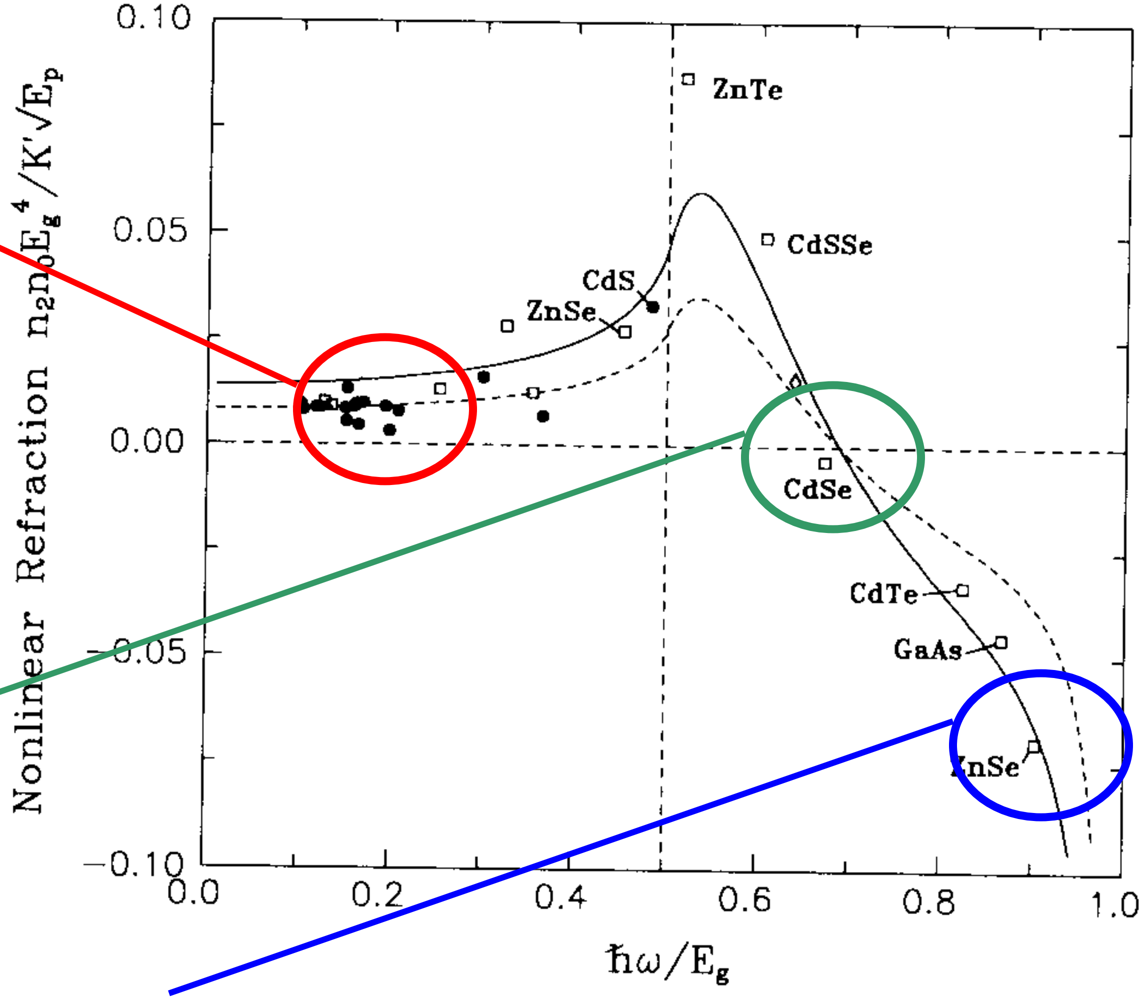
   same effect negative in semiconductors, $n_2=-10^{-16}... -10^{-18}$ m^2/W
   (careful: TPA at the same time, free electrons be generated)

Electronic Kerr effect

Silica, 0.2x bandgap

roll over at 0.7x bandgap

negative above 0.7x bandgap



Ref.: Sheik-Bahae et al., IEEE JQE 27, 1296 (1991)

The electronic Kerr effect

AlGaAs: -4×10^{-16} m²/W

Material	Wavelength (μm)	Bandgap (eV)	Refr. Index	n_2 (Exp.) $\times 10^{-13}$ (esu)
Ge	10.6	0.87†	4.00	2700
GaAs	1.06	1.35	3.47	−2700
CdTe	1.06	1.44	2.84	−2000
CdSe	1.06	1.74	2.56	−90
$CdS_{0.5}Se_{0.5}$	1.06	1.93	2.45	1000
ZnTe	1.06	2.26	2.79	830
CdS	0.53	2.42	2.34	−3400
ZnSe	1.06	2.58	2.48	170
ZnSe	0.53	2.58	2.70	−400
SBN	1.06	3.3	2.4	30
ZnS	1.06	3.54	2.40	48
KTP	1.06	3.54	1.78	13
BaF_2	1.06	9.21	1.47	0.67
BaF_2	0.53	9.21	1.47	0.85
AlGaAs	0.850	1.57	3.30	−2000
AlGaAs	0.840	1.57	3.30	−4000
AlGaAs	0.830	1.57	3.30	−7000
AlGaAs	0.825	1.57	3.30	−10000
AlGaAs	0.820	1.57	3.30	−14000
AlGaAs	0.815	1.57	3.30	−20000
AlGaAs	0.810	1.57	3.30	−26000
CdS	1.06	2.42	2.34	280
AgCl	1.06	3.10	2.07	23
ZnO	1.06	3.20	1.96	23
NaBr	1.06	5.63	1.64	3.3
$CaCo_3$	1.06	5.88	1.60	1.1
KBr	1.06	6.04	1.56	2.9
KCl	1.06	6.89	1.49	2.0
KDP	1.06	6.95	1.60	0.7
KH_2PO_4	1.06	7.12	1.50	0.8
NaC[illegible]	[illegible]	[illegible]	[illegible]3	1.6
Al_2C[illegible]	[illegible]	[illegible]	[illegible]5	1.2
KF	[illegible]	[illegible]	[illegible]6	0.75
MgO	1.06	7.77	1.70	1.6
SiO_2	1.06	7.80	1.40	1.1

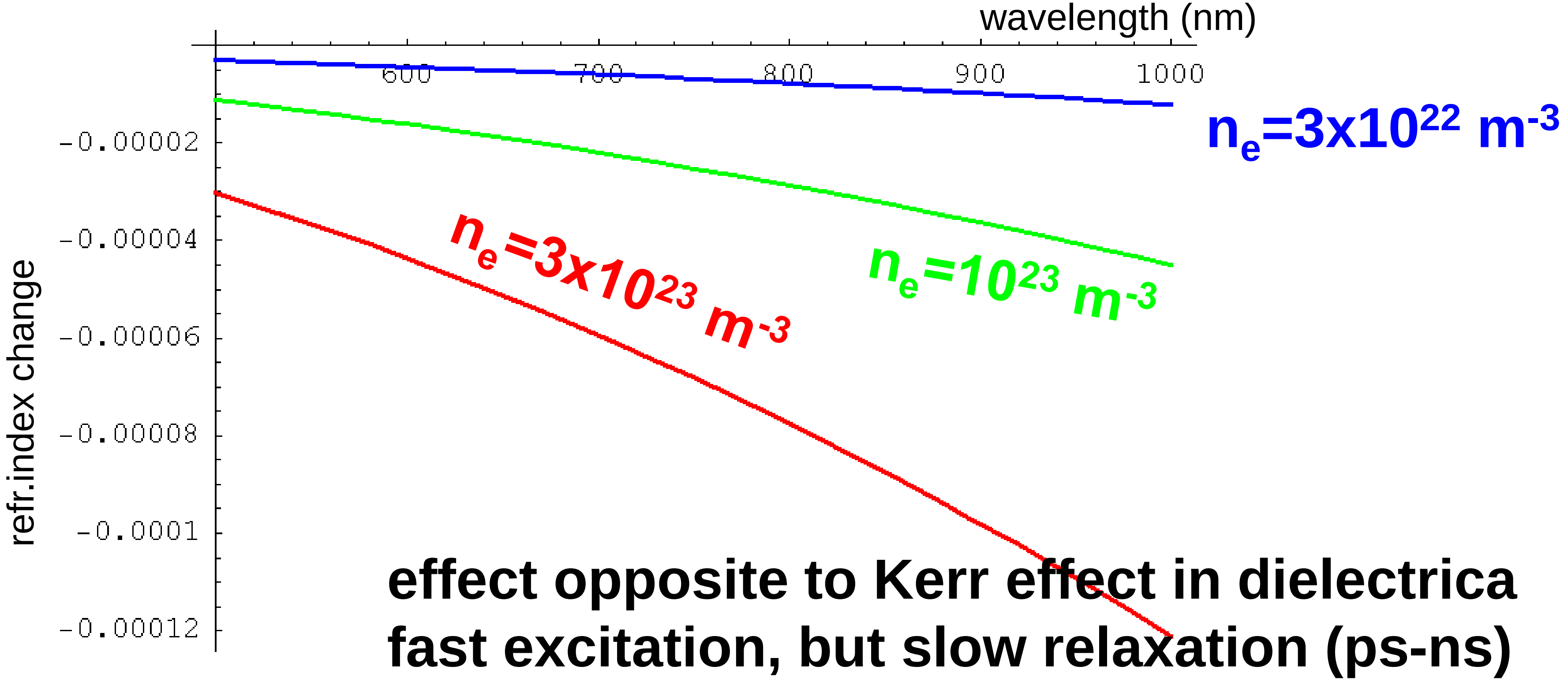
Silica: $+3.2\times10^{-20}$ m²/W

Ref.: Sheik-Bahae et al., IEEE JQE 27, 1296 (1991)

Drude-type contributions

$$n = \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2} \qquad \omega_p := e\sqrt{\frac{n_e}{\epsilon_0\, m_e}}$$

Refractive index decreases with increasing electron density



effect opposite to Kerr effect in dielectrica
fast excitation, but slow relaxation (ps-ns)

Other types of self-phase modulation

1. **electronic polarization type** 10^{-20} m^2/W, 1 fs

2. **resonant, molecular orientation, e.g., CS_2** 10^{-18} m^2/W, 1 ps

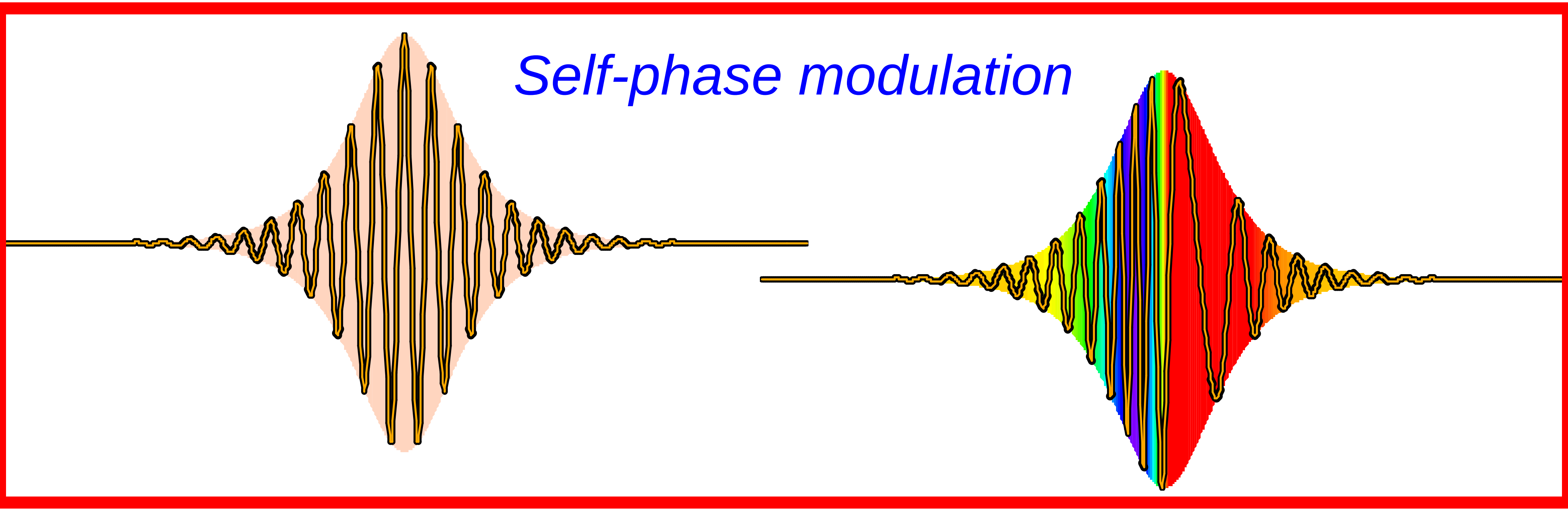
Interesting, yet useless for ultrafast...

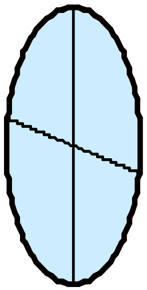
3. **Resonant atomic absorption, e.g. Na** 10^{-14} m^2/W, 100 ps

pulse compression - active spectral broadening

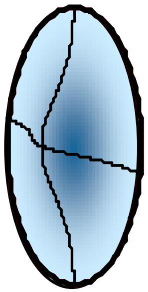
Refractive index depends on intensity:

$$n(I)=n+n_2I$$





Self focussing due to transverse beam profile



Spectral broadening via SPM

Index increases with intensity

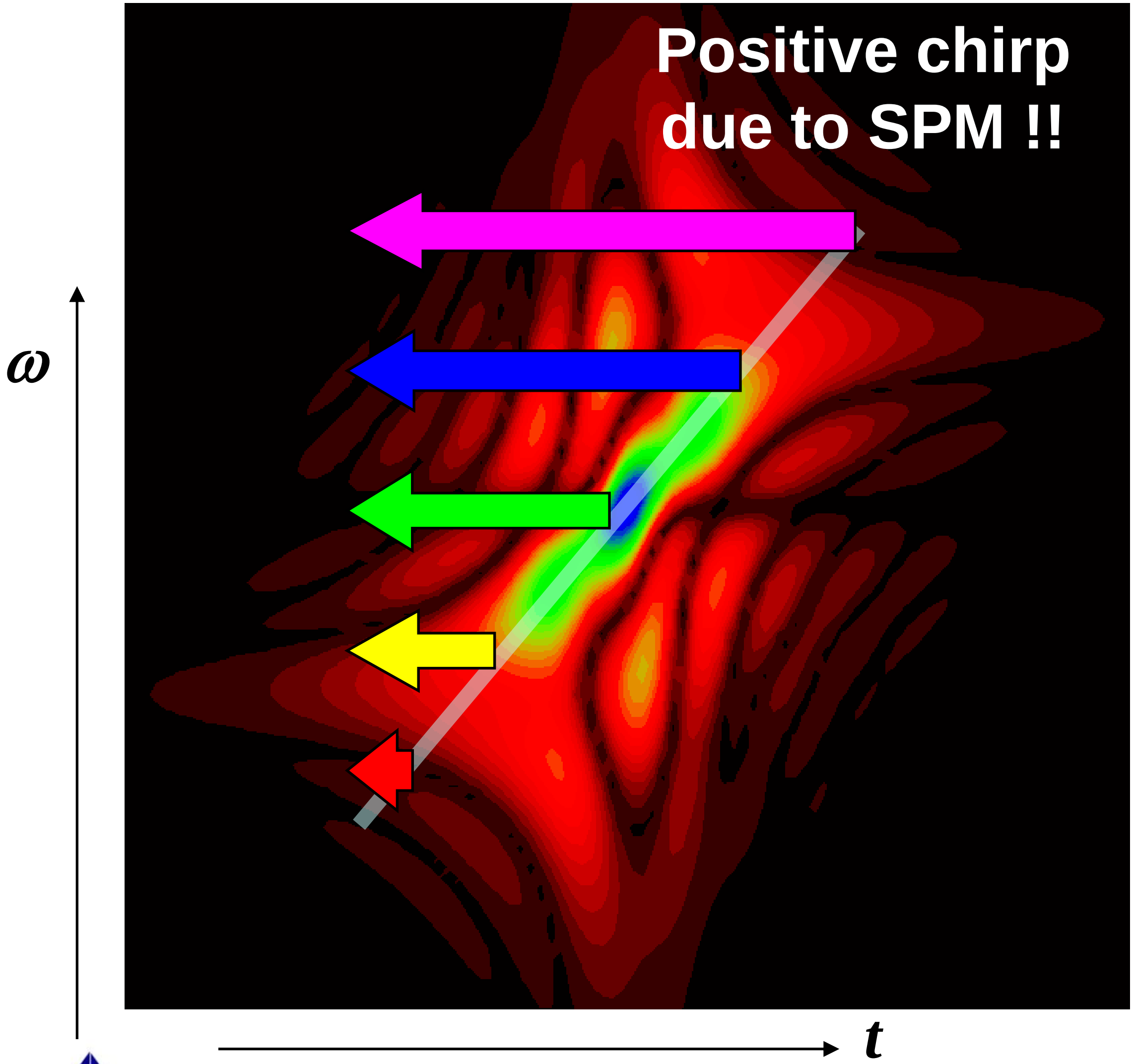
Pulse center is retarded

Compression of cycles in the trailing part $\Rightarrow$ blue shift
Expansion in the leading part $\Rightarrow$ red shift

Newly generated spectral content !

Negative dispersion required for obtaining the shortest pulse

SPM in the spectrogram picture



Negative dispersion required for maximum temporal localization of pulse energy

Can only be provided by material disp. at λ>1.3μm

Localization will never be perfect, pedestal formation

(active) Pulse compression

Combining SPM with dispersion compensation yields a shorter pulse

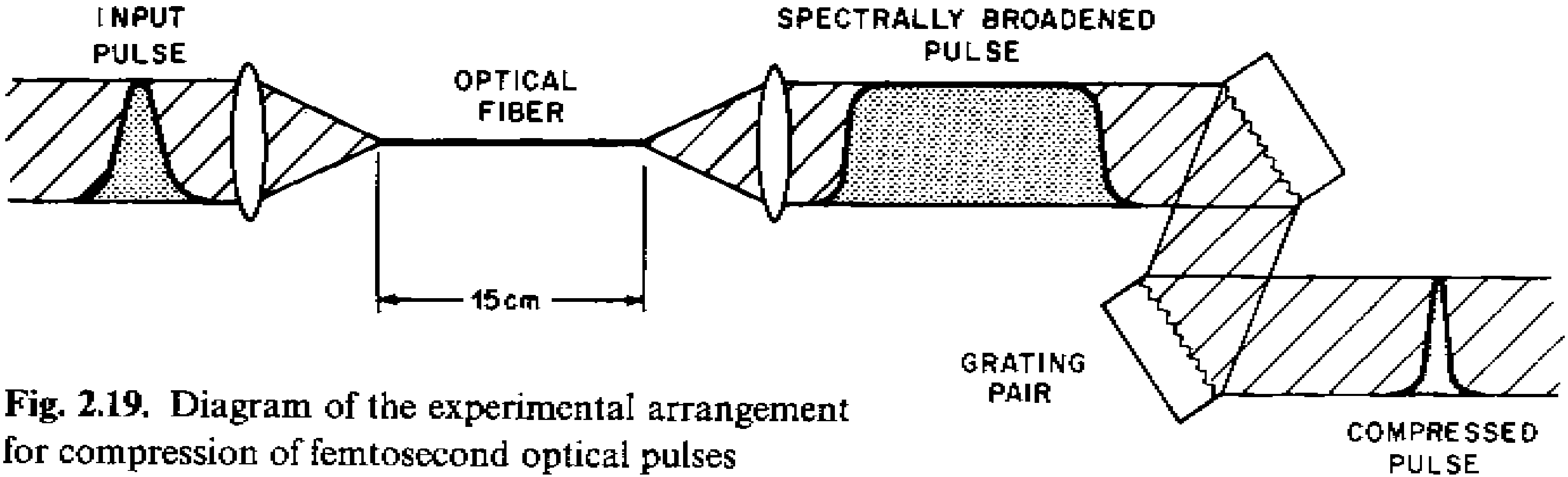
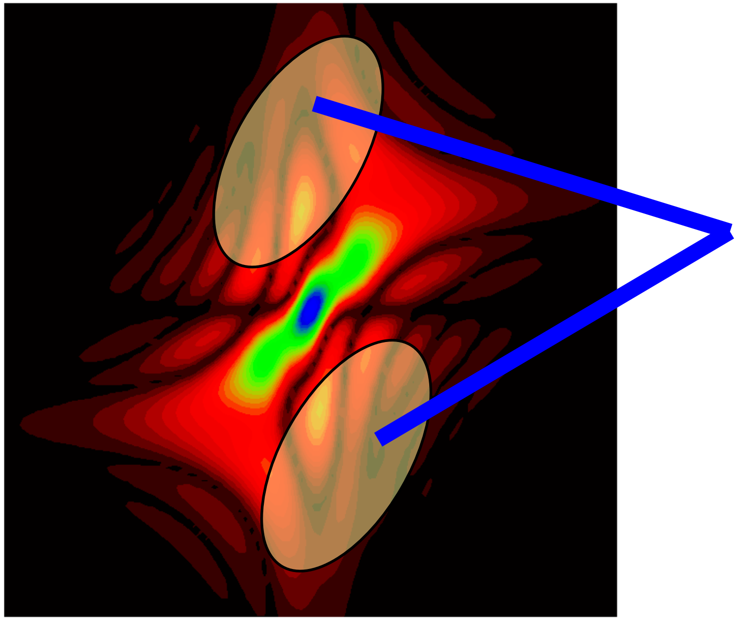


Fig. 2.19. Diagram of the experimental arrangement for compression of femtosecond optical pulses

Ref.: C.V.Shank et al., Appl.Phys.Lett. **40**, 761 (1982)

SPM + GVD provided in discrete steps



energy in pedestals will effectively be lost...

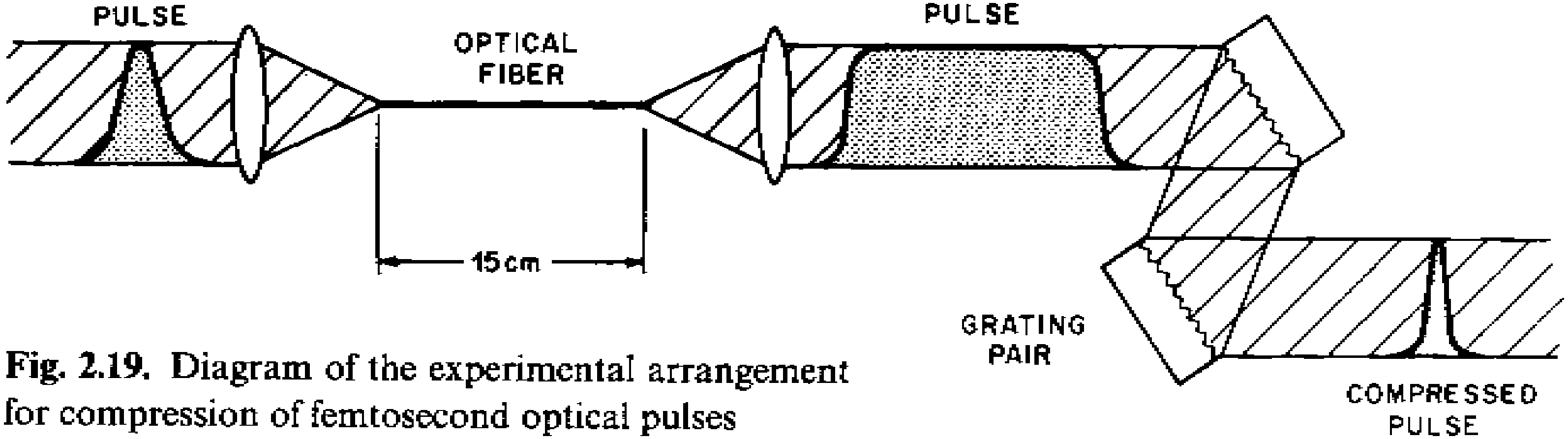


Fig. 2.19. Diagram of the experimental arrangement for compression of femtosecond optical pulses

Pedestal-free balance b/t GVD and SPM

Solitons

$$\frac{\partial A}{\partial z} = -\frac{i}{2}\beta_2 \frac{\partial^2 A}{\partial t^2} + i\gamma |A|^2 A$$

Nonlinear Schrödinger Equation

$\gamma := n_2\, \omega_0 / c\, A_{eff}$

Ref.: Zakharov & Shabat,
Sov. Phys. JETP **34**, 62 (1972)

Some solutions are of the form

$$A(t) = A_0 \,\mathrm{sech}(t/t_0)$$

Balance of dispersive phase effects and self-phase modulation

Solitons

``I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large **solitary** elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the **Wave of Translation**".

John Scott Russell, "Report on Waves" (Report of the 14th meeting of the British Association for the Advancement of Science, York, September 1844 (London 1845), pp 311-390, Plates XLVII-LVII).

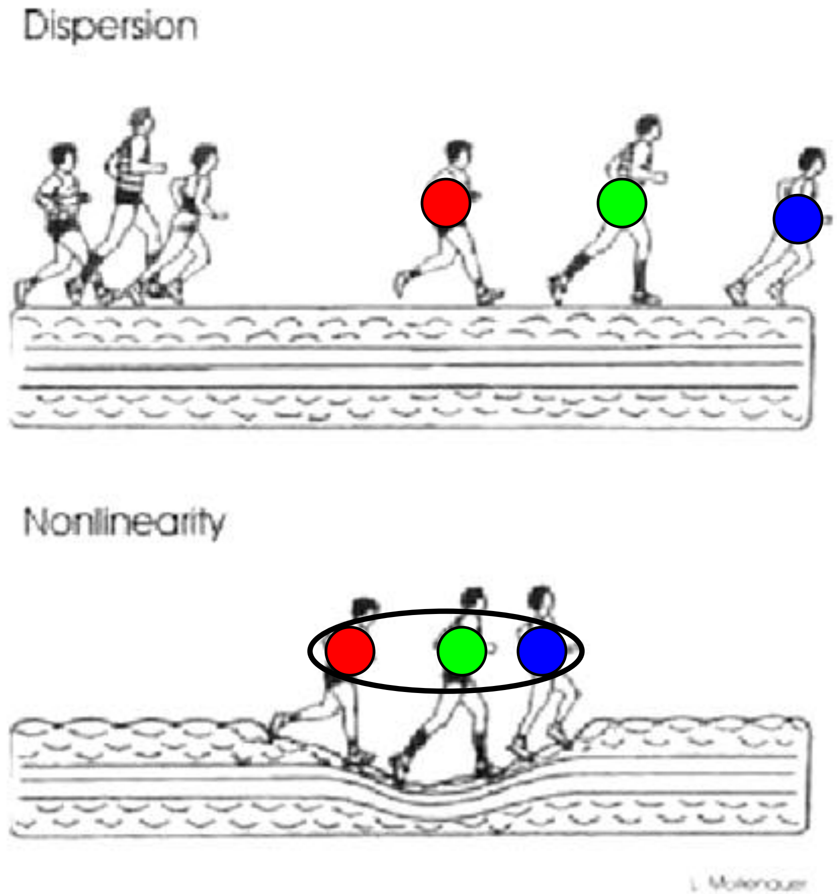
A intuitive picture for solitons

Group delay effects due to dispersion and nonlinearity cancel each other.

Stable shape despite dispersive medium

Negative dispersion +

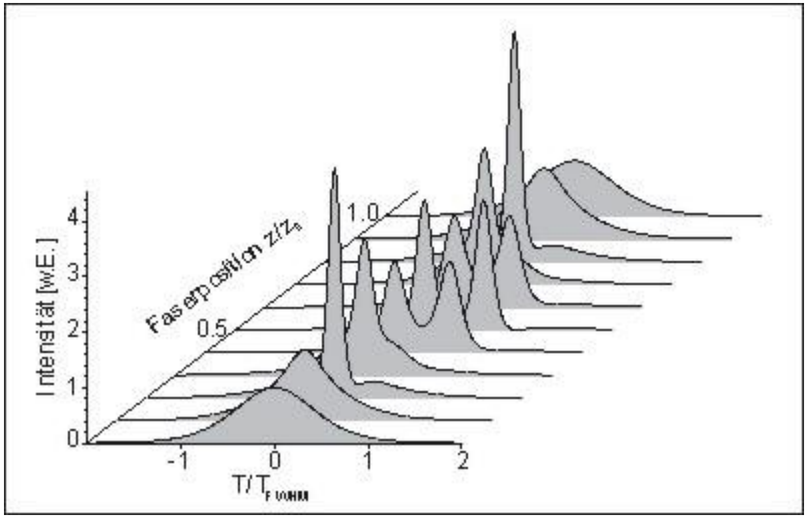
Sign of SPM as in dielectrics (retarding Kerr)

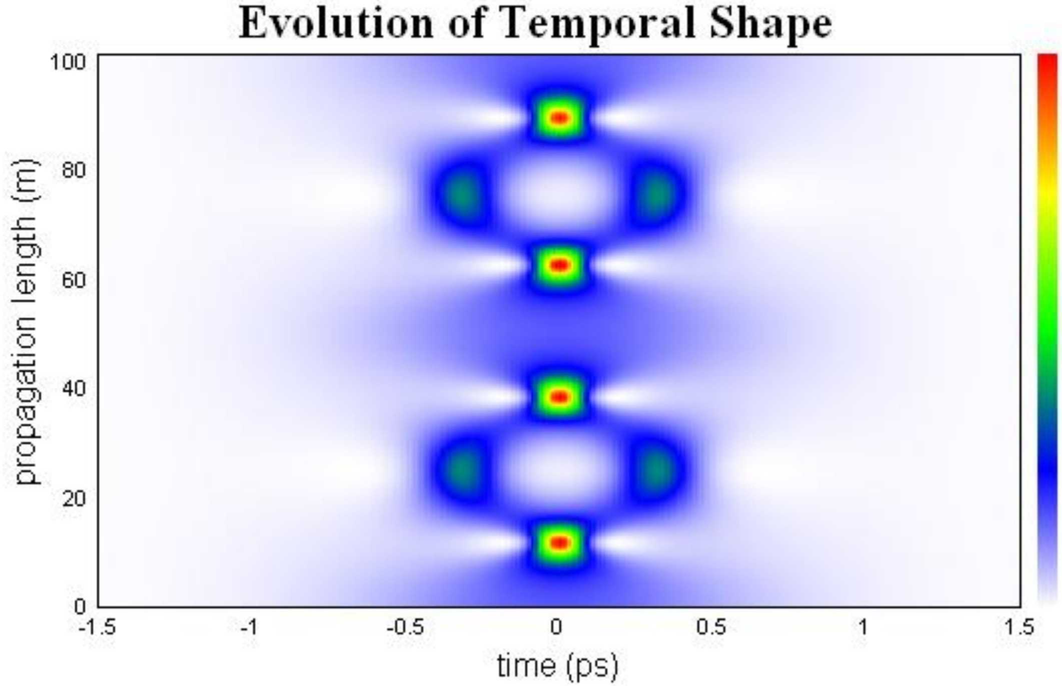


A soliton of runners

picture Ref. Linn F. Mollenauer

Higher-order solitons





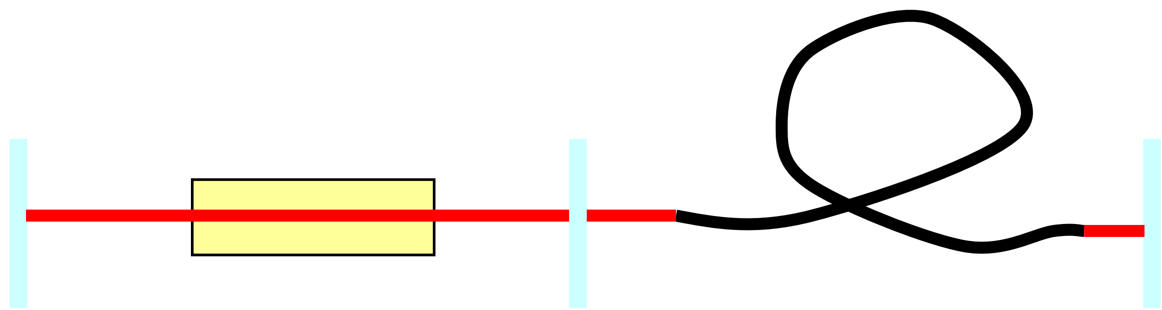
Fundamental soliton (sech) propagates without changing its shape.

Higher-order solutions exist, display breathing behavior

Can be employed for pulse compression

Translating SPM into SAM

Interferometer – APM (additive-pulse mode-locking)

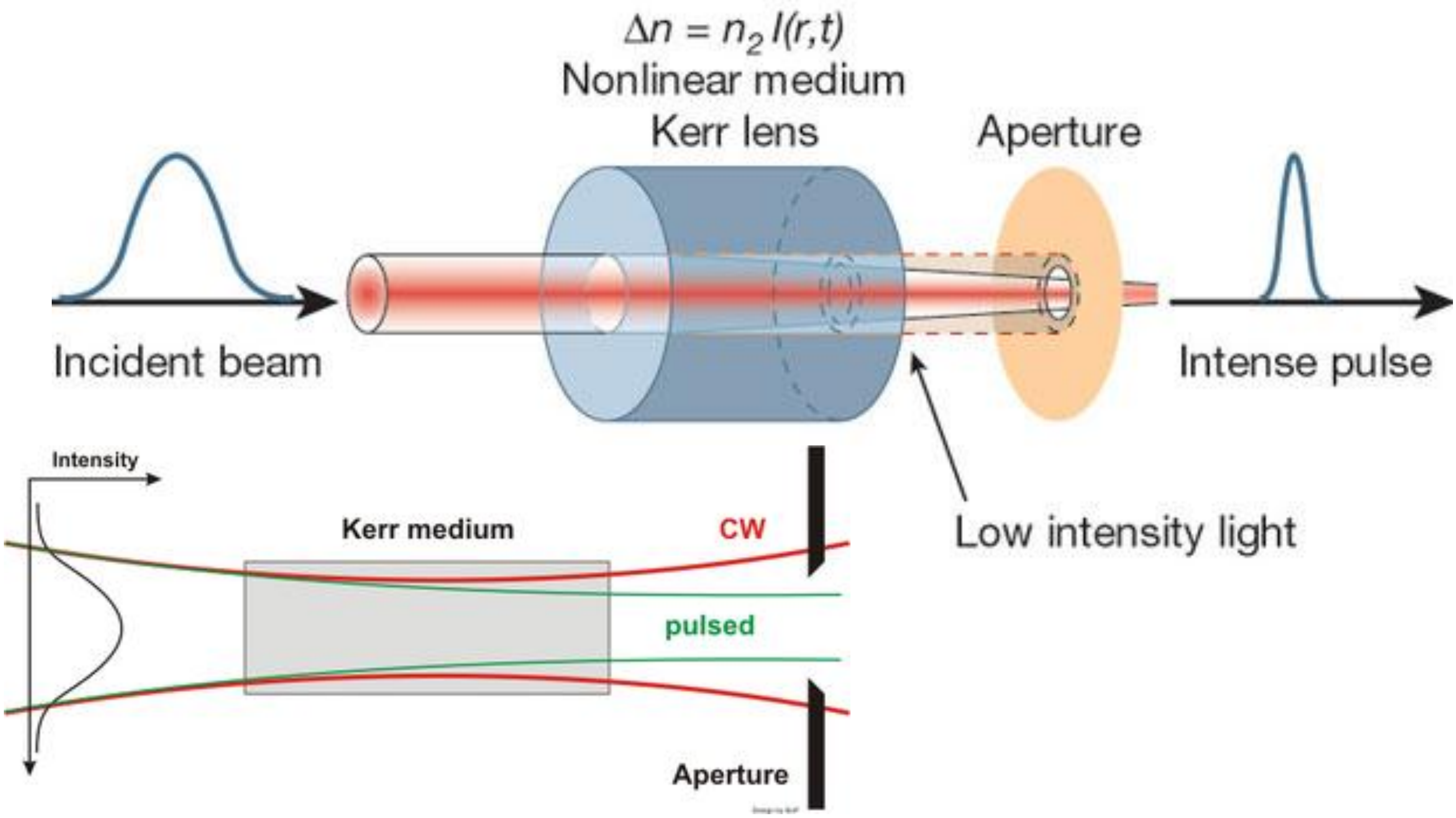


Coupled cavities

Ref.: Ippen et al., JOSA B **6,** 1736 (1989)
NOLM: Doran & Wood, Opt. Lett. **13**, 56 (1988)

Translating SPM into SAM (II)

Kerr – lensing mechanism



How the KLM laser really works: V. Magni et al., JOSA B **12** 476 (1995).

The main building blocks of ultrafast optics

1. **Group Delay Dispersion** **GDD**

2. **Self-phase modulation** **SPM**

3. **Saturable absorption (or self-amplitude modulation)** **SAM**

4. **Laser gain, dynamic gain saturation**