Generation of high-energy, few-cycle optical pulses
PART II : Methods for generation

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Max-Born-Institute

Overview

First Lecture:

Foundations of Ultrashort Pulse Compression

- Description of short laser pulses (duration, chirp, spectrum)
- Group Delay Dispersion and its compensation
- Nonlinear optical effects (SPM and SAM)
- Soliton and solitonlike pulse shaping

Second lecture:

Practical methods for ultrafast pulse compression

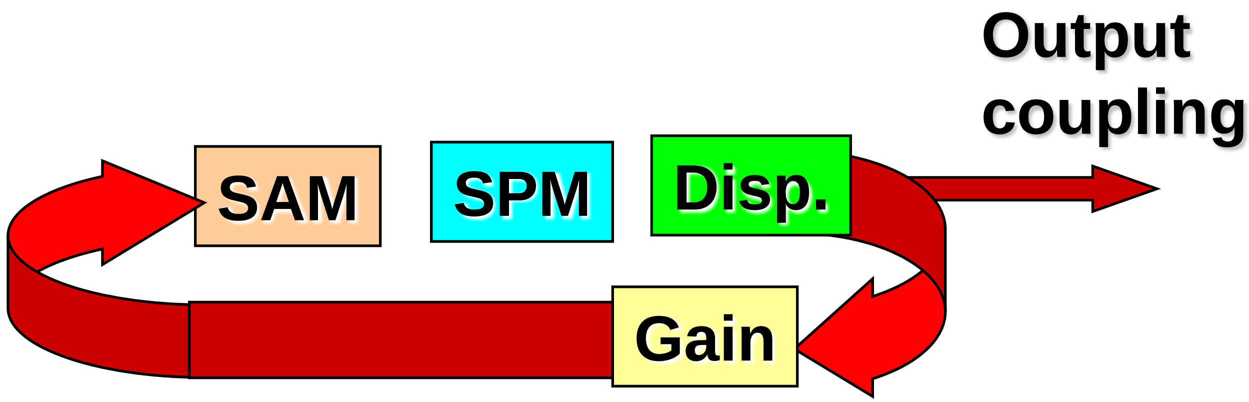
- Laser oscillators
- Amplification of short pulses, CPA
- Active pulse compression (fiber-grating, hollow fiber, filament)

The main building blocks of ultrafast optics

1. **Group Delay Dispersion** **GDD**
2. **Self-phase modulation** **SPM**
3. **Saturable absorption** **SAM**
   (or self-amplitude modulation)
4. **Laser gain, dynamical gain saturation**

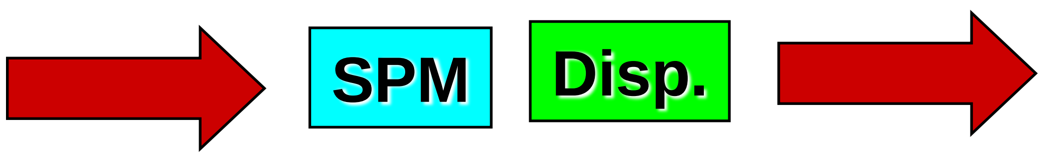
The three basic schemes

1. **Oscillator**



2. **Amplification**

3. **Compression**



Femtosecond Oscillators

SAM vs. gain saturation –
mode-locking stability

The „mode-locking force“

Fast and slow saturable absorbers

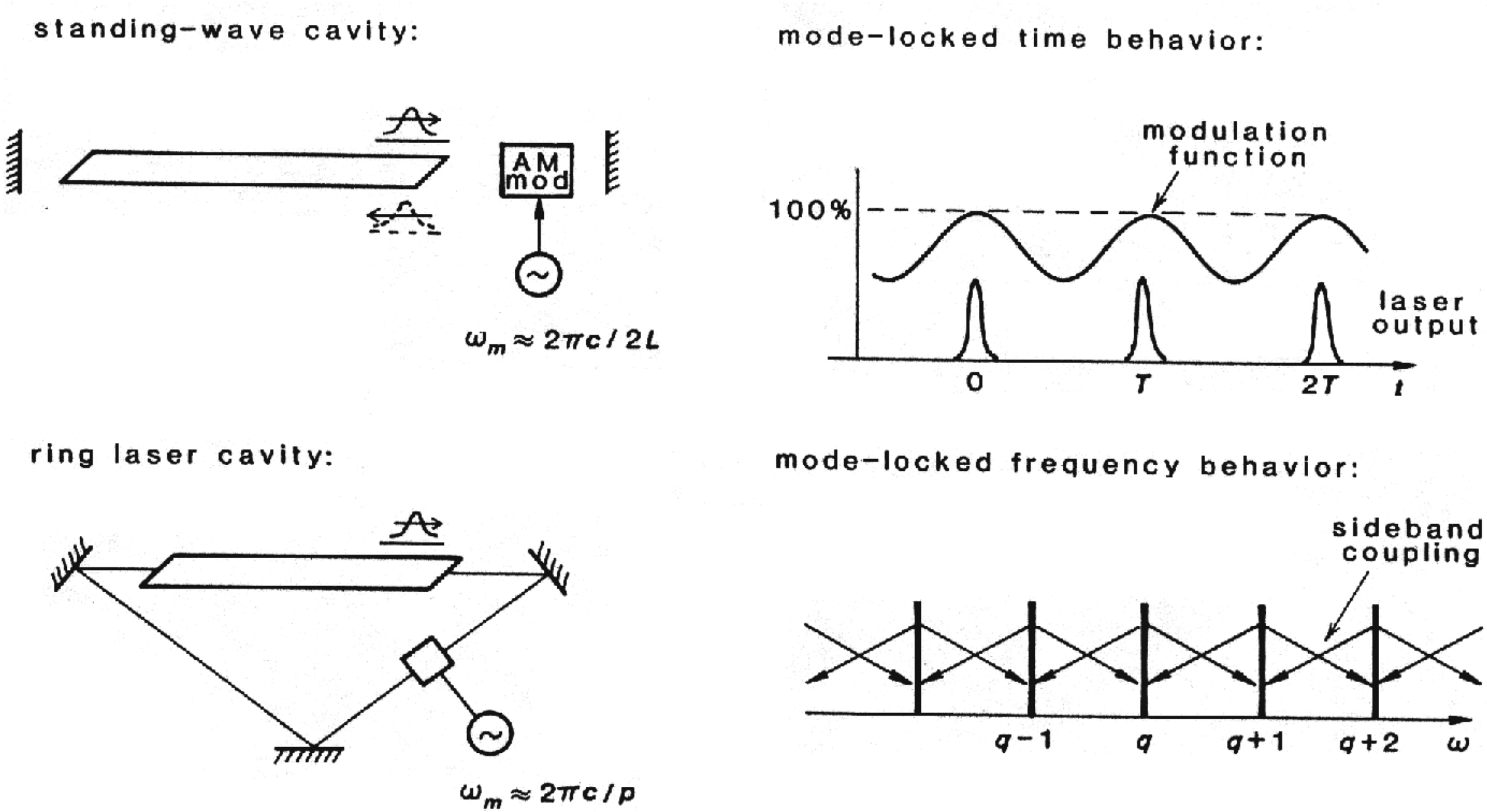
Soliton mode-locking

Net gain window

Shortest pulse durations

Mode-locking

Intracavity optical switch opens and closes synchronously with the propagating optical pulse



Ref: A.E. Siegman, *Lasers,* University Science Books, 1986

Understanding Mode-Locking

Time domain:
Synchronous switch

Steady-state:
Pulse broadening and shortening effects cancel out

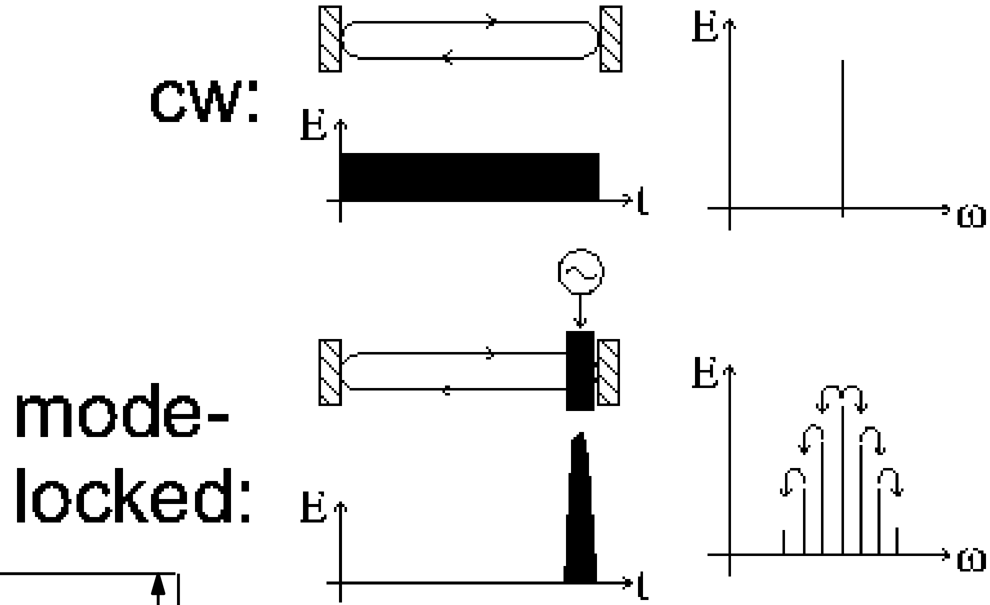
Start-up:
Is there an advantage for short-pulse operation ?

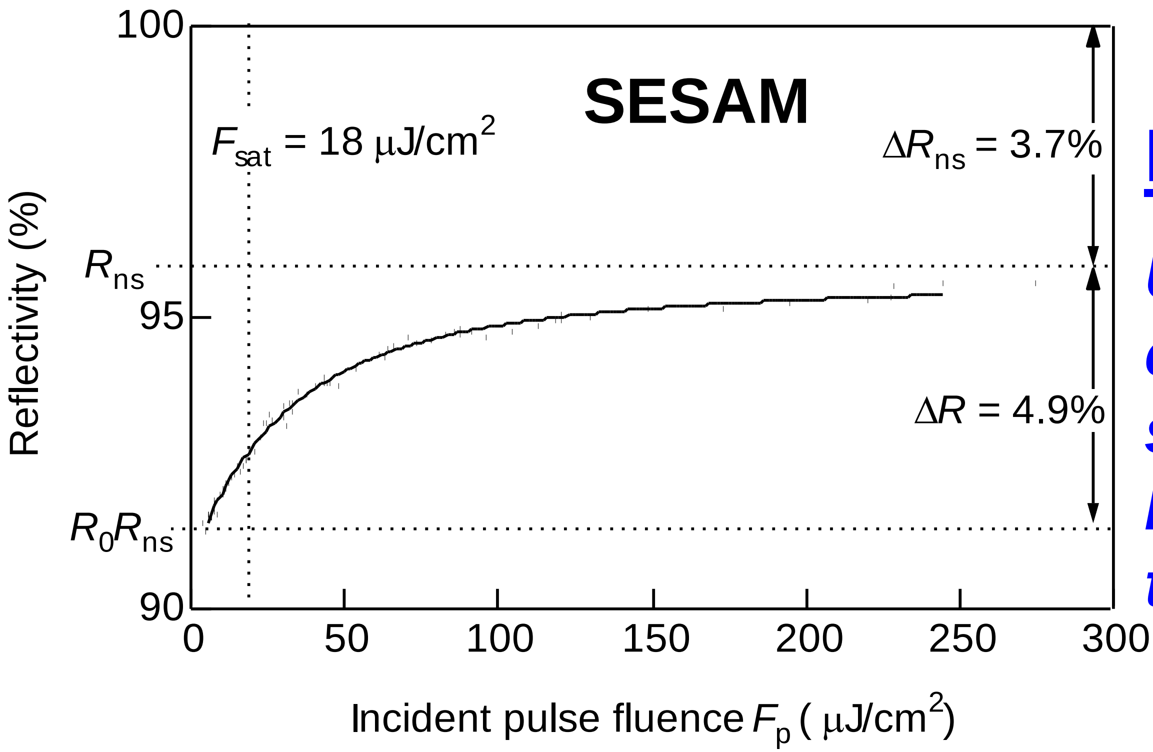
Frequency domain:
Coincidence of modulator sidebands and cavity modes

Passive vs. active mode-locking

Active mode-locking:

Drive modulator with rf wave; generated sidebands coinciding with optical cavity modes



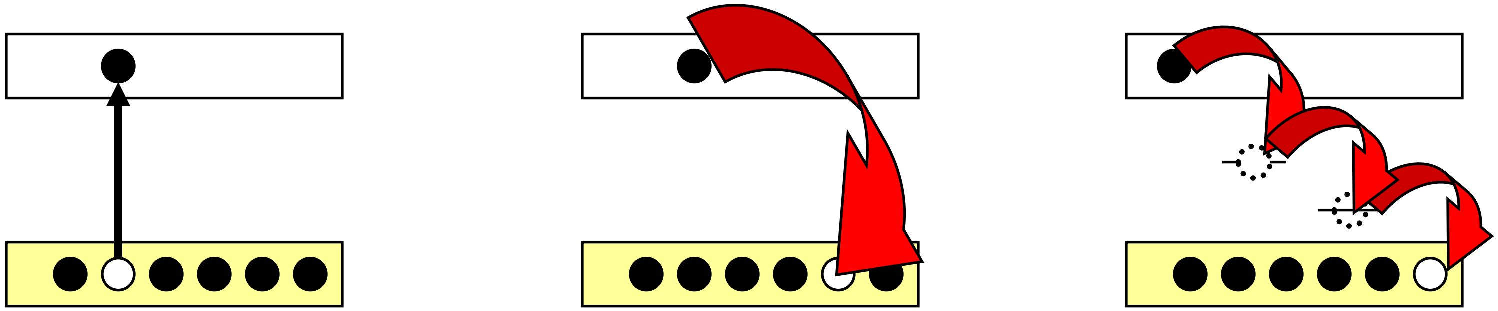


Passive mode-locking:

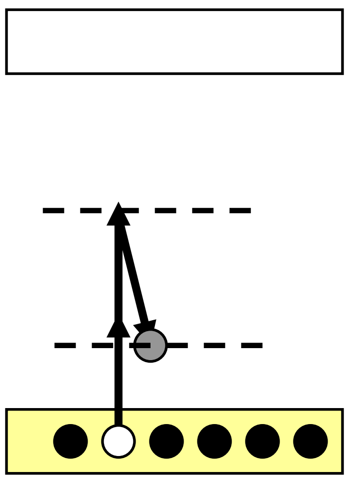
Use nonlinear transmission characteristics of a saturable absorber. Pulse modulates its own transmision

Virtual vs. real transition nonlinearity

	excitation	direct relaxation	intermediate states
real transitions semiconductor (dye, SESAM)		***very slow*** **≈ 100ps**	***somewhat faster*** **≈ 1ps**



virtual states wide bandgap material (opt. Kerr)



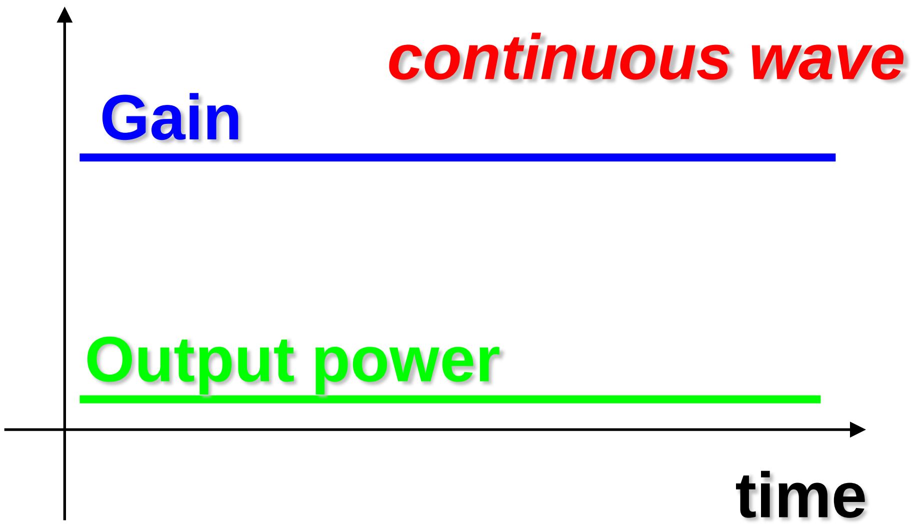
ultrafast

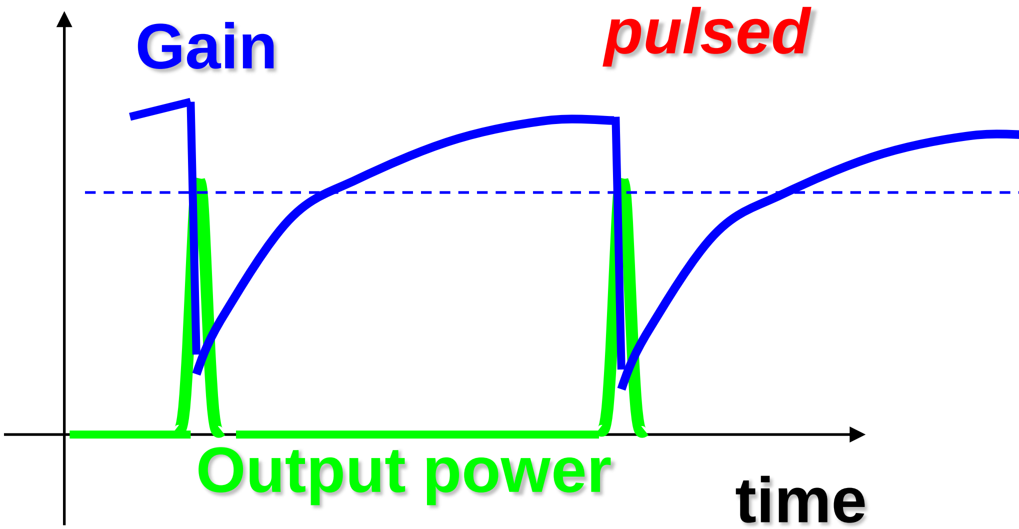
$\tau_{relax} \approx (E_{gap}/h)^{-1}$
< 1 fs

reactive nonlinearity

Gain saturation

Gain saturation normally renders pulsed operation less favorable than cw operation:



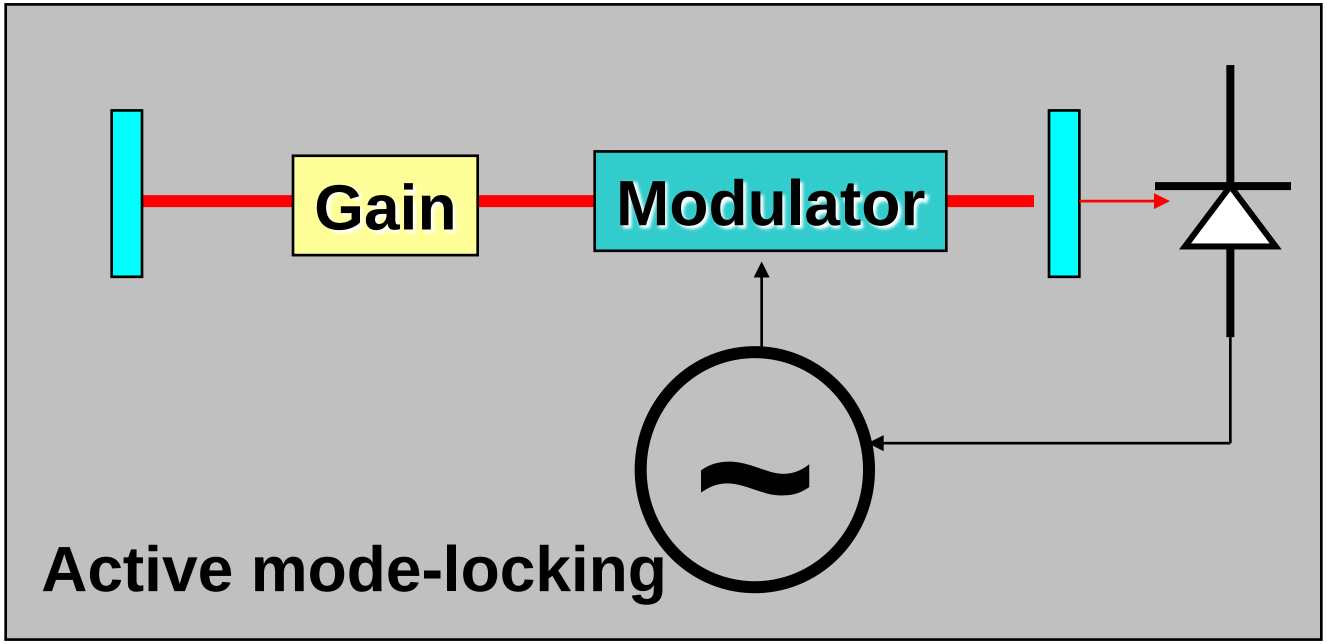


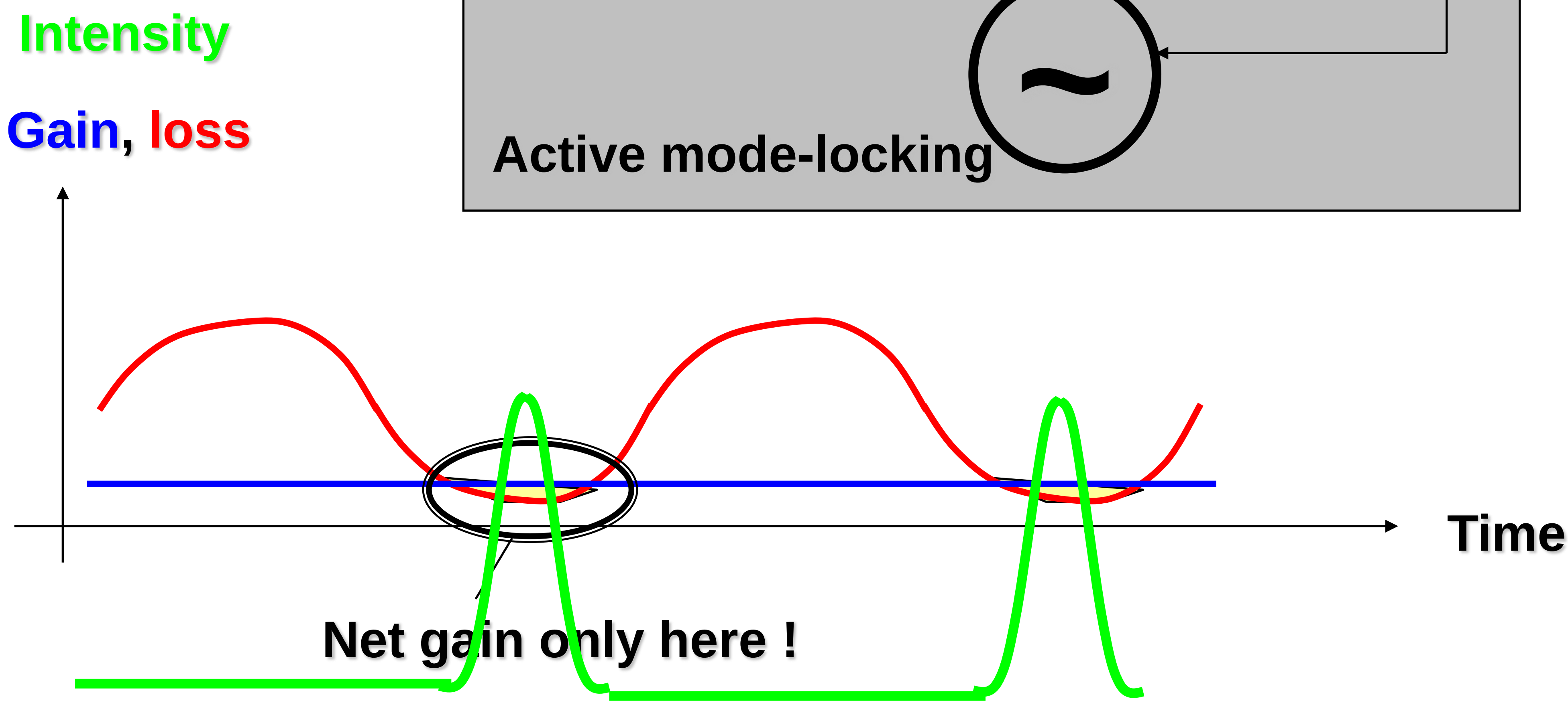
Frantz-Nodvik eq.:

$$E_{out} = E_{sat} \log\left[1 + \exp\left(g_0 L\right)\left[\exp\frac{E_{in}}{E_{sat}} - 1\right]\right]$$

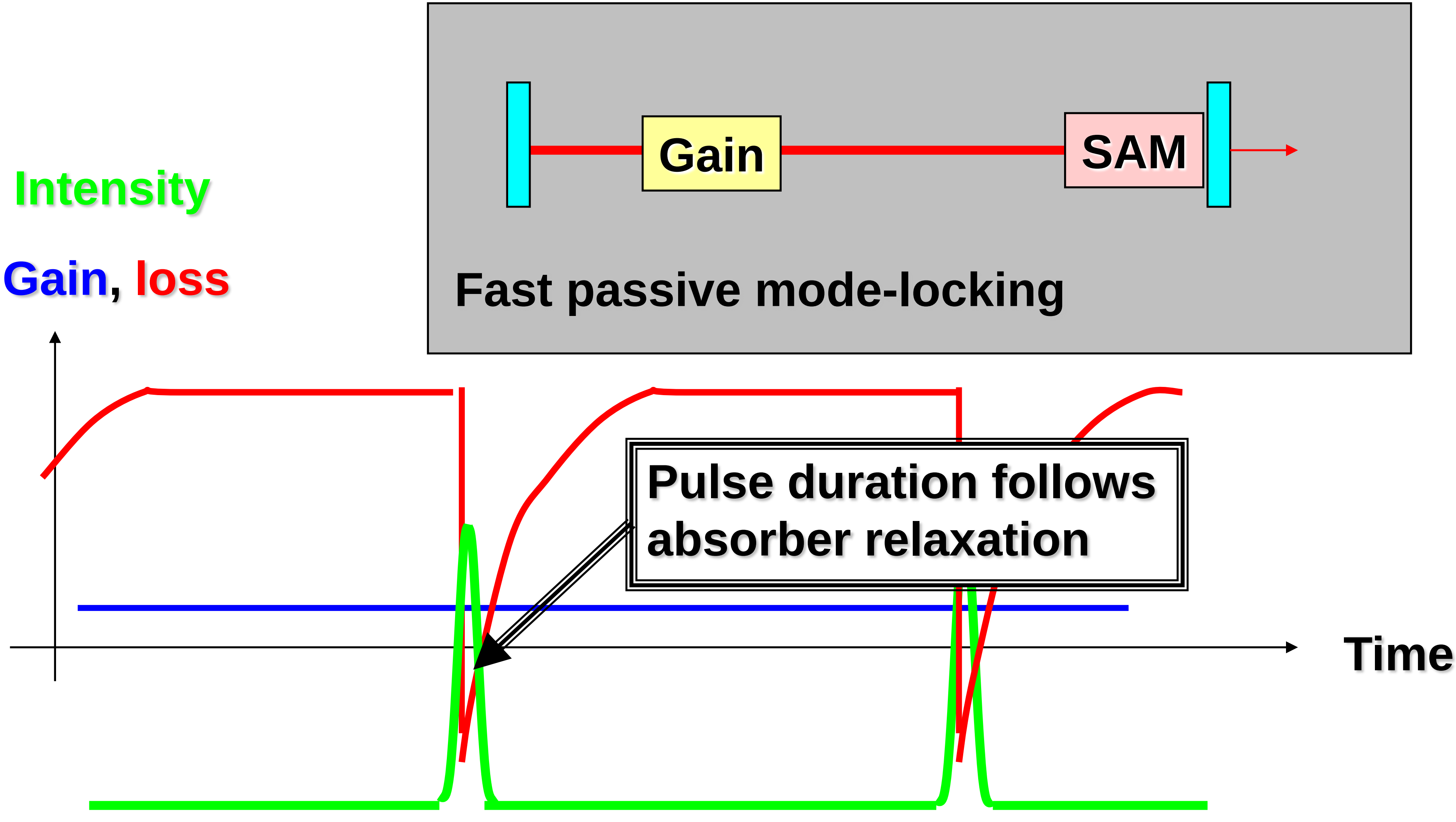
Krausz, Brabec, Spielmann, Opt. Lett. **16**, 235 (1991).

Net Gain Window (active mode-locking)



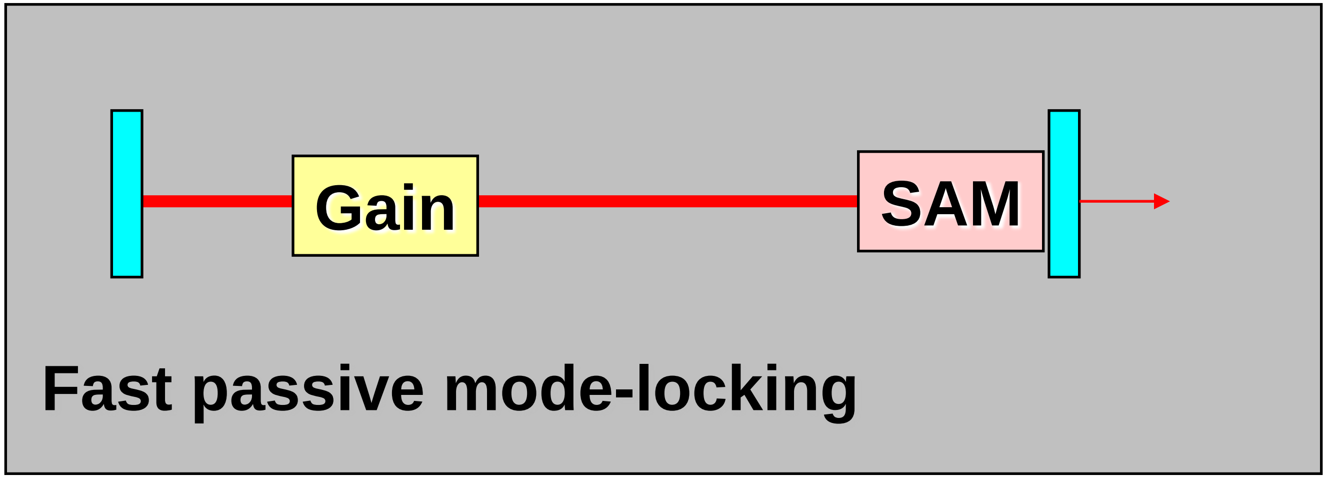


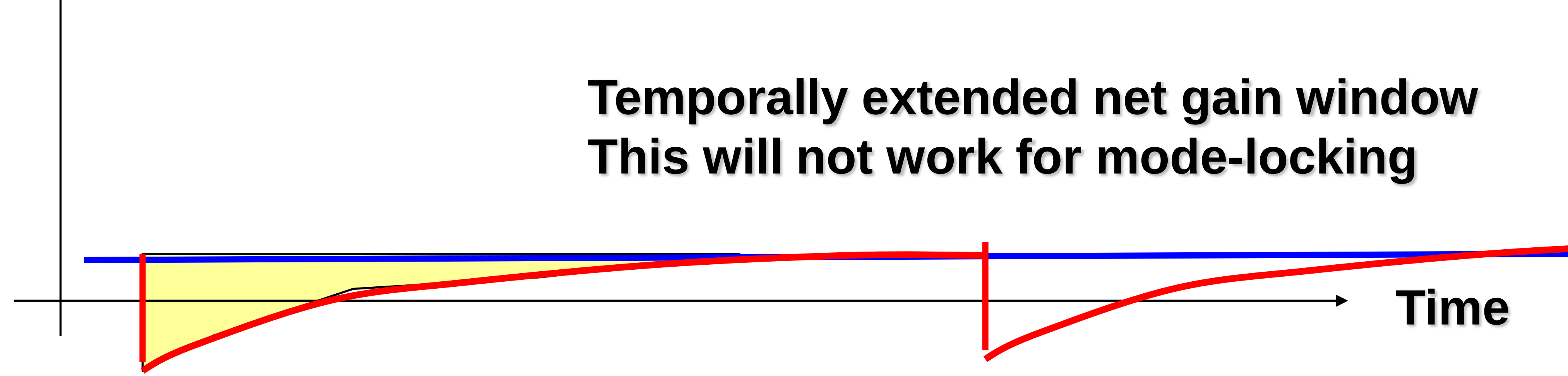
Net Gain Window (fast absorber)



H. A. Haus, *J. Appl. Phys.* **46**, 3049 (1975).

Net Gain Window (slow absorber)





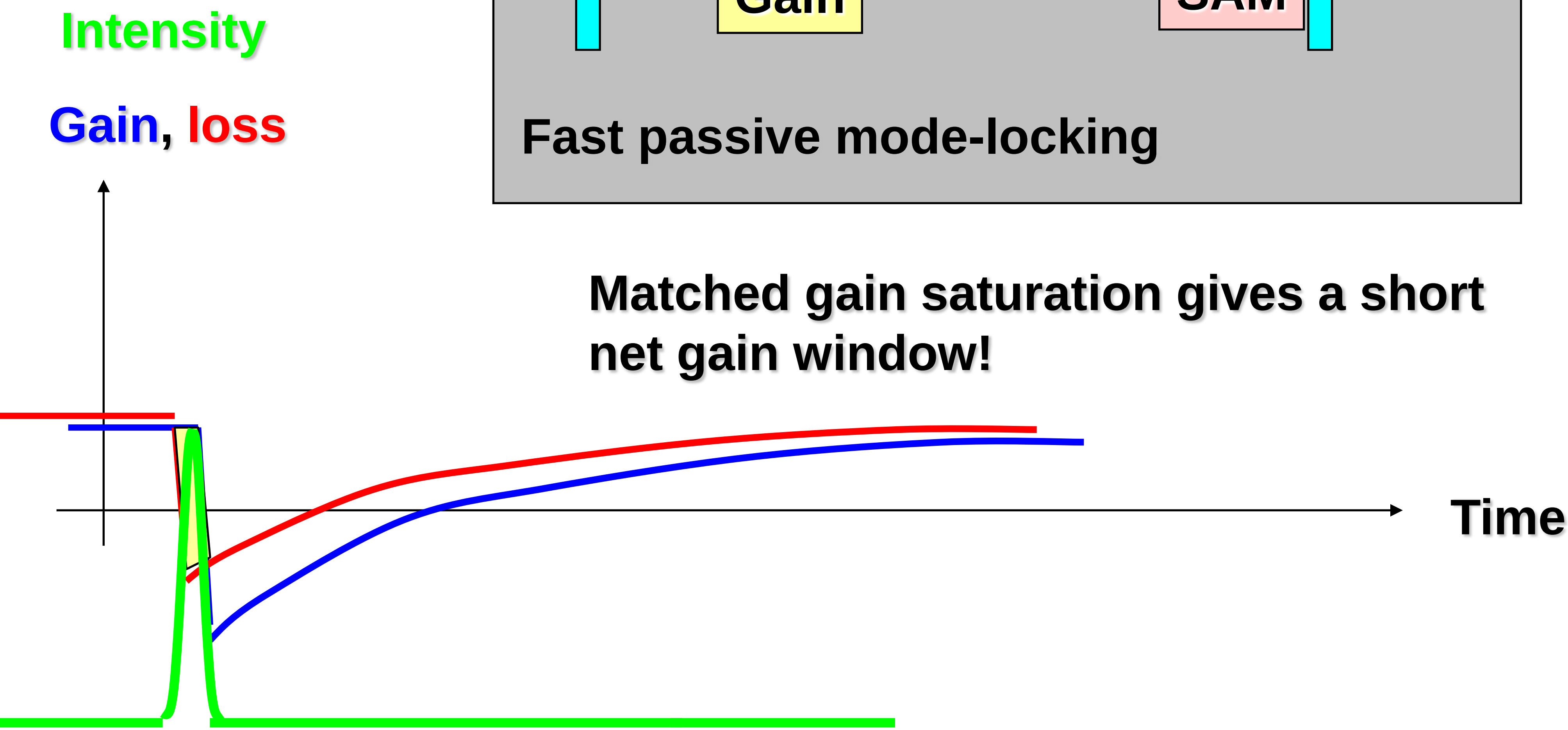
Temporally extended net gain window
This will not work for mode-locking

H.A. Haus, IEEE J. Quantum Electron. **QE-11**, 736 (1975)

Net Gain Window (slow absorber II)

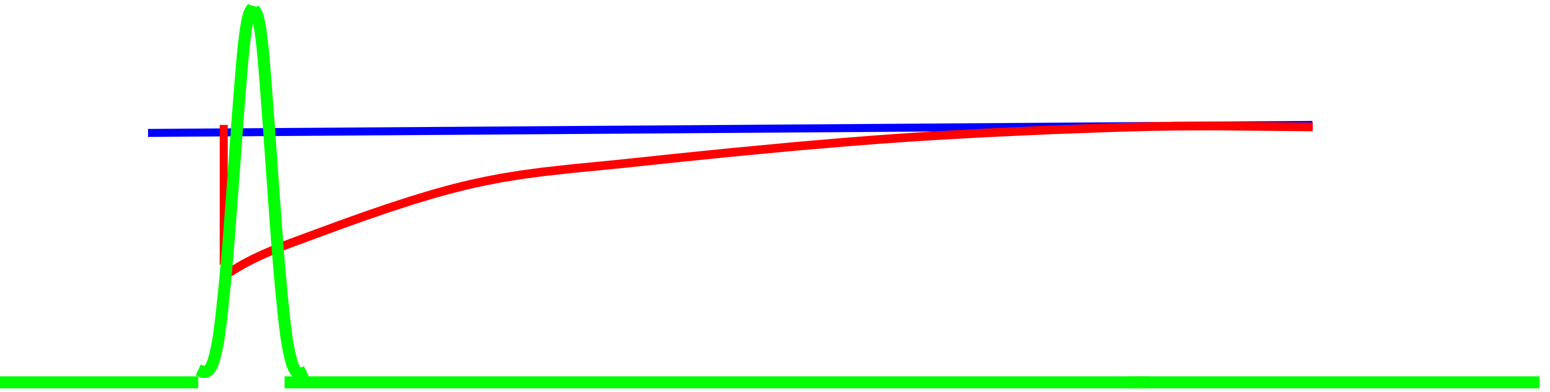
Fast passive mode-locking

Matched gain saturation gives a short net gain window!



H.A. Haus, IEEE J. Quantum Electron. **QE-11**, 736 (1975)

Soliton mode-locking



Idea: arrange cavity such that fundamental soliton can propagate (balance of SPM and dispersion)

Everything not belonging to the soliton will be eventually stripped off into „continuum“ outside the net gain window

Pulses more than 10x shorter than absorber response

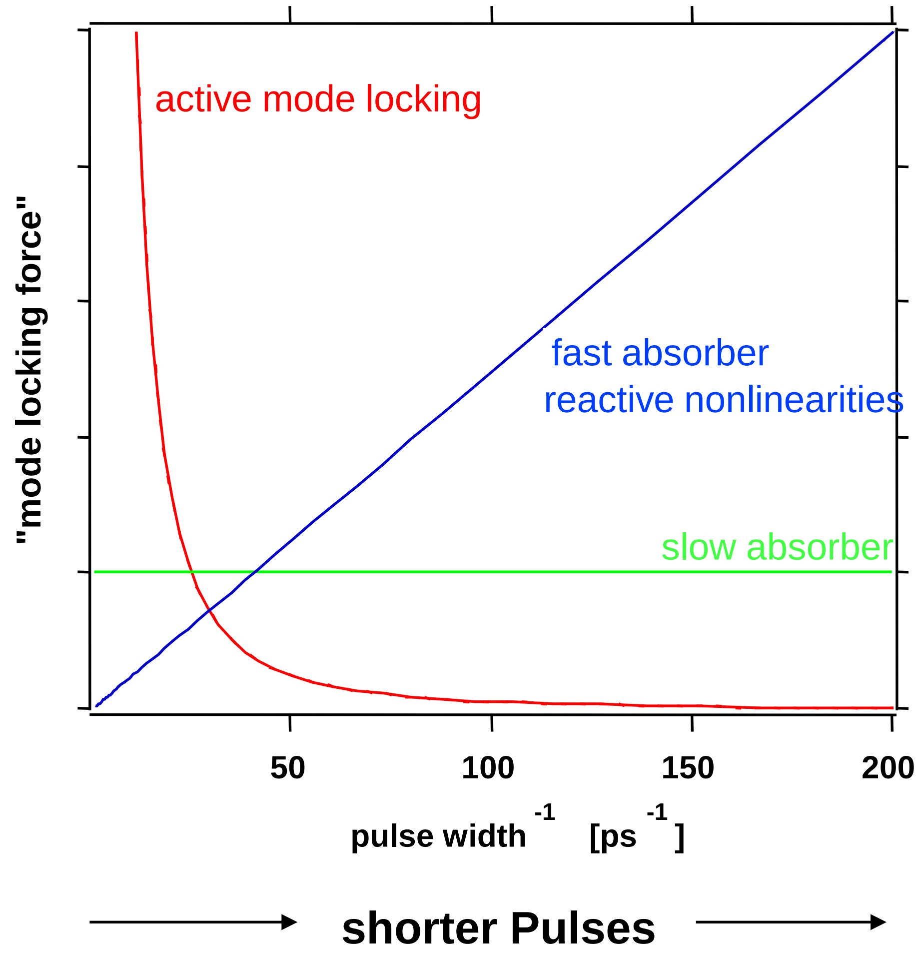
The most successful mode-locking method to date !!!

F. X. Kärtner *et al.*, “IEEE J. STQE. **2**, 540 (1996)

Mode-locking force

- Mode-locking force = pulse shortening / roundtrip
- Balance with gain bandwidth / dispersive broadening

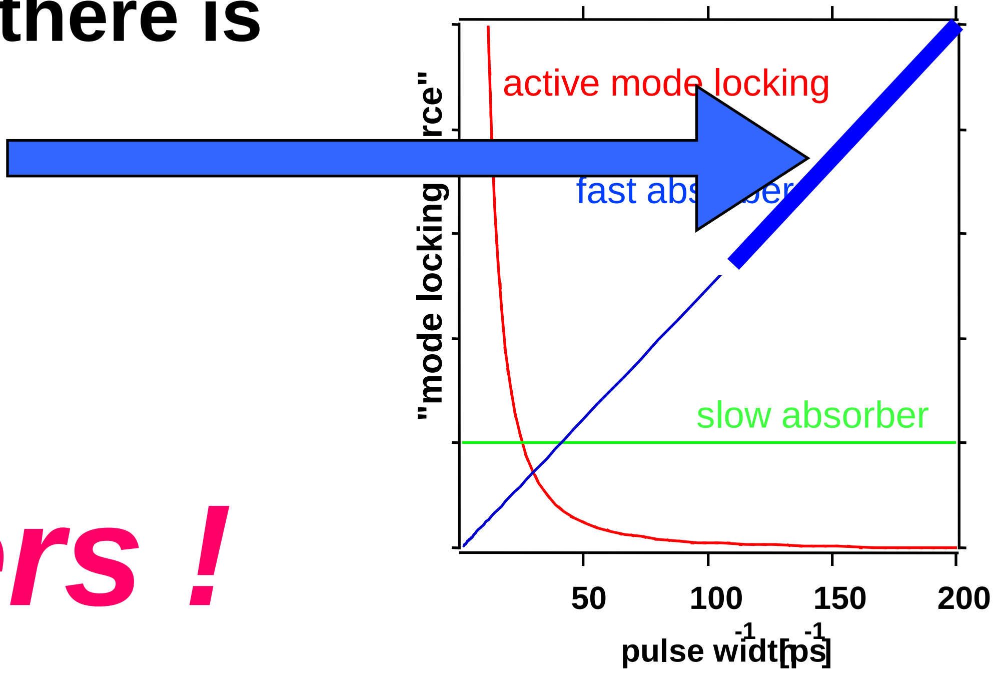
fast absorbers for shortest pulses, slow absorbers for reliable start-up



Ref.: E.P.Ippen, *Appl. Phys.* **B 58,** 159 (1994)

Initiating and sustaining mode-locking

- **For the shortest pulses, there is no way around a fast saturable absorber**



Speed matters !

- **But: it may prove useful to add a slow effect to get mode-locking started.**

(combine fast absorber with active mode-locking or SESAM...)

Net gain window conclusions

For mode-locking you need to constrict your net gain window to the minimum duration possible!

If this does not work sufficiently, your laser will fall back into cw-operation (cw-spike), double pulse etc.

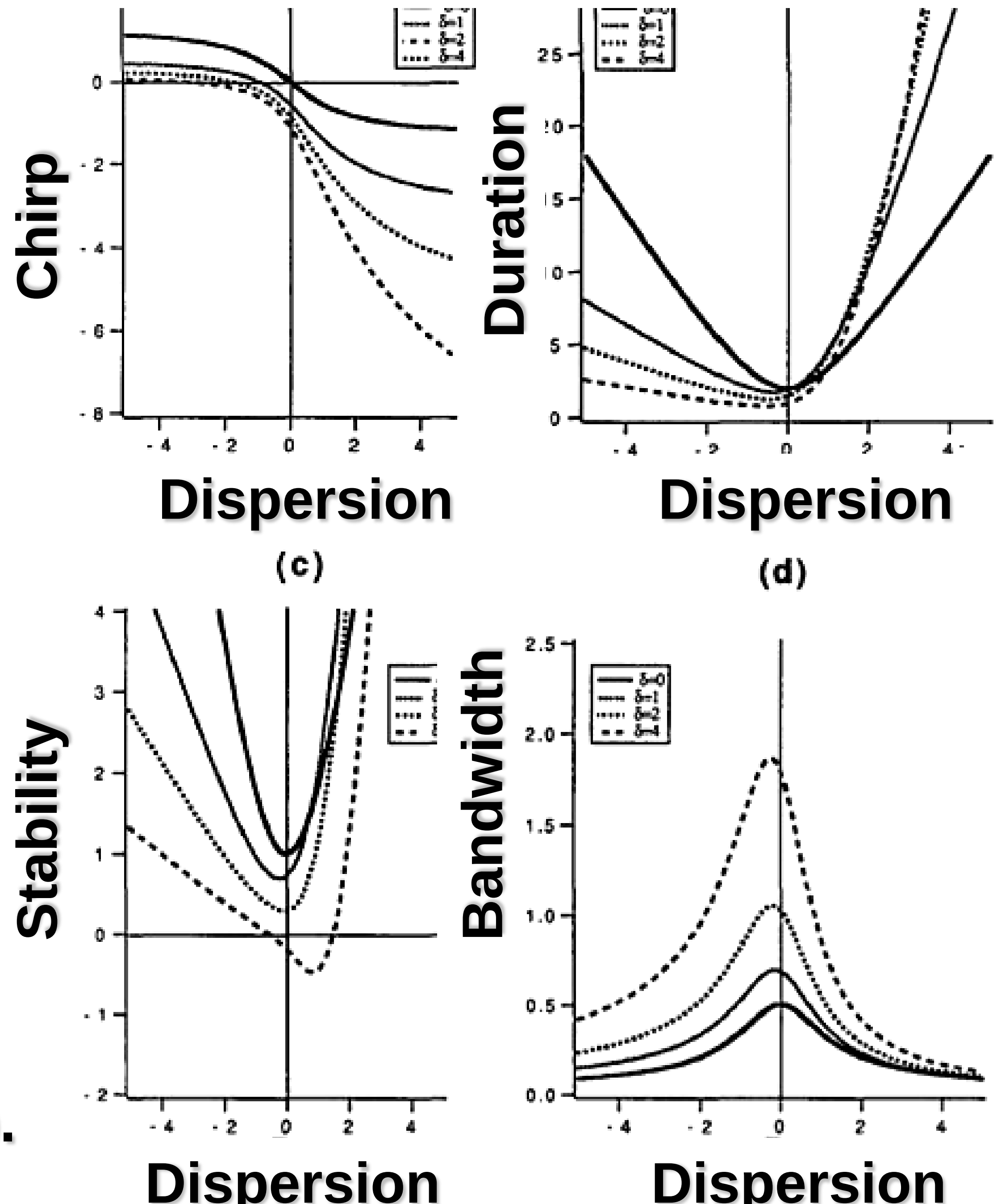
Ideally you want the fastest possible absorber.

However, even slow absorbers may work if you manage to find something that stabilizes your trailing wing.

A more extensive view

δ=SAM coefficient

Solid=weak
dashed=strong

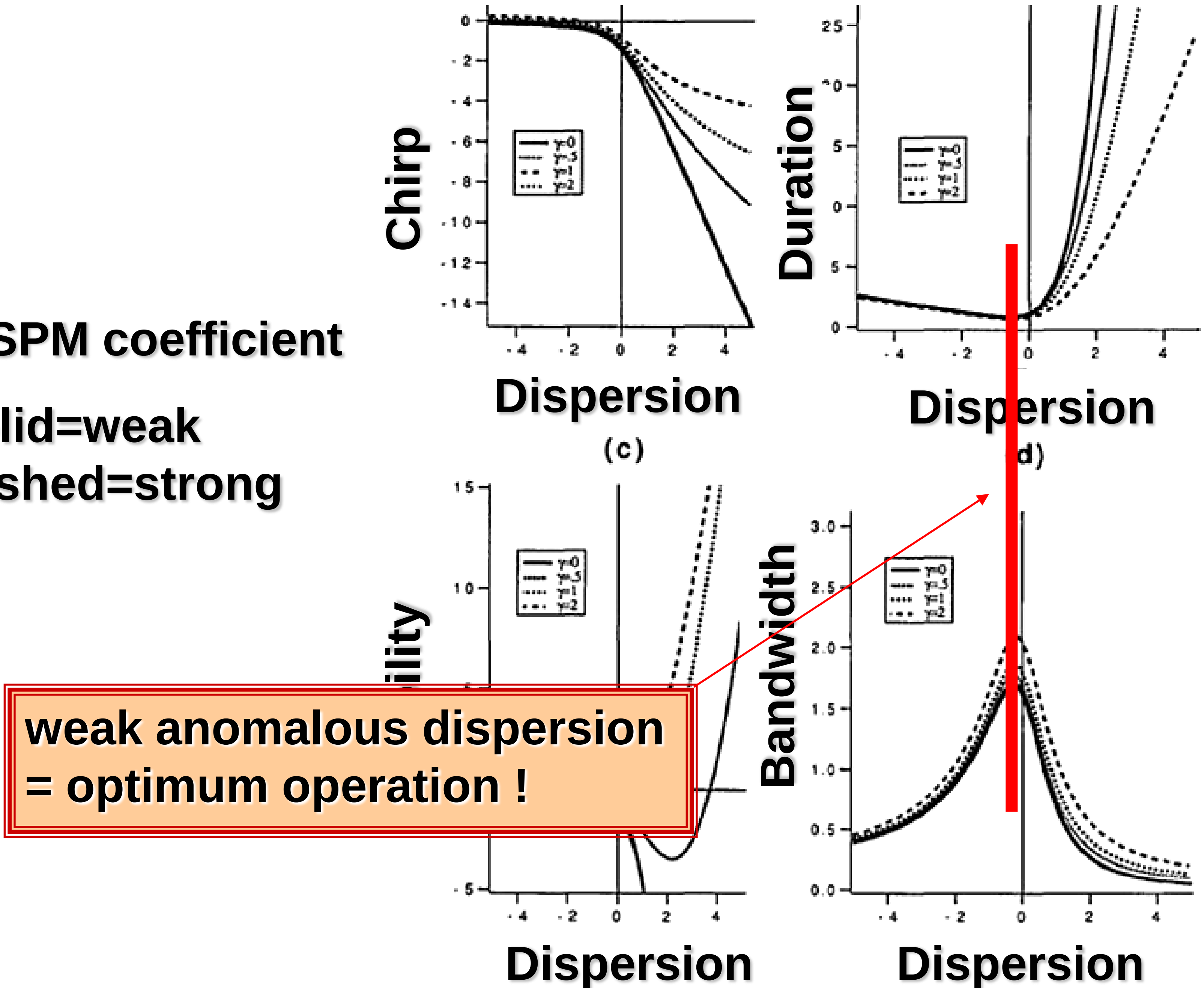


H.A.Haus et al.
JOSA B 8, 2068 (1991).

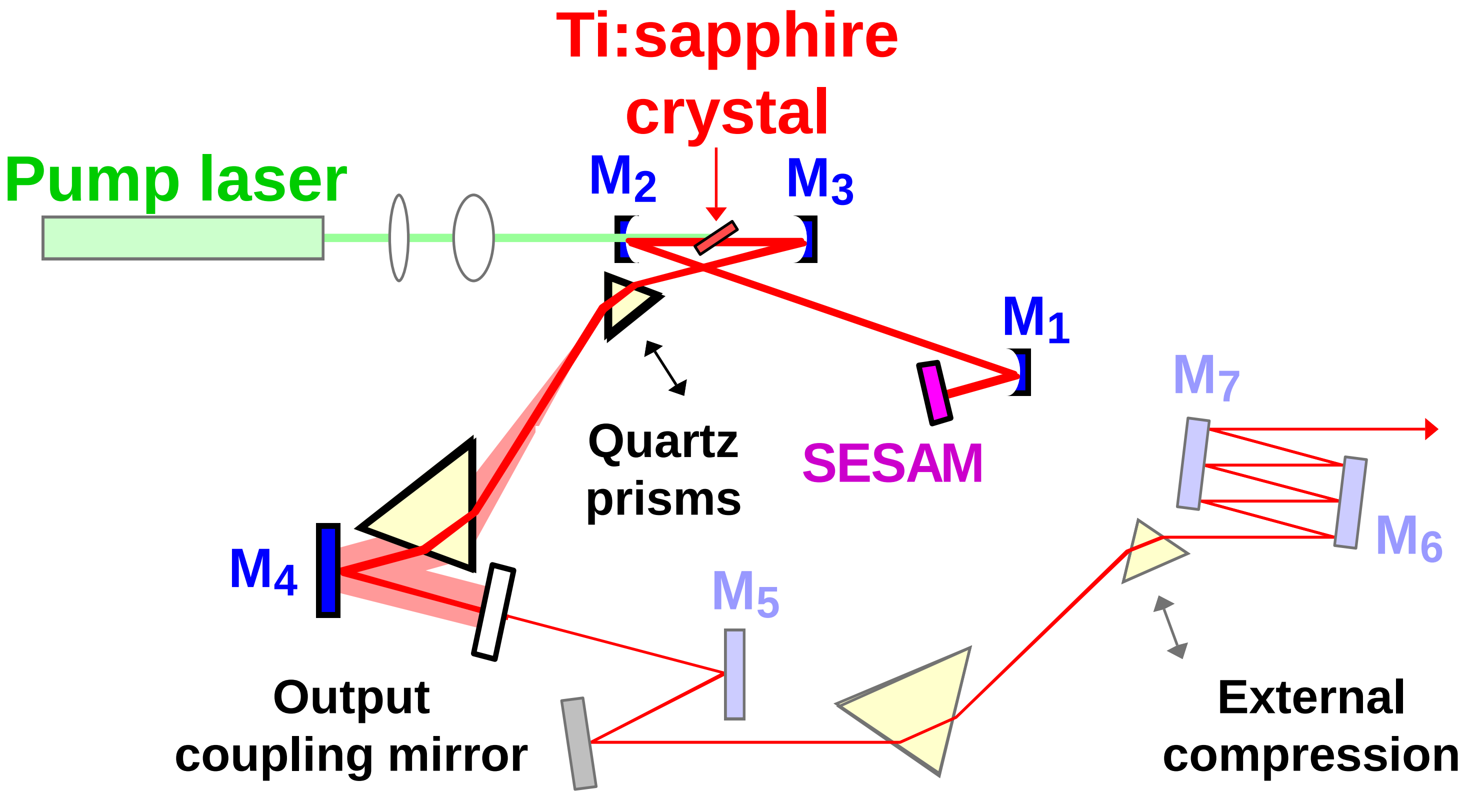
Dependence on SPM

γ=SPM coefficient

Solid=weak
dashed=strong

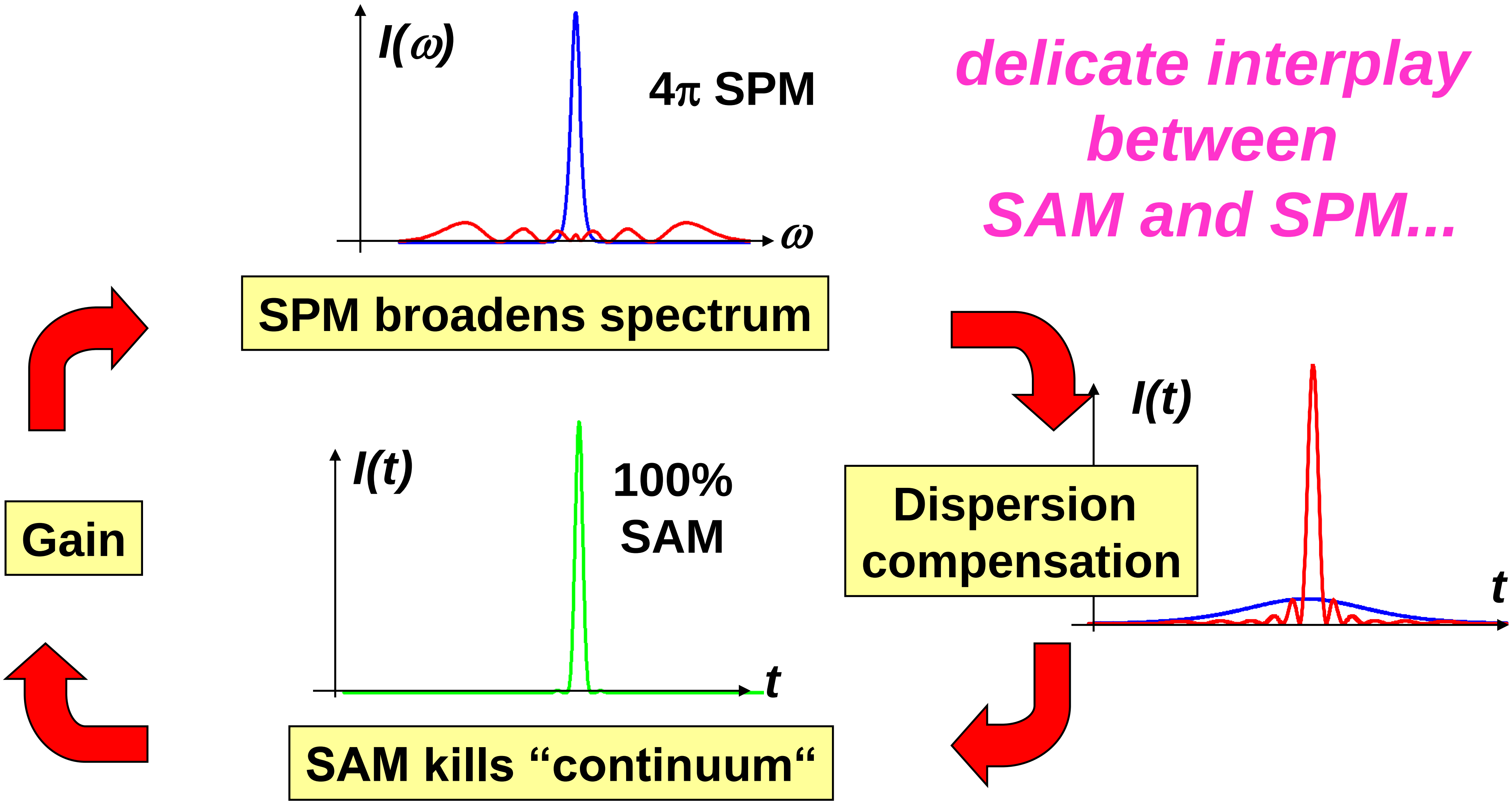


An ultrashort-pulse Ti:sapphire laser



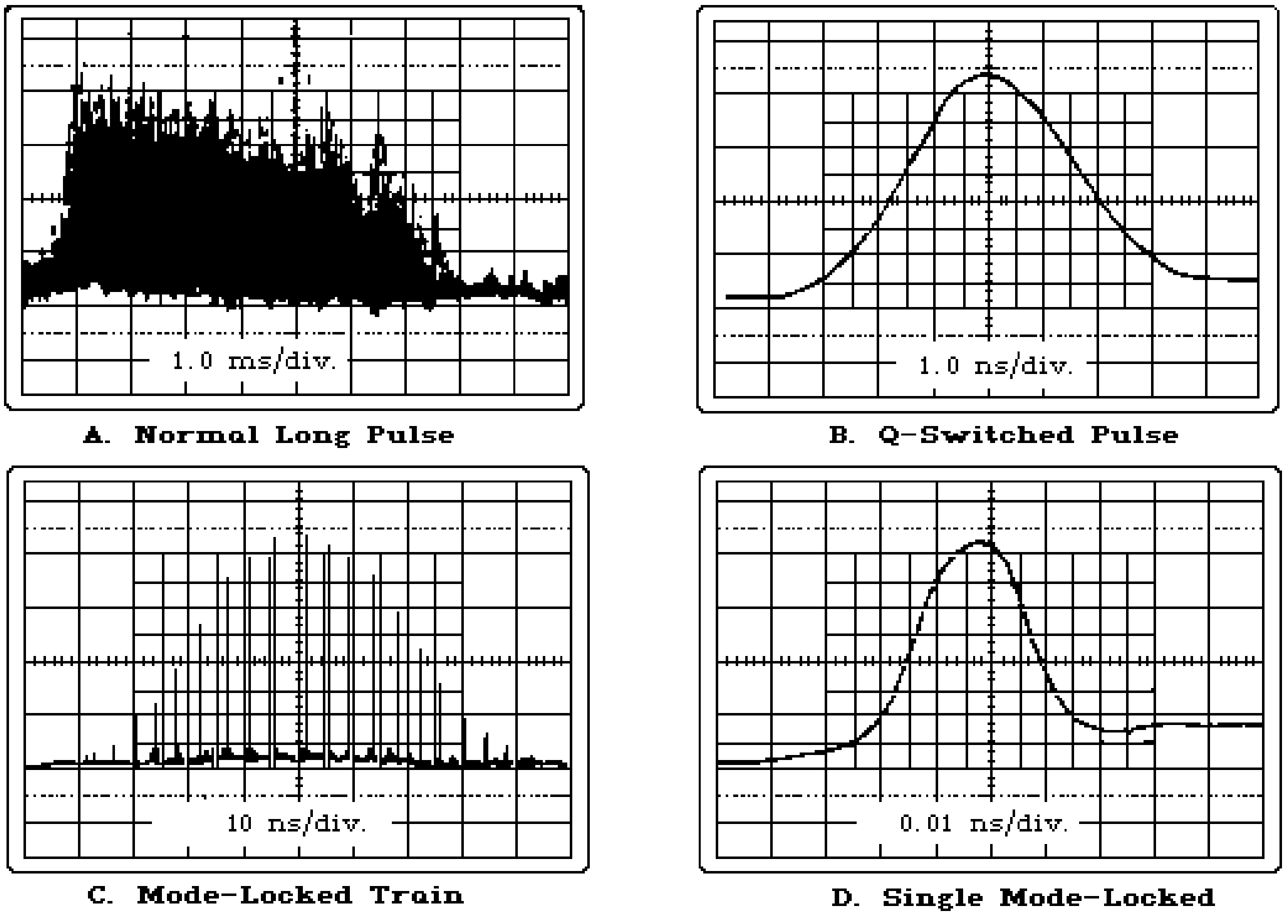
Ref.: D.H. Sutter et al., *Opt. Lett.* **24**, 631 (1999)

How a KLM laser really works...



Haus et al., „Structures for additive pulse modelocking,“ ***JOSA* B <u>8</u>, 2068 (1991)**
Brabec et al. , „Mode-locking in solitary lasers,“ ***Opt. Lett.* <u>16</u> 1961 (1991)**

Q-switched mode-locking



Oscilloscope traces of a long-pulse (normal-pulse) laser (A), a Q-switched pulse (B), and mode-locked laser outputs (C and D)

(From: Sliney & Wolbarsht, 1980). Note changes in time scales in each trace (1 ms = 10^6 ns).

Certain lasers tend to emit bursts of short pulses rather than a continuous train...

Avoiding q-switched mode-locking

$$E_p^2 > E_{sat,abs} E_{sat,gain} \Delta R$$

$$E_{sat} = F_{sat} A_{\mathrm{mode}} \qquad F_{sat,gain} = h\nu / \sigma$$

Laser needs to be driven into saturation to obtain stable mode-locking

Increase cavity length

Focus tighter!

Specific problem of rare-earth doped solid-state lasers.

Ref.: C. Hönninger et al., JOSA B 16, 46 (1999).

Conclusions

SAM is important to provide mode-locking stability (prevent fallback into cw operation, q-switching)

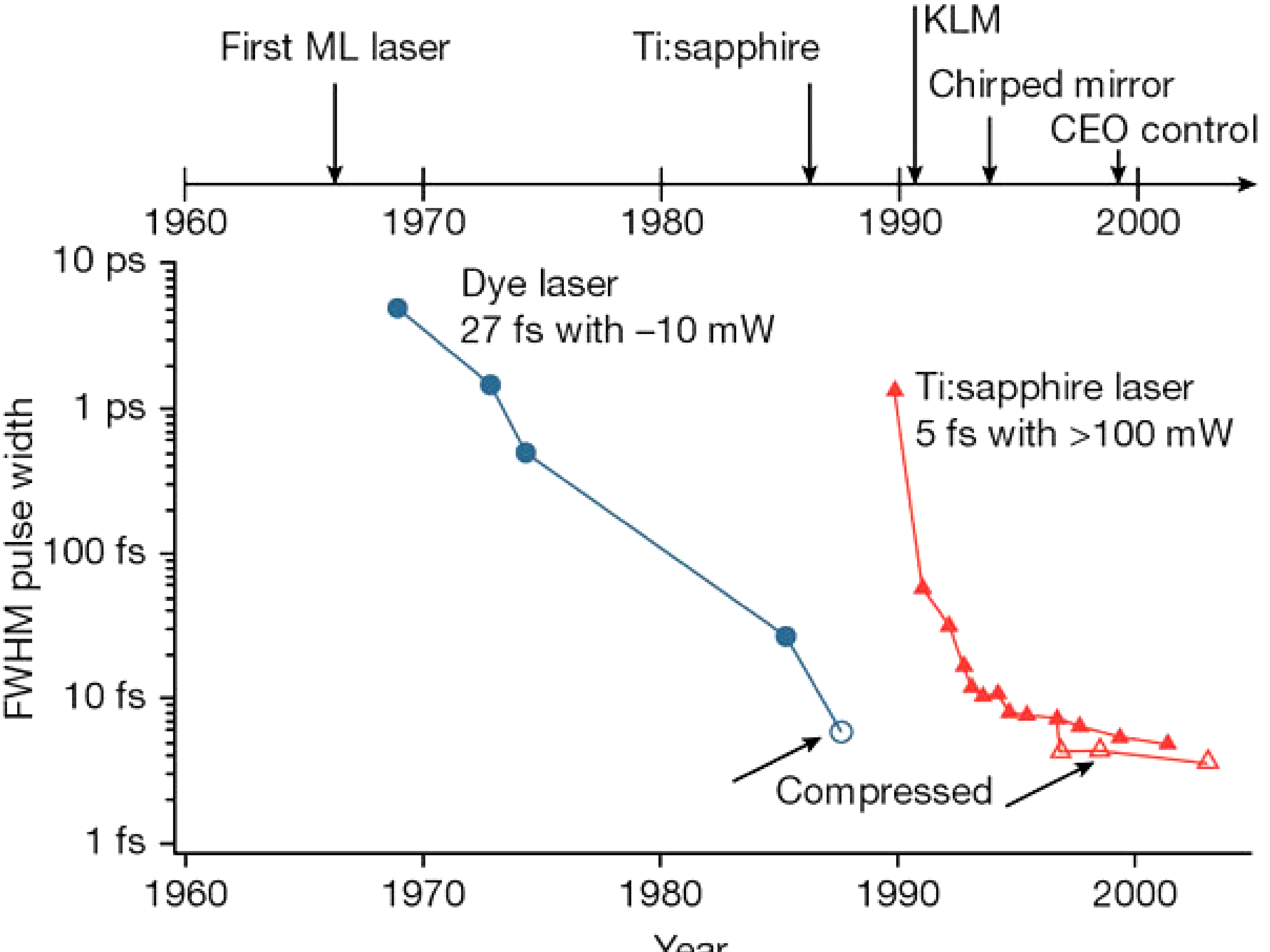
Fast absorbers provide poor start-up.

Slow absorbers may require extra tricks to provide short pulses.

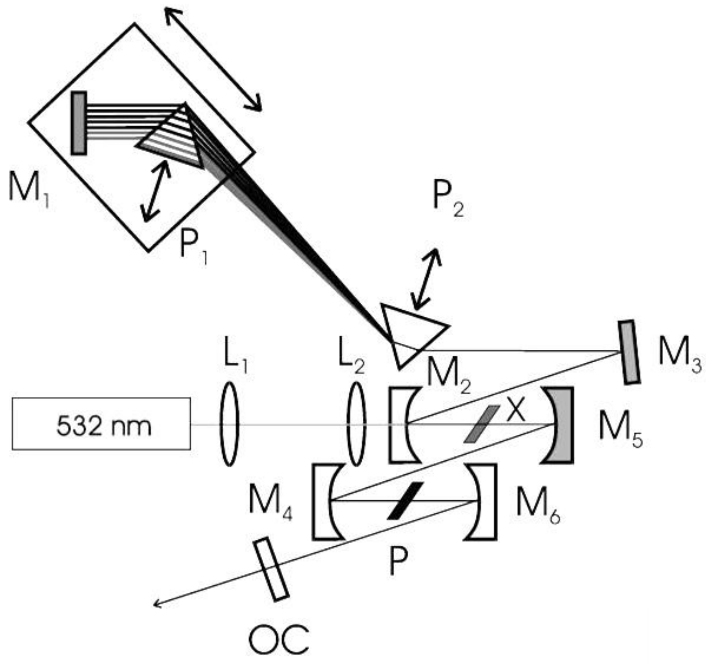
For optimum exploitation of gain bandwidth, you need to exploit (soliton-like) compression in every cavity roundtrip (negative β_2)

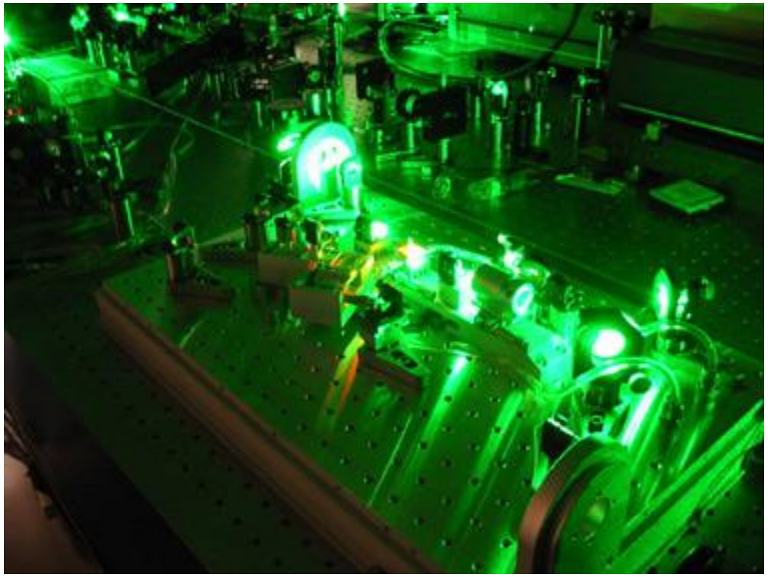
Kerr-lens mode-locking is, in fact, soliton mode-locking!

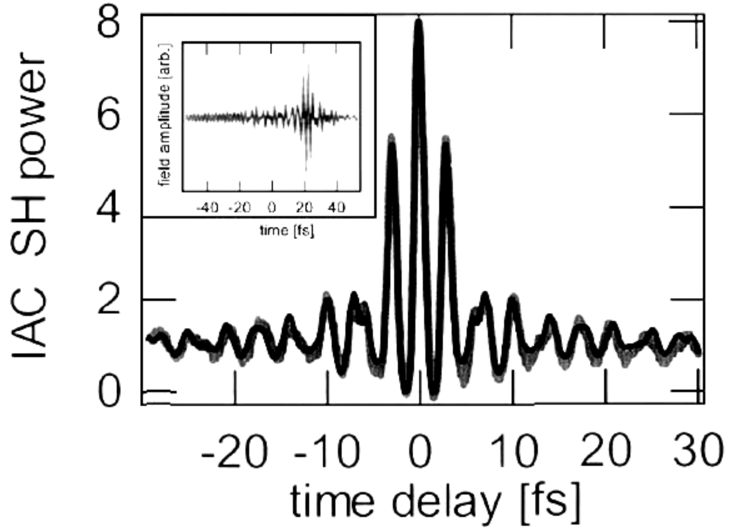
World records

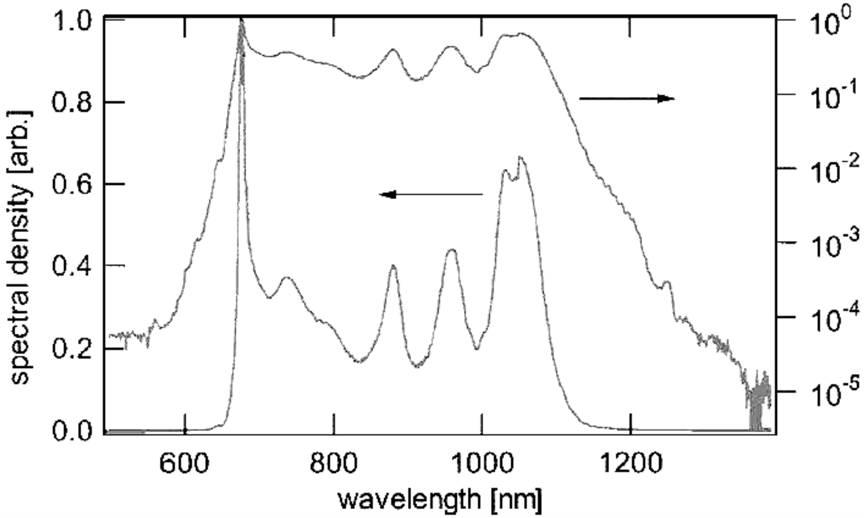


2-cycle laser pulses directly from oscillator





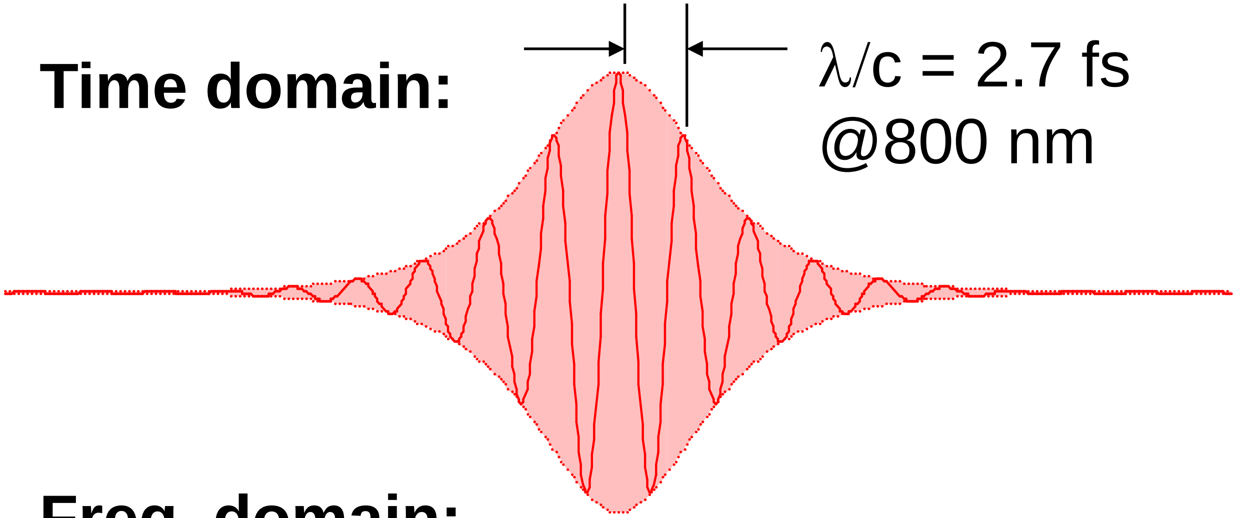




R.Ell et al., Opt.Lett. **26**, 373 (2001)

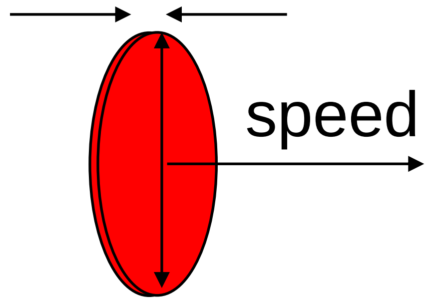
Two-cycle pulses – octave-spanning light membranes

Time domain:



λ/c = 2.7 fs
@800 nm

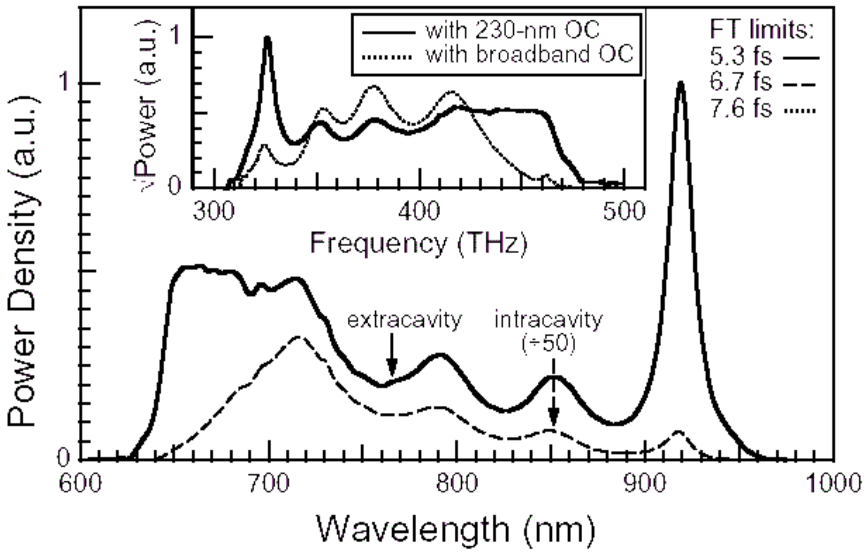
1.6 µm

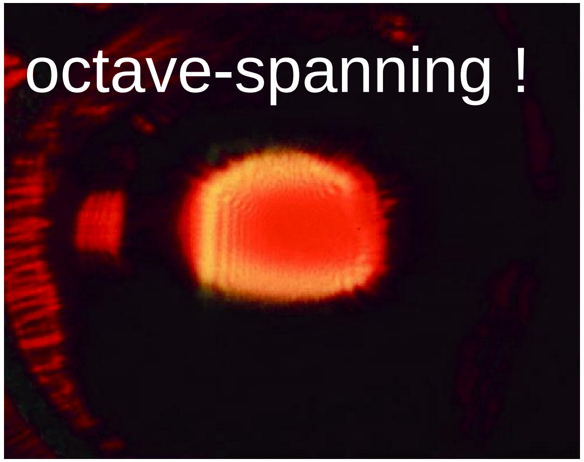


speed of light

Ø ≈ 1 mm

Freq. domain:

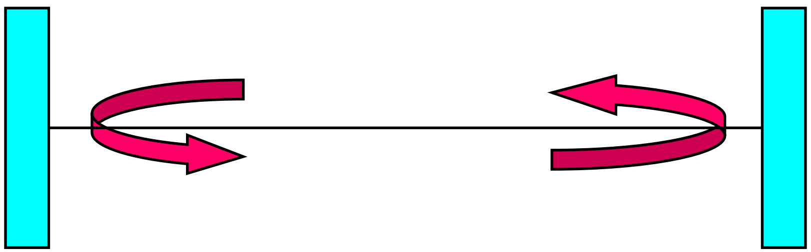




Refs.: G.Steinmeyer, *Science* **286**, 1507 (1999); D.H. Sutter, *Opt. Lett.* **24**, 631 (1999)

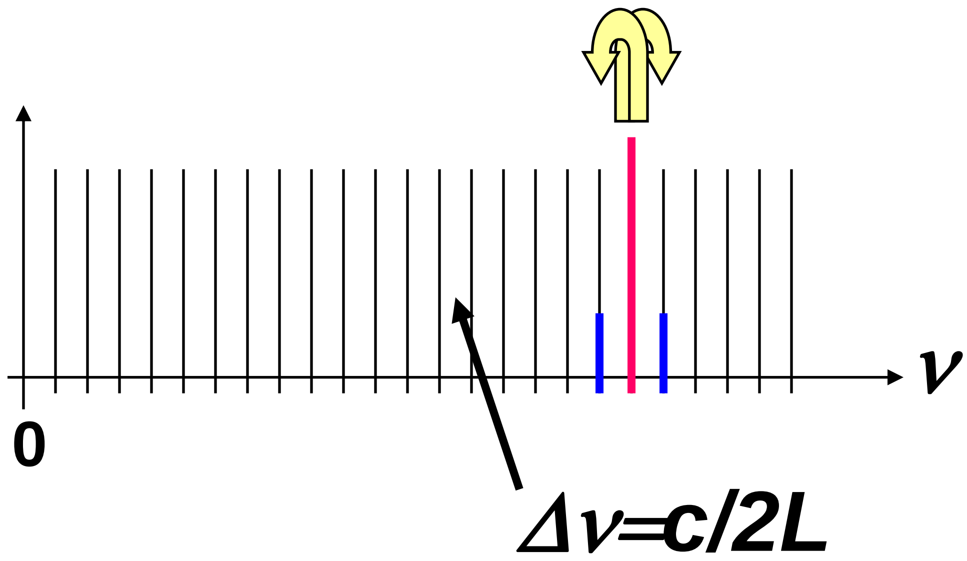
The mode comb

Siegman's picture of the laser



Cavity eigenfrequencies:

$$\nu_m = m\frac{c}{2L}$$

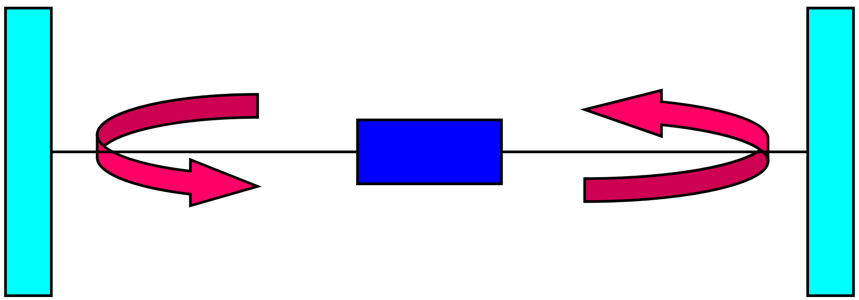


Modulator creates sidebands at neigboring modes

Ref: A.E. Siegman, *Lasers,* University Science Books, 1986

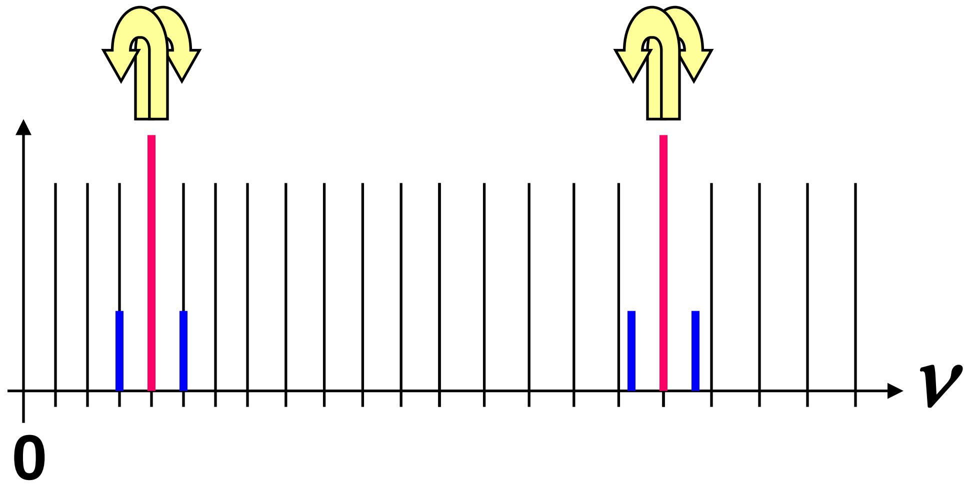
The real world

The cavity contains dispersive material



L is now a function of λ !

Cavity eigenfrequencies
no more equidistant !

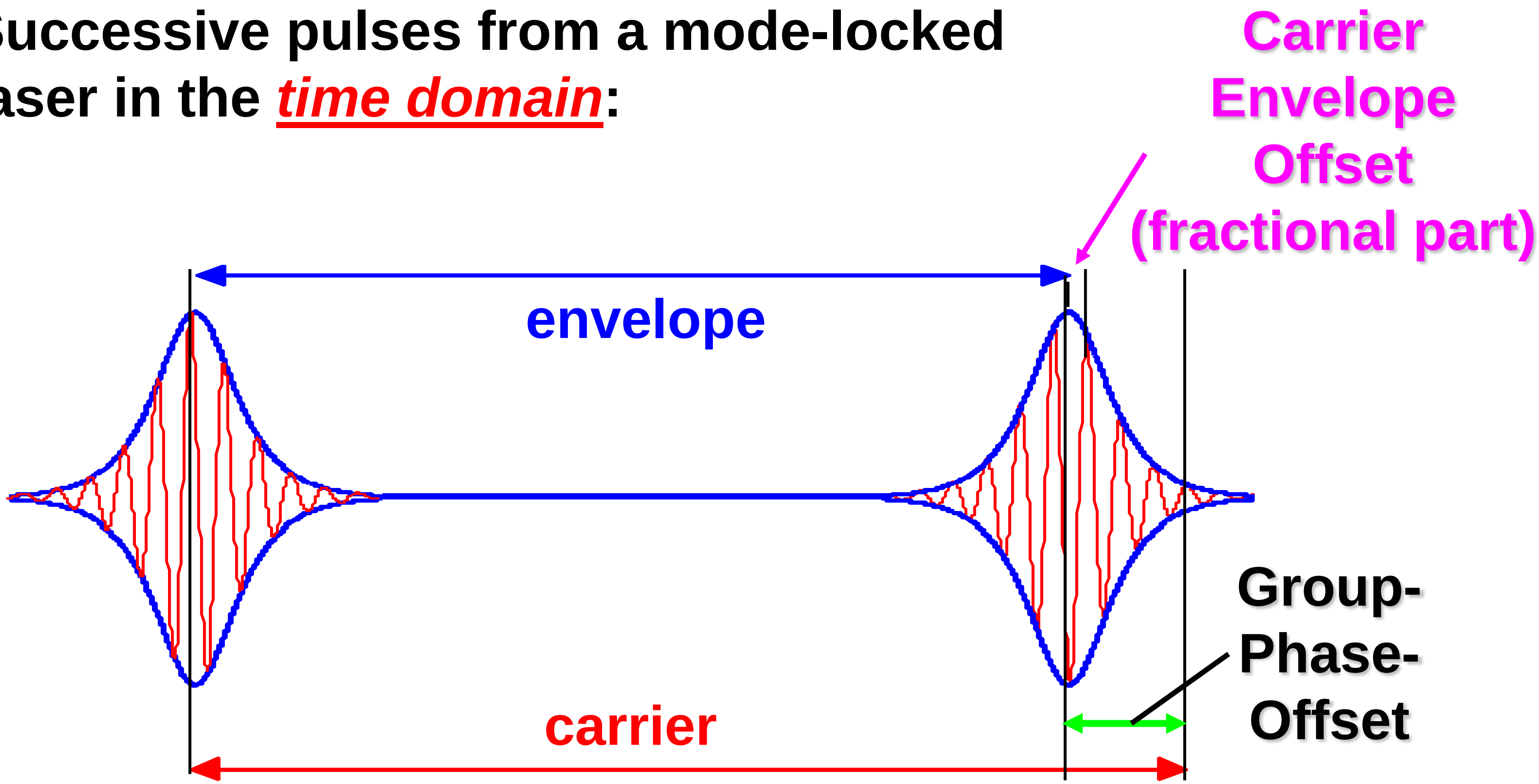


If the modulations fits
on one end of the spectrum
it does not at the other ...

How can we solve this dilemma ?

Carrier Envelope Offset (CEO)

Successive pulses from a mode-locked laser in the *time domain*:

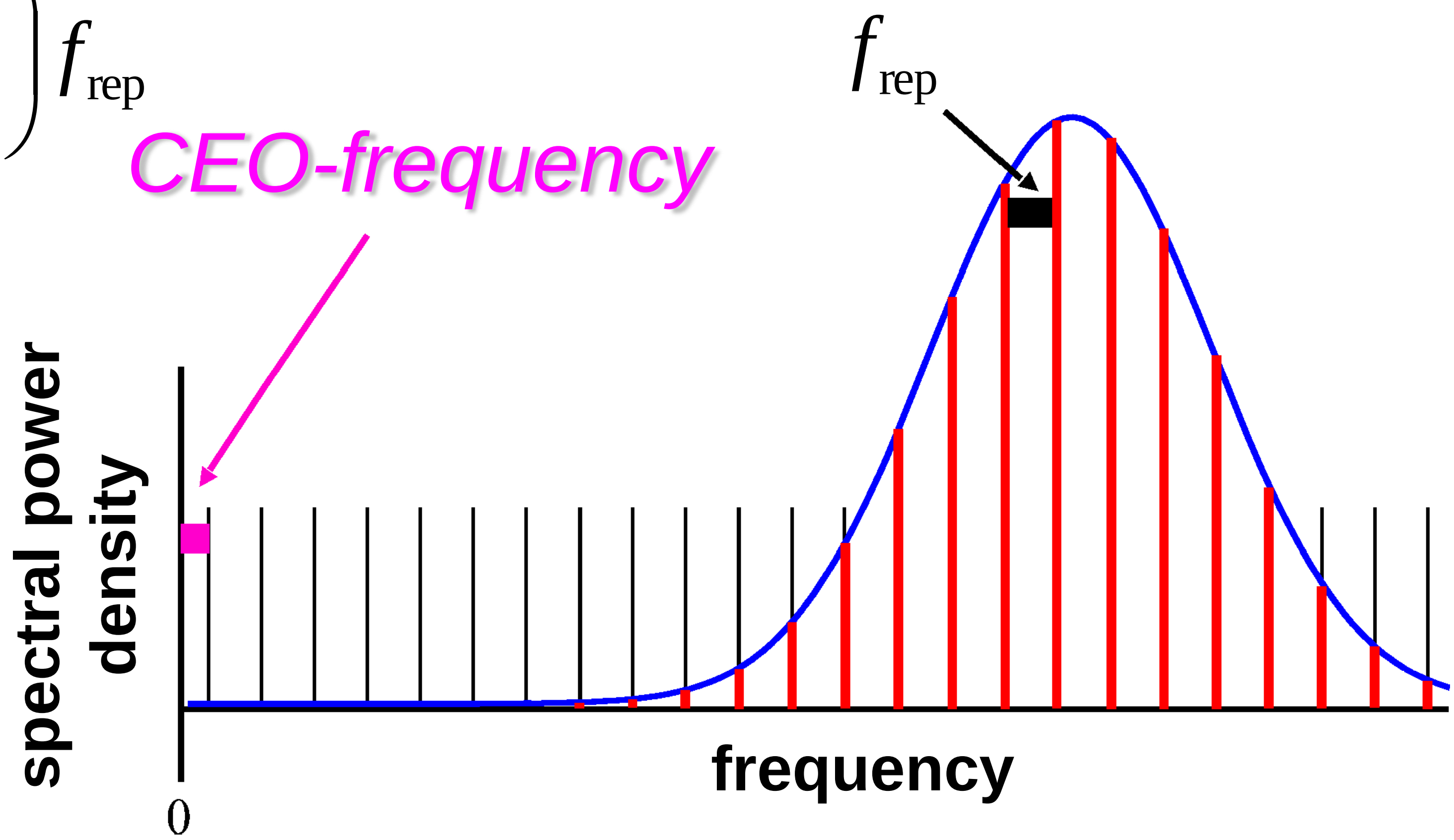


$$\varphi_{\mathrm{GPO}} = \frac{2\pi}{\lambda} L(n_{\mathrm{g}} - n)$$

Ref.: H.R. Telle et al., *Appl. Phys. B* **69**, 327 (1999)

Carrier-Envelope Offset (CEO)

$$\nu_m = \left(\frac{\varphi_{\mathrm{GPO}}}{2\pi} + m \right) f_{\mathrm{rep}}$$



mode-locked laser = optical frequency ruler

- **mode comb uniformity better than 10^{-15}**
- **otherwise rep-rate would be function of wavelength**
- **2 degrees of freedom: "translation" and "breathing"**

Ref: T.Udem et al., *Opt. Lett.* **24**, 881 (1999)

Comb parameters vs. Intracavity disp.

1. **Frequency spacing of the comb determined by the group delay of the cavity**

2. **Zero-offset (CEO) of the comb determined by group-phase offset**

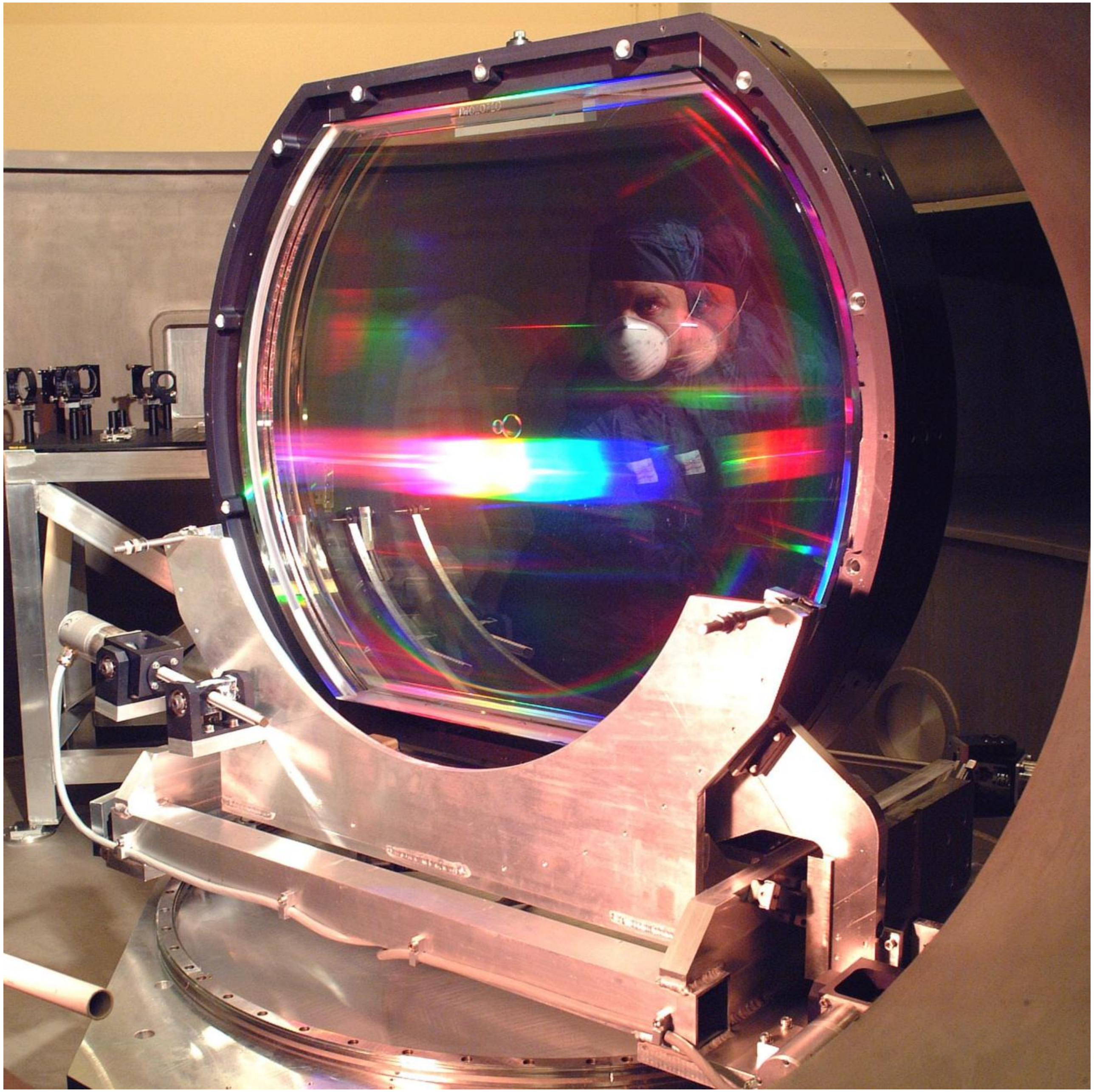
3. **All higher-order effects lead pedestal formation in pulse shaping**
   (to be counteracted by SAM)

Comb frequencies ≠ Cavity eigenfrequencies

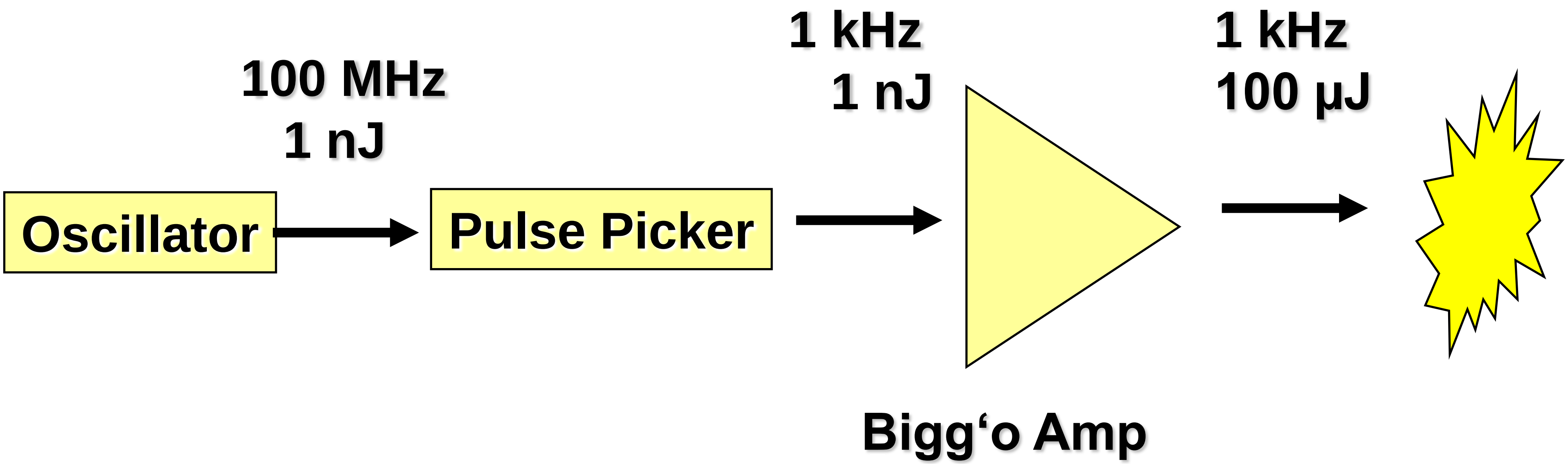
Summary oscillators

- For the generation of few-cycle pulses, a fast absorber together with soliton-like pulse shaping is required!
- SAM acts to prevent fall-back into continuum SPM fights bandwidth reduction of the gain medium
- Real absorbers are often not fast enough
- Artificial absorbers translate phase nonlinearities into amplitude nonlinearities
- On the shortest time scales, the CE phase starts to matter

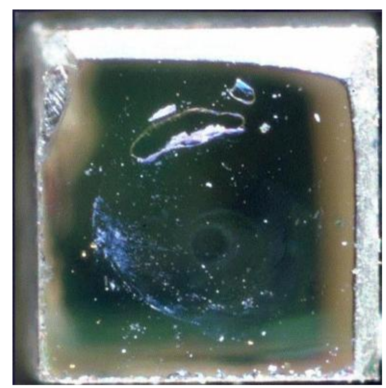
Chirped pulse amplification



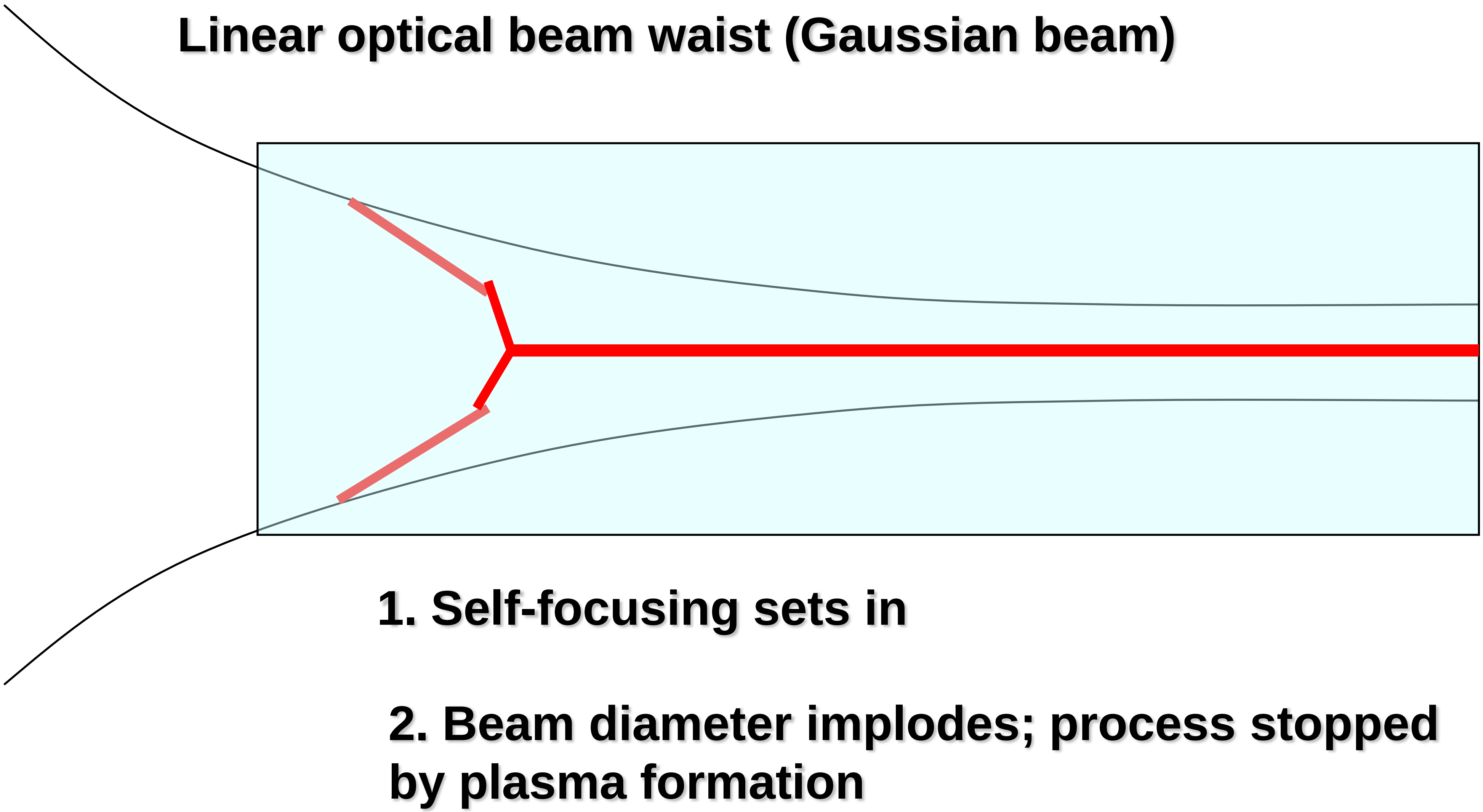
Amplification of Laser Pulses



This way, you easily fry the crystal…



Why is damage occuring ?



The B integral

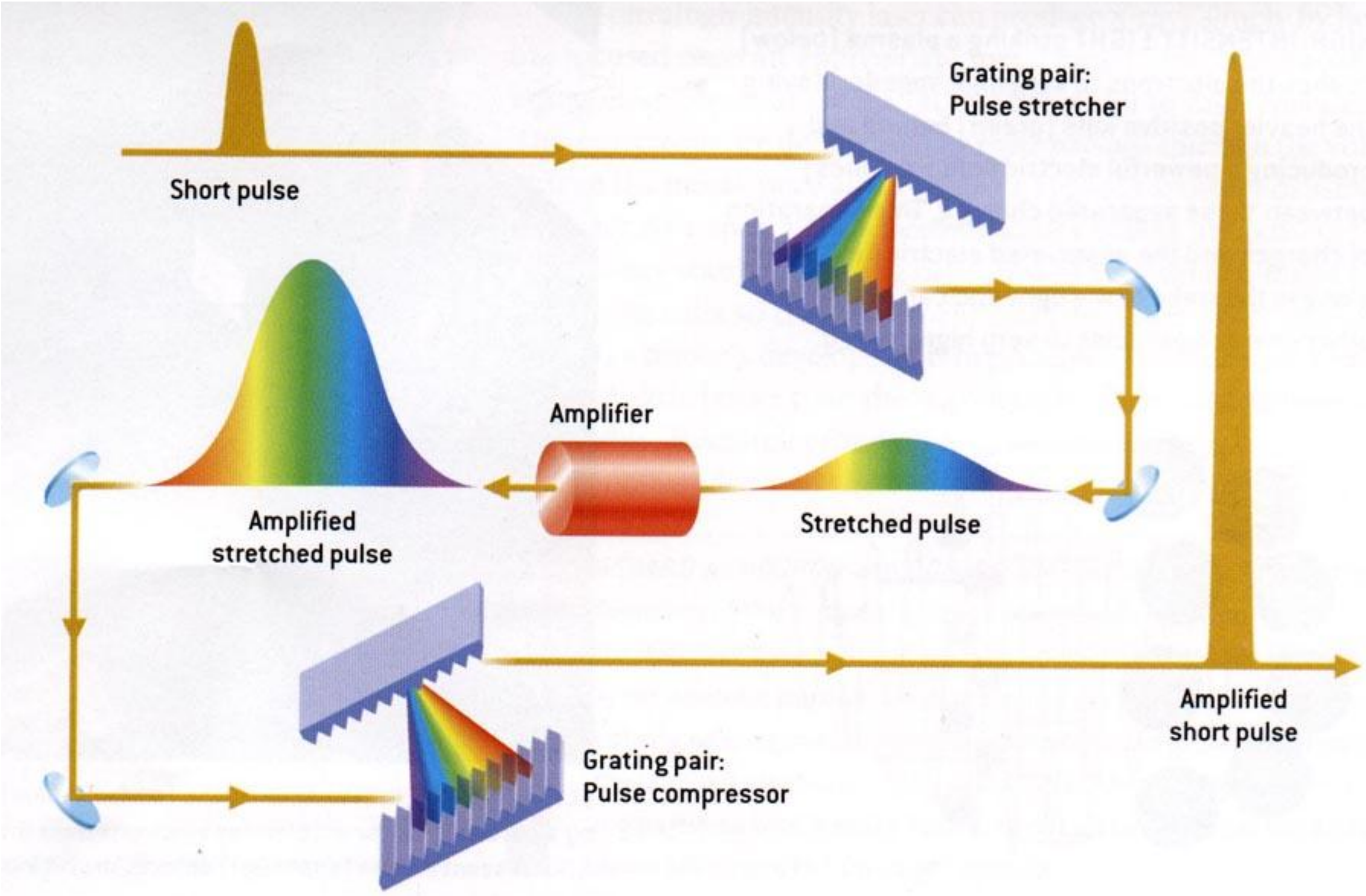
$$B = \frac{2\pi}{\lambda} \int n_2 I(z) dz$$

should be smaller than ≈2π to avoid catastrophic self-focusing

This becomes limiting when exceeding pulse energies of 1 µJ with sub-100 fs pulse duration !

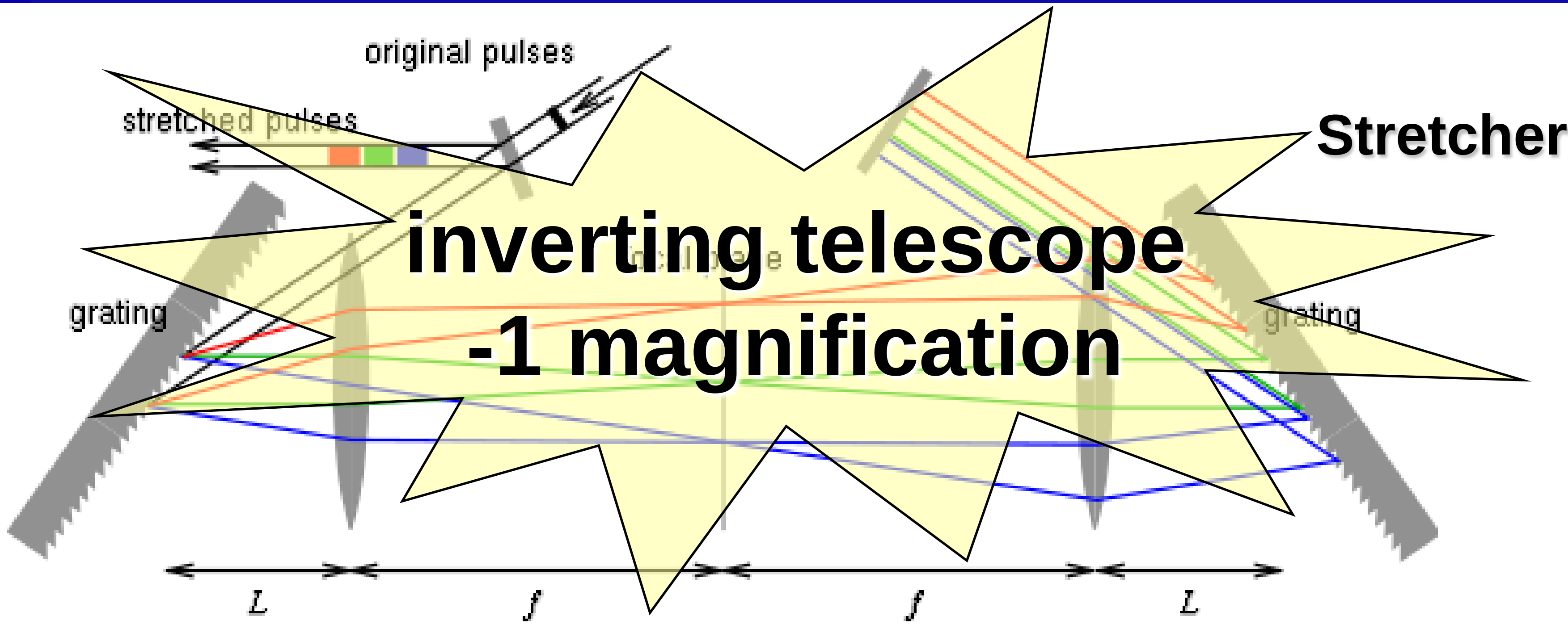
Two possible solutions: ***Scale up diameter or***
increase pulse duration

Chirped pulse amplification

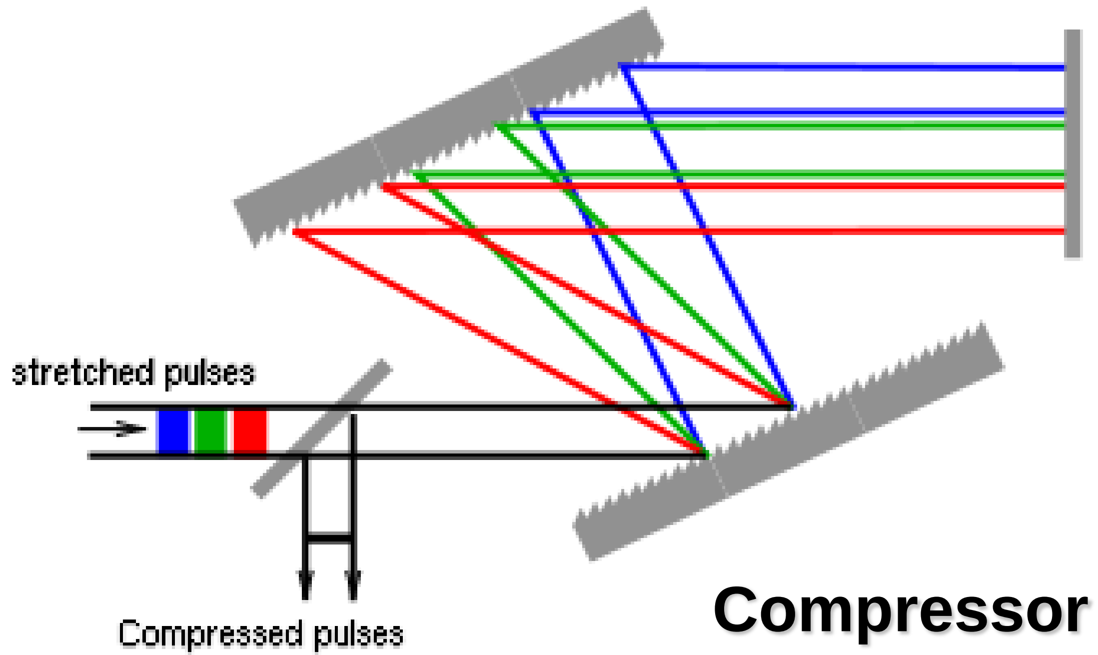


Strickland & Mourou, Opt. Commun. **56**, 219 (1985)

Stretcher - compressor



Important: You want to have two elements with large dispersion that exactly cancel out!



CPA performance and limitations

Stretching from 100fs to about 1ns (10000x)

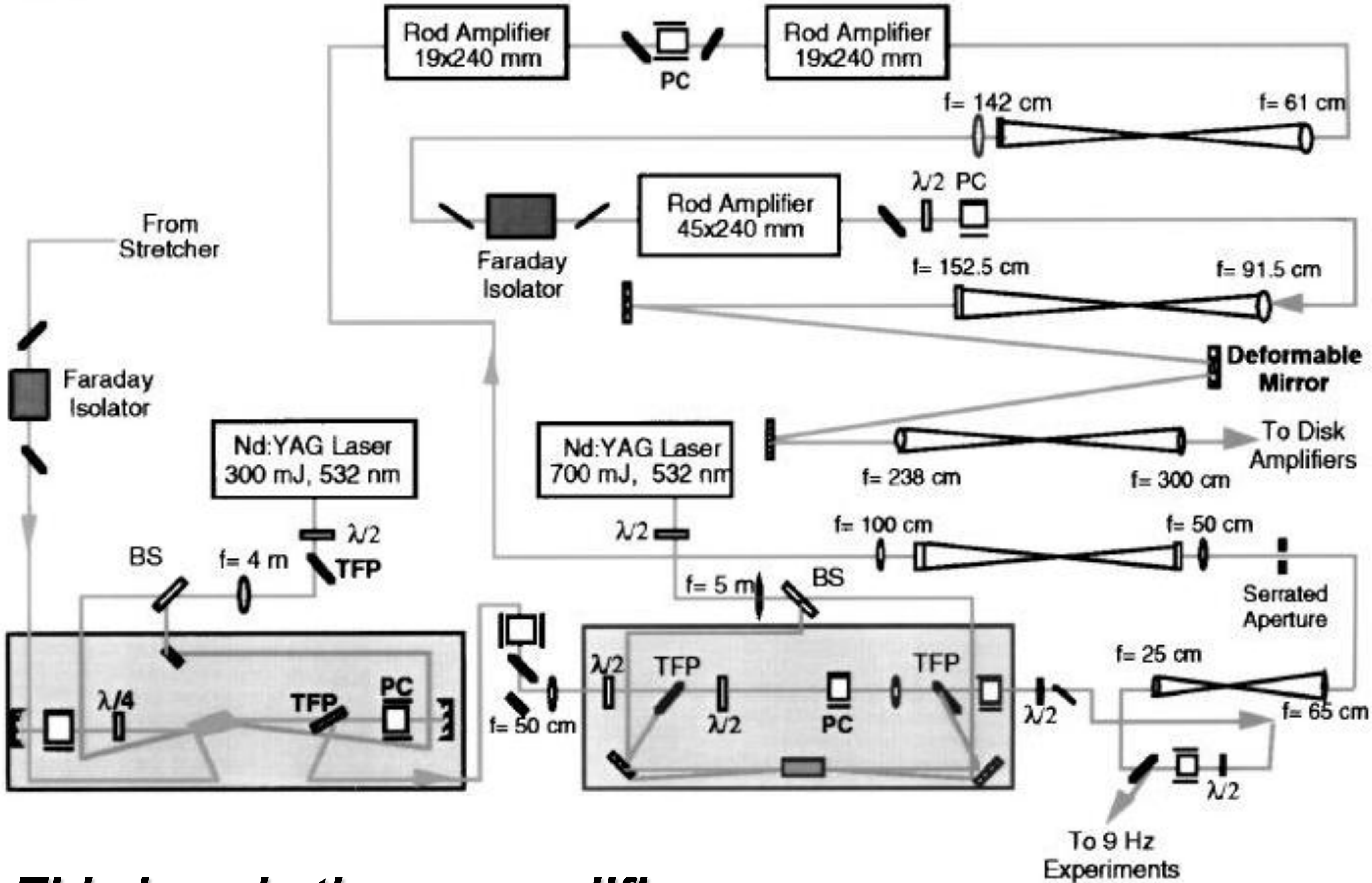
Peak powers of several 100 TW possible

Pulse energies of several Joules

Focusing >10^{20} W/cm^2

Bottleneck: Grating size

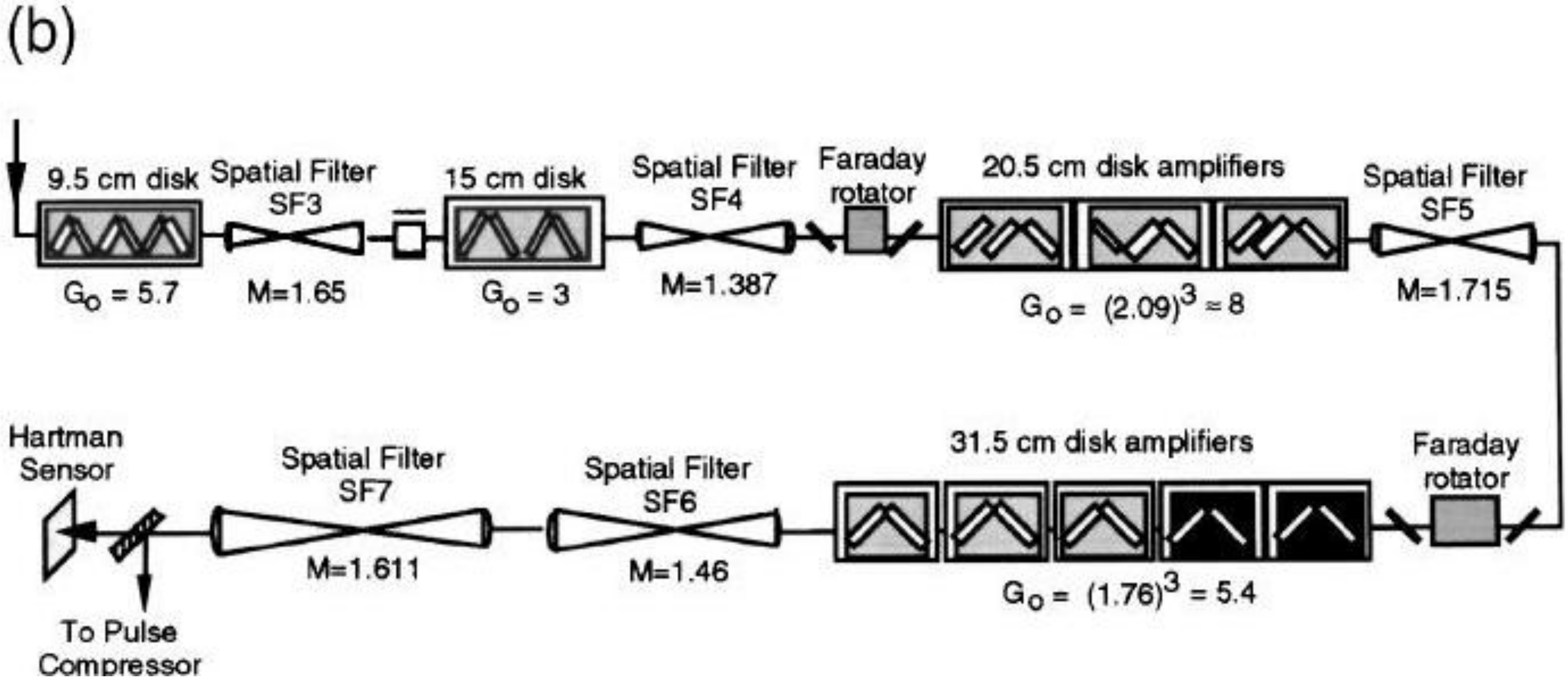
The pre-amp (front end) of a PW laser



This is only the preamplifier…

Perry et al., OL **24** 160 (1999); 660J, 450fs, >7x10^{20} W/cm^2

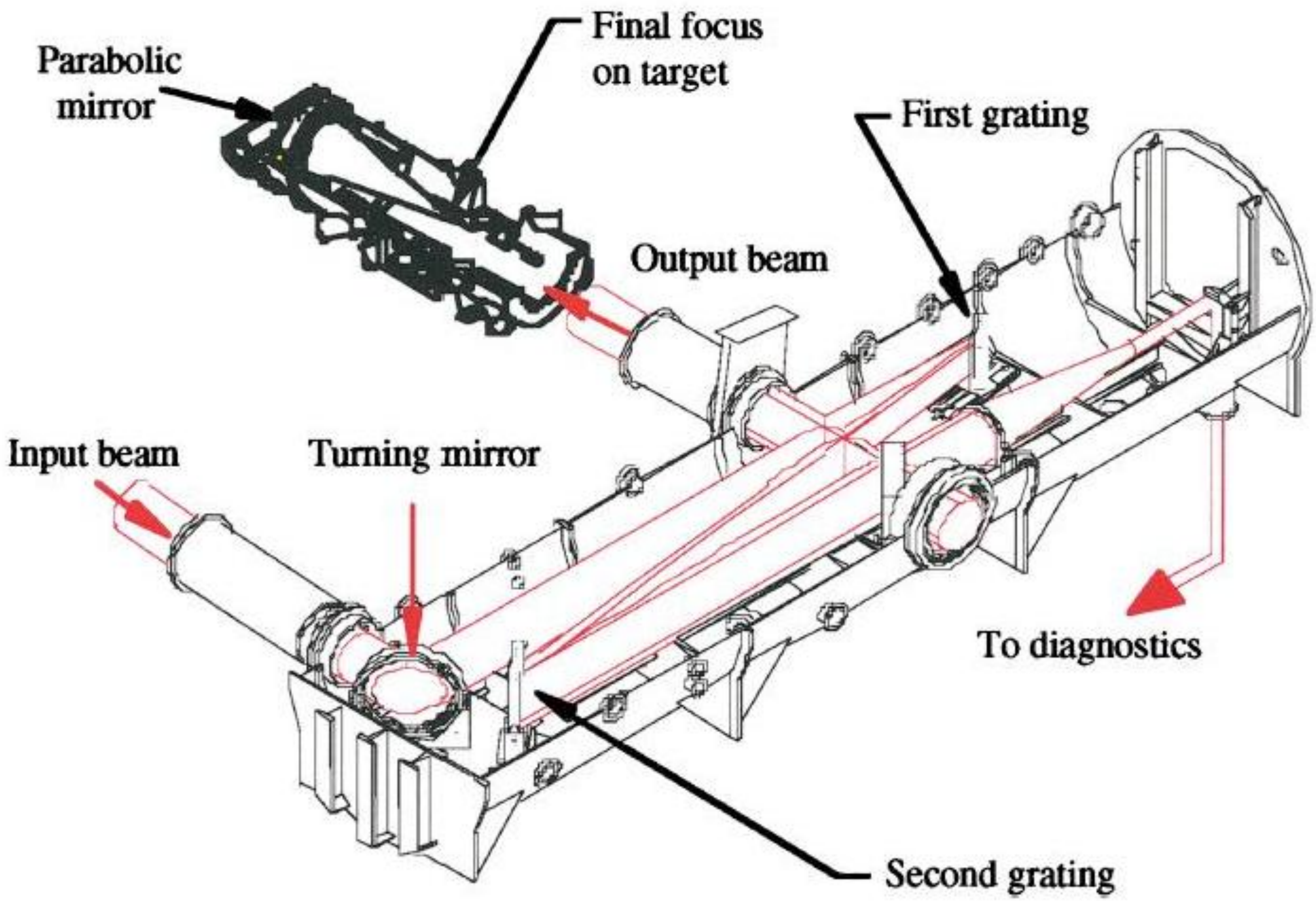
The power amplifier



You will need this to get some real power out…

Perry et al., OL **24** 160 (1999); 660J, 450fs, >7×10^{20} W/cm^2

Compressor chamber of PW laser

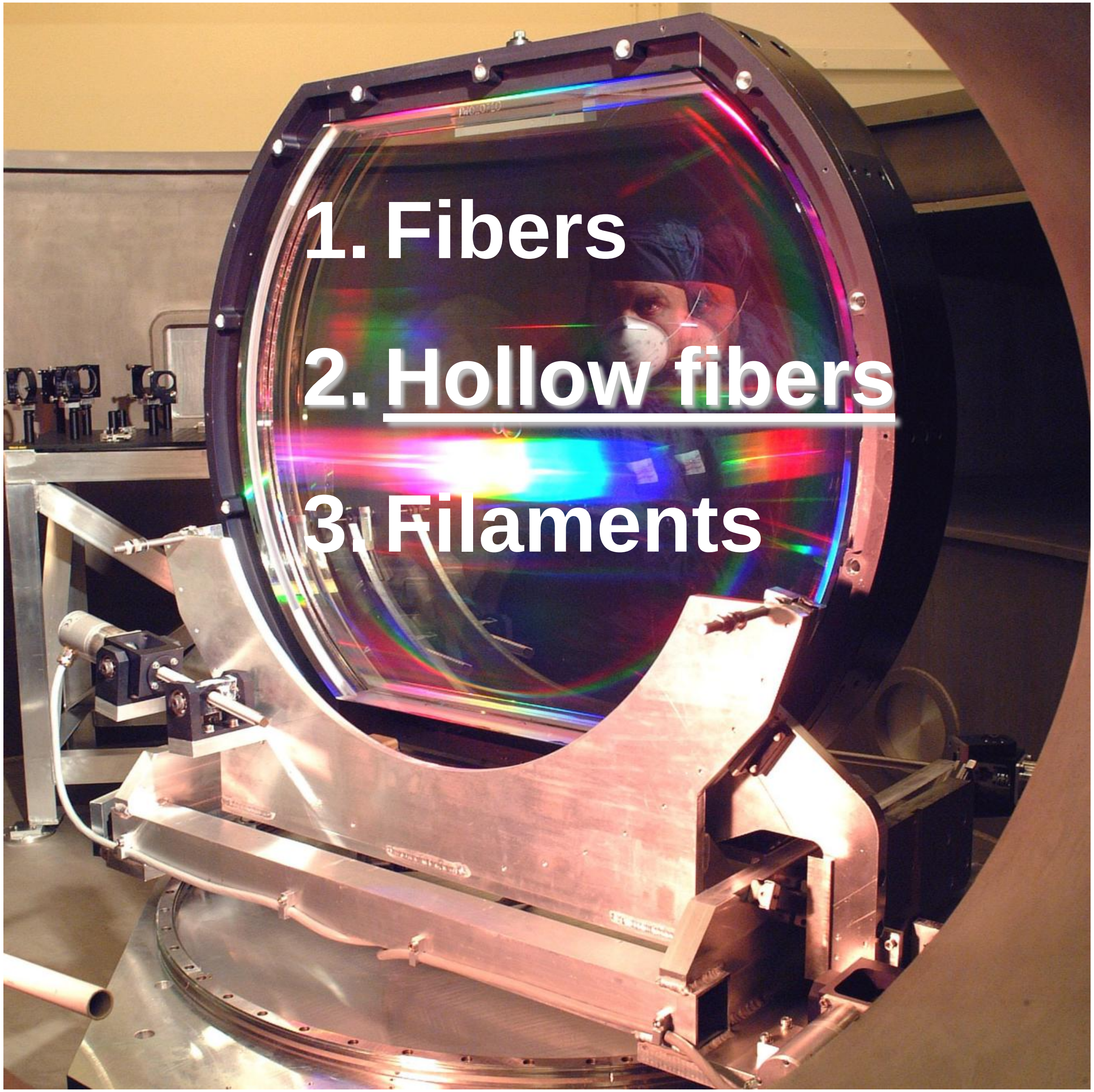


The compressor chamber looks like a small submarine…

Conclusions

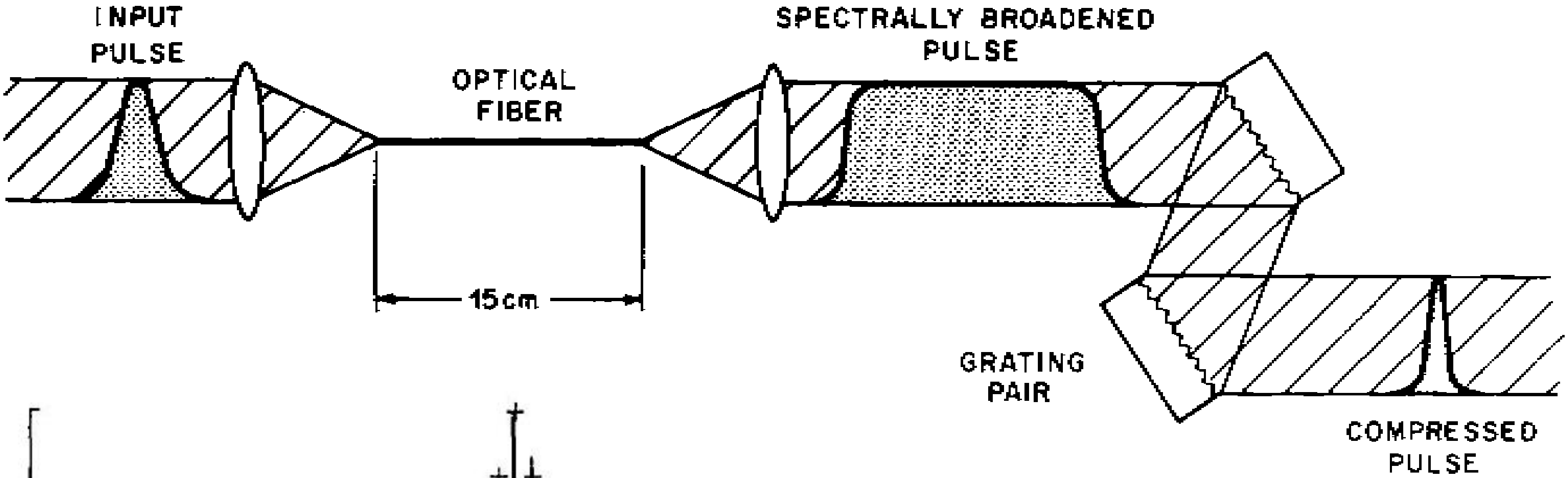
- **CPA indispensable for generating short energetic pulses**
- **Dispersion management with stretcher / compressor arrangements**
- **B-integral needs to be smaller than a few π**
- **Intensities of 10^{20} W/cm^2 have been demonstrated, exceeding inner-atomic binding forces by orders of magnitude**

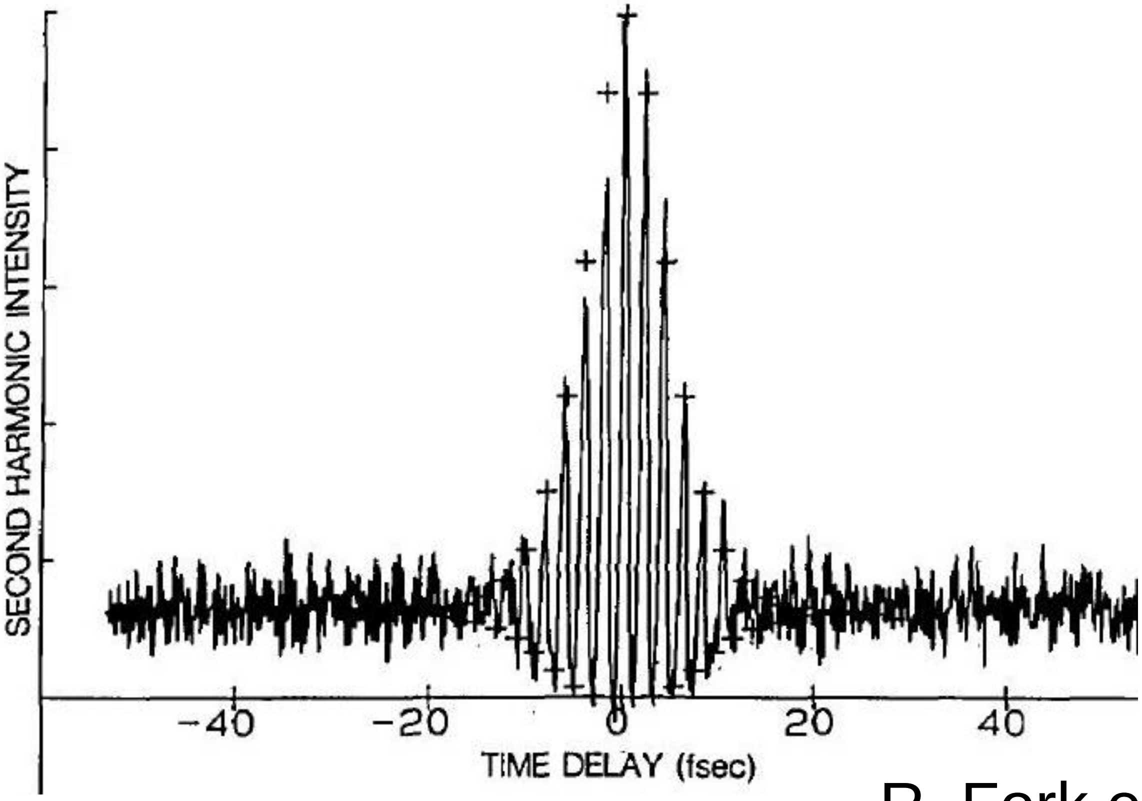
Compression techniques



1. Fibers
2. <u>Hollow fibers</u>
3. Filaments

Traditional fiber – grating compressor





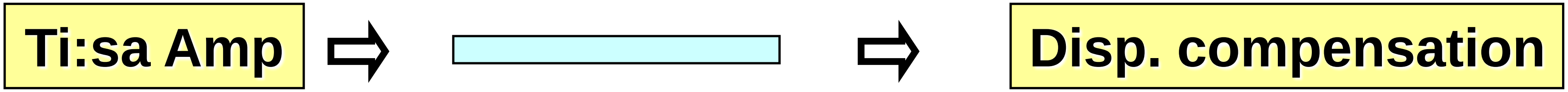
Combined prism and grating compressor

amplified dye laser

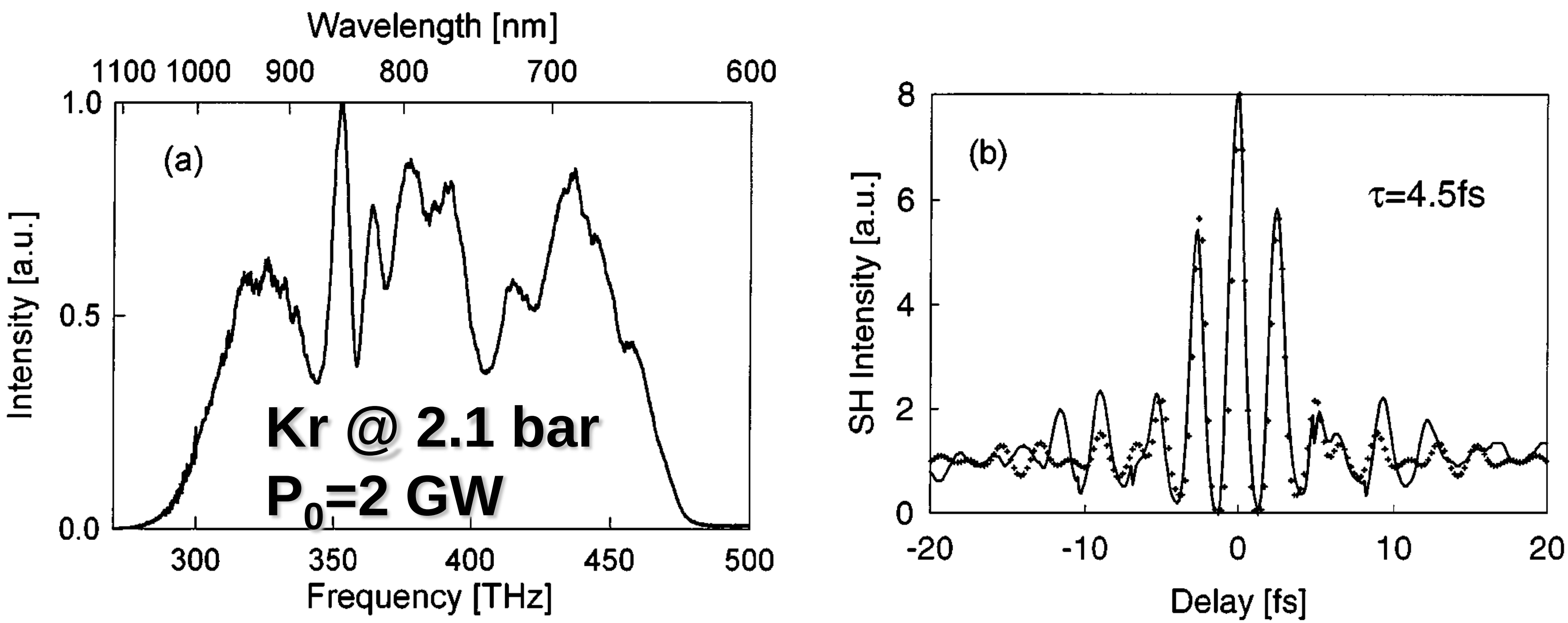
long-standing world record of 6 fs pulse duration

R. Fork et al., OL **12**, 483 (1987)

The hollow fiber compressor



hollow waveguide filled with noble gas (neon, argon, krypton)



Ref.: M. Nisoli et al., Opt. Lett. **22**, 522 (1997)

The hollow fiber compressor

Noble gases are most indestructible medium
He: ionization energy 21.4 eV

Nonlinearity adapted to high peak powers
of CPA laser systems (several GW)

Focused field strength can exceed binding
forces for any given atom!

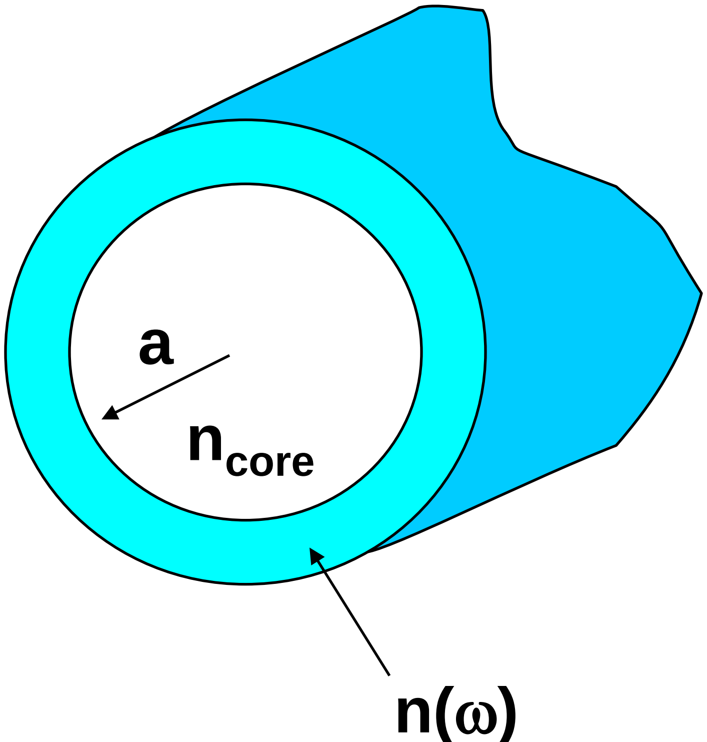
Hollow geometry provides quasi-guiding
and confines the beam to a small interaction
volume

Avoid ionization of gas, no other nonlinearities,
simple dispersion

Hollow fiber compressor

Wave equation:

$$\nabla^2 E - \frac{\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$



Solutions are of the form

$$E_{l,\nu}(\vec{r},t) = \mathrm{Re}\{\frac{\exp[i(\omega t - \beta z)]}{2} \times$$

$$[\tilde{E}_{l,\nu}(r)\exp(i\nu\vartheta) + \tilde{E}_{l,-\nu}(r)\exp(-i\nu\vartheta)]\}$$

Hybrid modes $EH_{\nu m}$ with radial behavior related to Bessel functions of 1st order $J_\nu(r)$

β=propagation constant

ν,m=integer, specifying mode

l={r,z,ϑ}

Ref.: S. de Silvestri et al., „Few-cycle-pulses by external compression“ in: Few-Cycle Laser Pulse Generation and its application, F.X.Kärtner ed.

Hollow fiber compressor

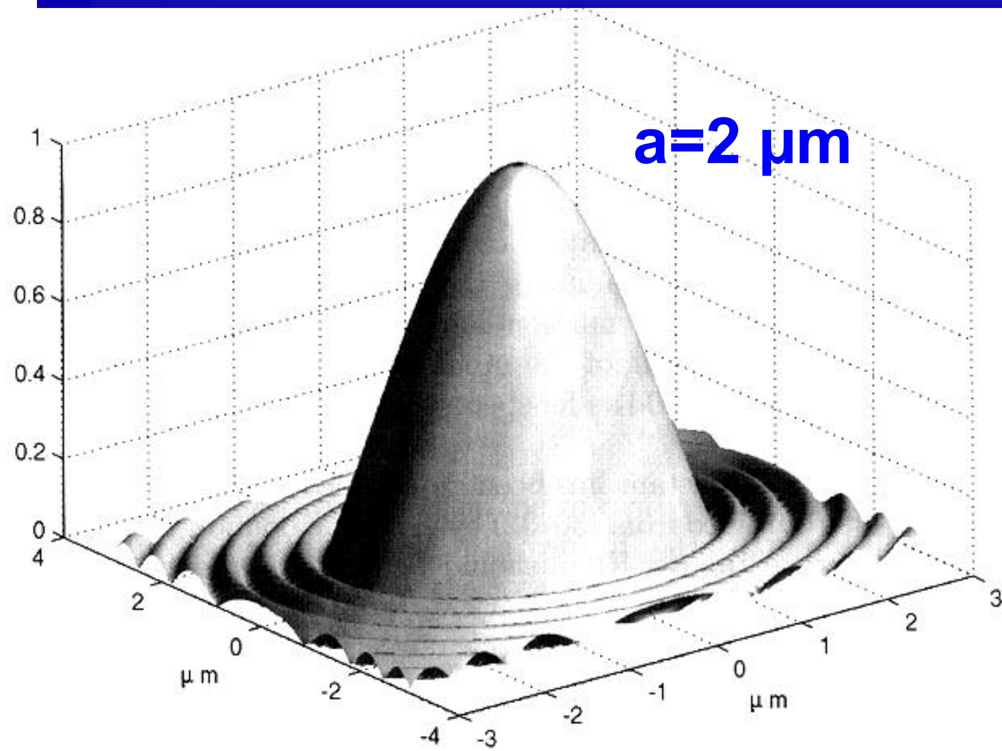


Fig. 5. EH_{11} mode profile calculated for capillary inner radius $a = 2\,\mu$m. In calculations, it was assumed that $n_{core} = 1$, $n_{clad} = 1.45$

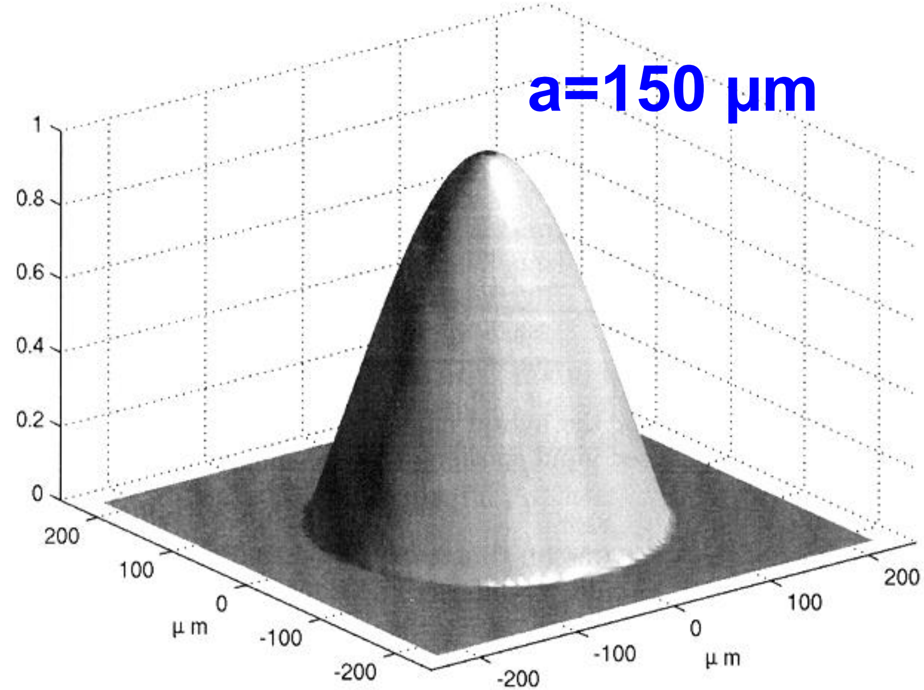
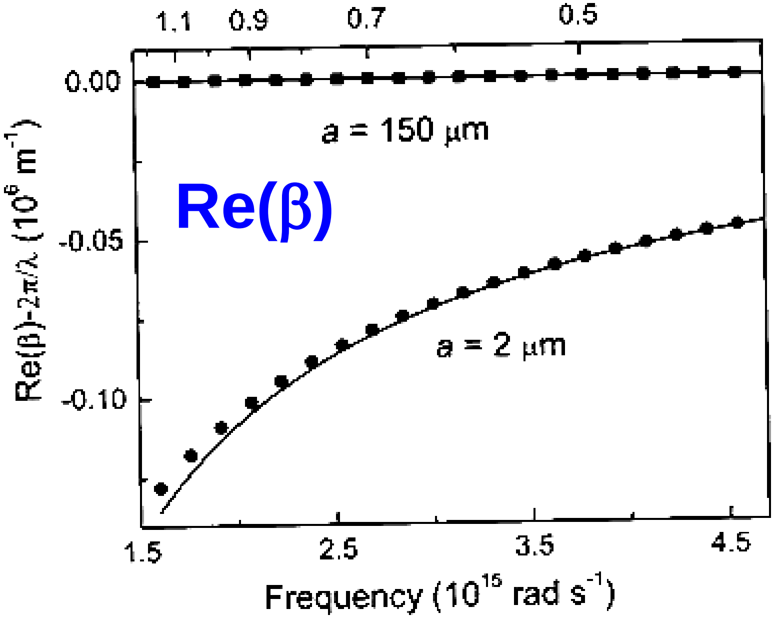


Fig. 6. EH_{11} mode profile calculated for a capillary inner radius $a = 150\,\mu$ the calculations, it was assumed that $n_{core} = 1$, $n_{clad} = 1.45$

Ref.: S. de Silvestri et al., „Few-cycle-pulses by external compression“
in: Few-Cycle Laser Pulse Generation and its application, F.X.Kärtner ed.

Hollow fiber compressor



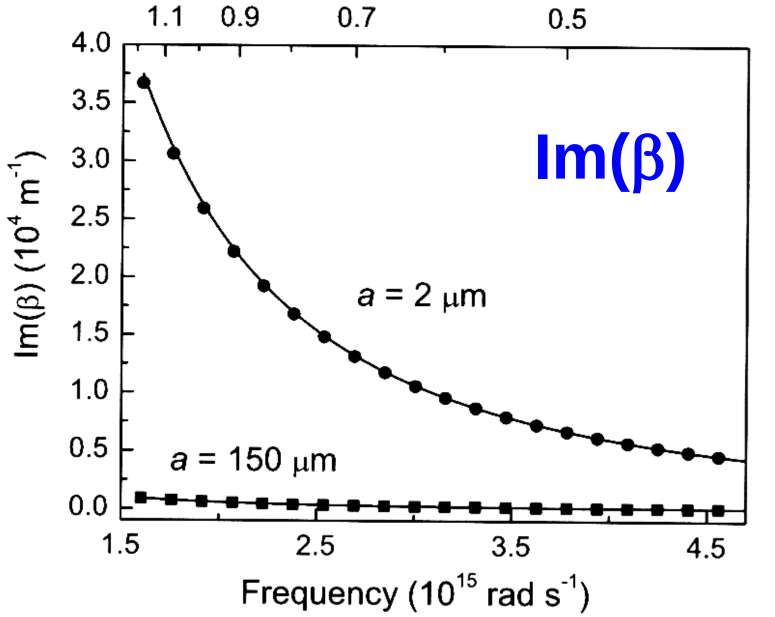


Fig. 2. Difference between the real part of the propagation constant β of the EH_{11} mode and the free propagation constant $2\pi/\lambda$ as a function of frequency for $a = 2\,\mu m$ (*solid dots*) and for $a = 150\,\mu m$ (*solid squares*). *Solid lines*: analytical approximation of the same quantity. In both cases, it was assumed that $n_{core} = 1$, $n_{clad} = 1.45$

u_m = Bessel zeros (2.4048, 5.52008, 8.65373...)

$$\beta(\omega) = \frac{\omega n_{core}(\omega)}{c}\left[1 - \frac{1}{2}\left(\frac{u_m c}{\omega n_{core}(\omega) a}\right)^2\right] + \frac{i}{a^3}\left(\frac{u_m c}{\omega n_{core}(\omega)}\right)^2 \frac{n^2(\omega)+1}{\sqrt{n^2(\omega)-1}}, \quad (33)$$

Ref.: S. de Silvestri et al., „Few-cycle-pulses by external compression“ in: Few-Cycle Laser Pulse Generation and its application, F.X.Kärtner ed.

Hollow fiber compressor

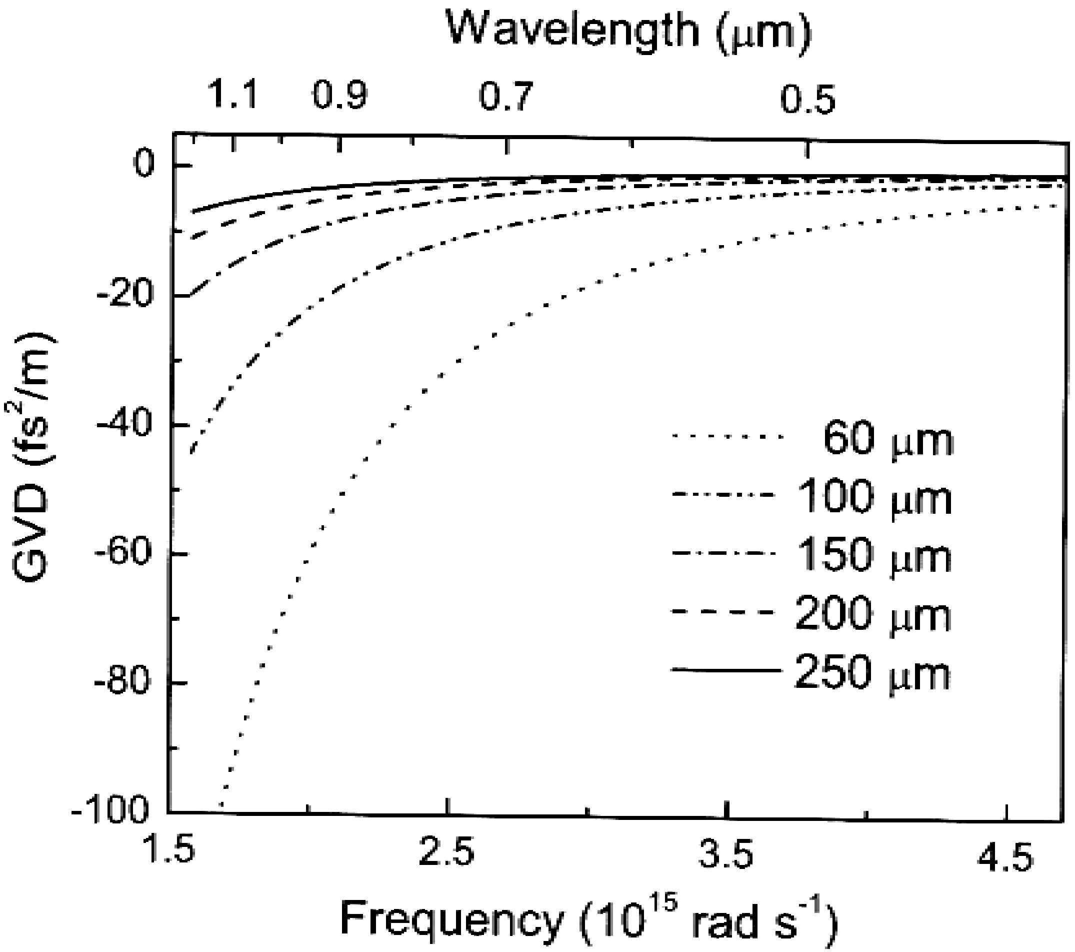


Fig. 8. Group velocity dispersion, GVD, of the EH_{11} mode as a function of frequency for various fiber radii

Ref.: S. de Silvestri et al., „Few-cycle-pulses by external compression“
in: Few-Cycle Laser Pulse Generation and its application, F.X.Kärtner ed.

Hollow fiber compressor

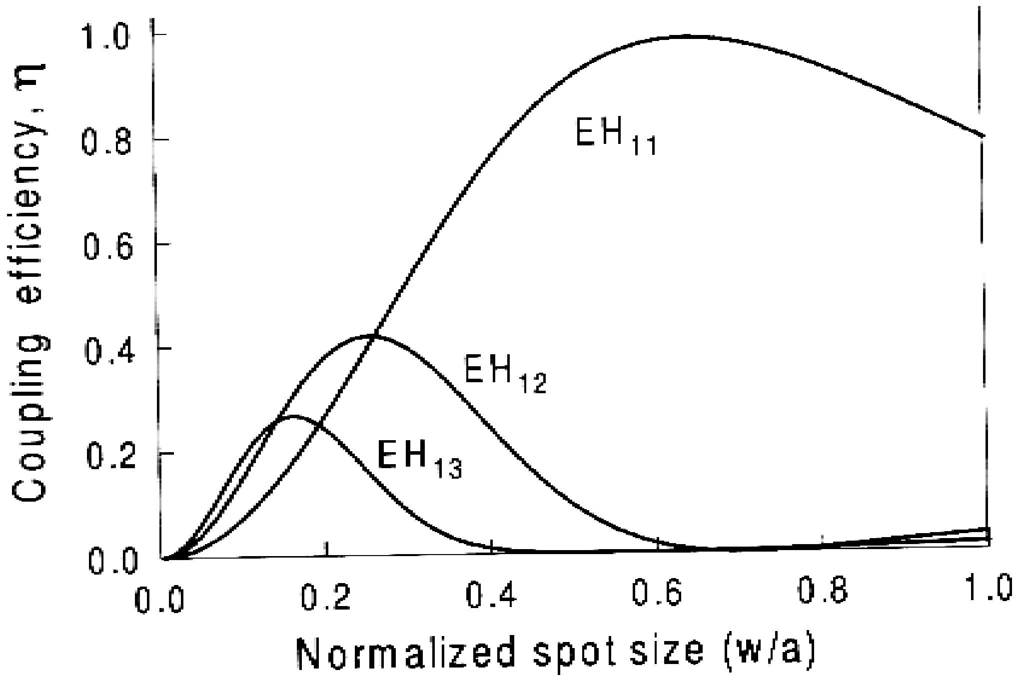
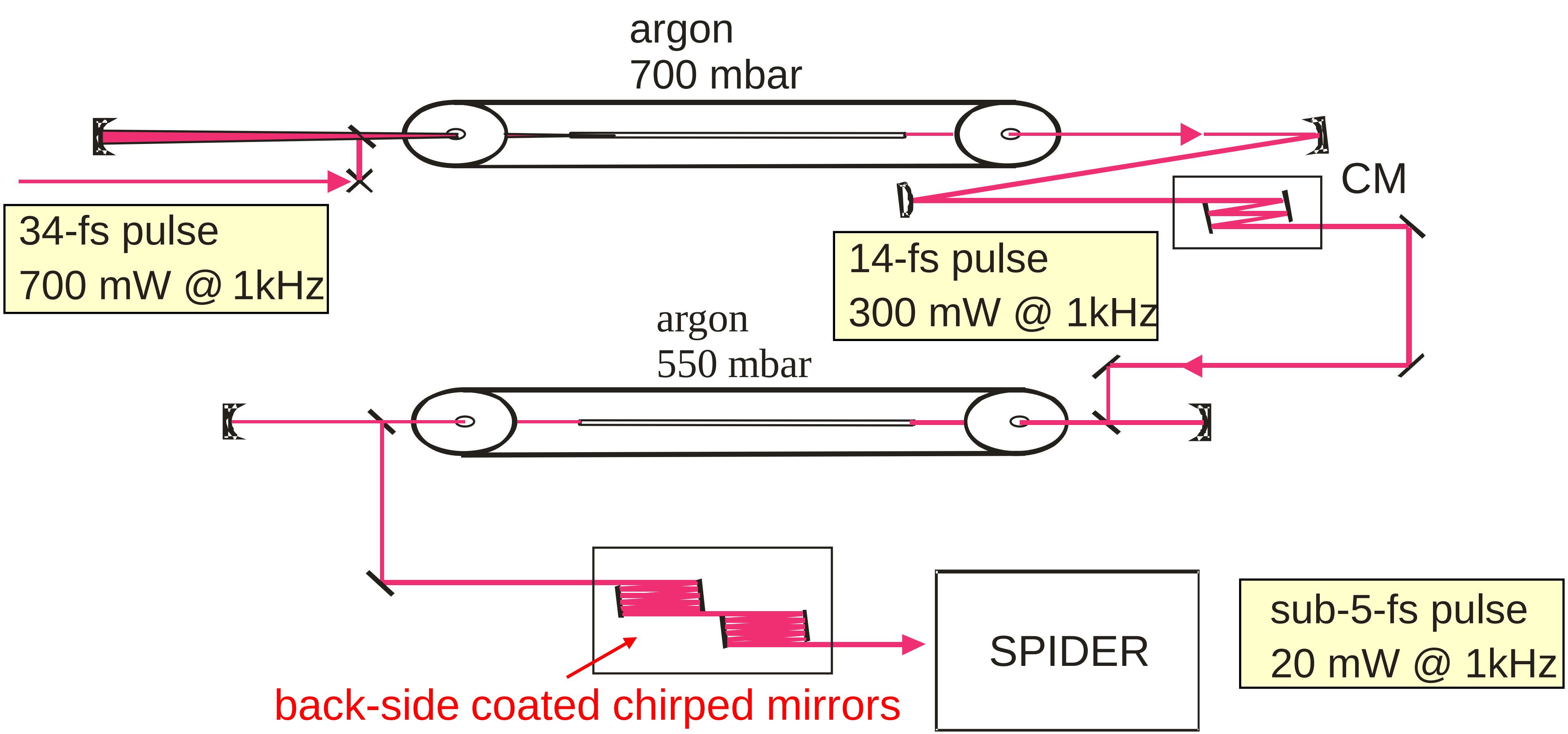


Fig. 7. Coupling efficiency η for the hybrid modes EH_{1m} excited by a Gaussian beam as a function of normalized input spot size w/a

$$\eta = \frac{4\left[\int r J_0(u_{1m} r/a) \exp(-r^2/w_l^2)\, \mathrm{d}r\right]^2}{w_l^2 \int r J_0^2(u_{1m} r/a)\, \mathrm{d}r} .$$

Ref.: S. de Silvestri et al., „Few-cycle-pulses by external compression“ in: Few-Cycle Laser Pulse Generation and its application, F.X.Kärtner ed.

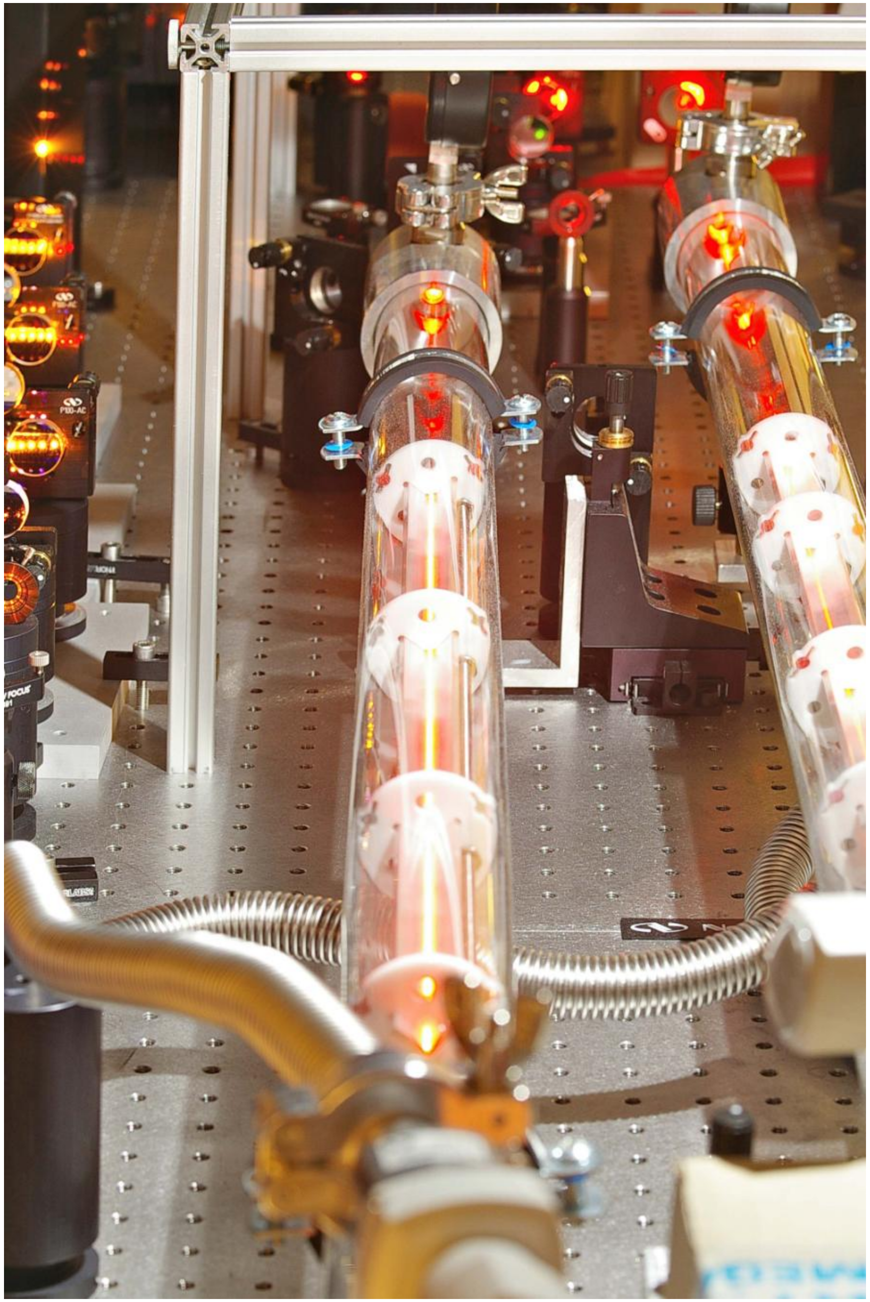
Our double-stage compressor at MBI



(dispersion compensation range: 520-1000 nm)

G. Sansone, G. Steinmeyer et al., Appl. Phys. B **78**, 551 (2004).
N. Matuschek et.al., Appl. Phys. B **71**, 509 (2000).
G. Stibenz and G. Steinmeyer, Appl. Phys. B, in press

Hollow fiber compressor

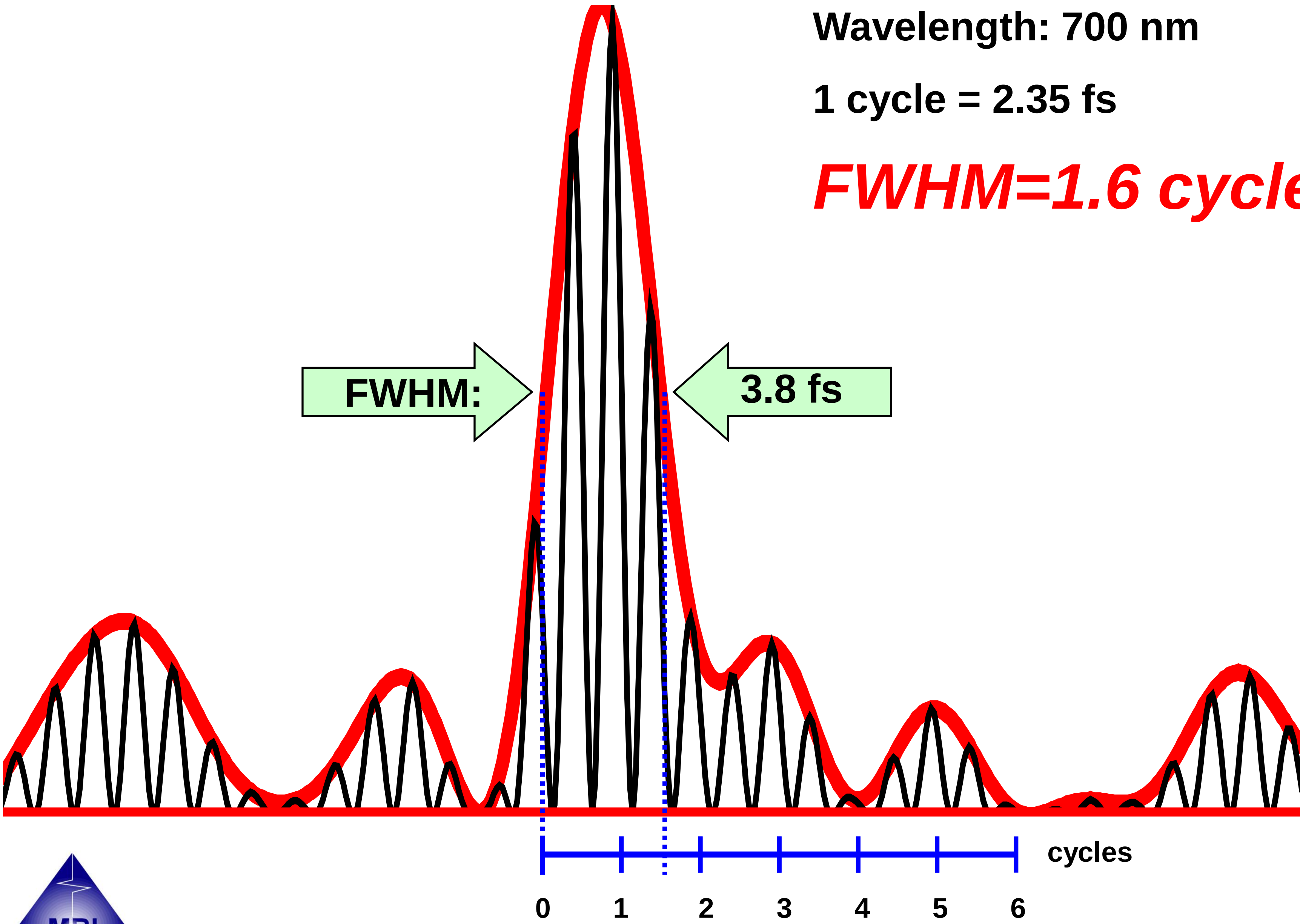


Shortest pulses with chirped mirrors

Wavelength: 700 nm

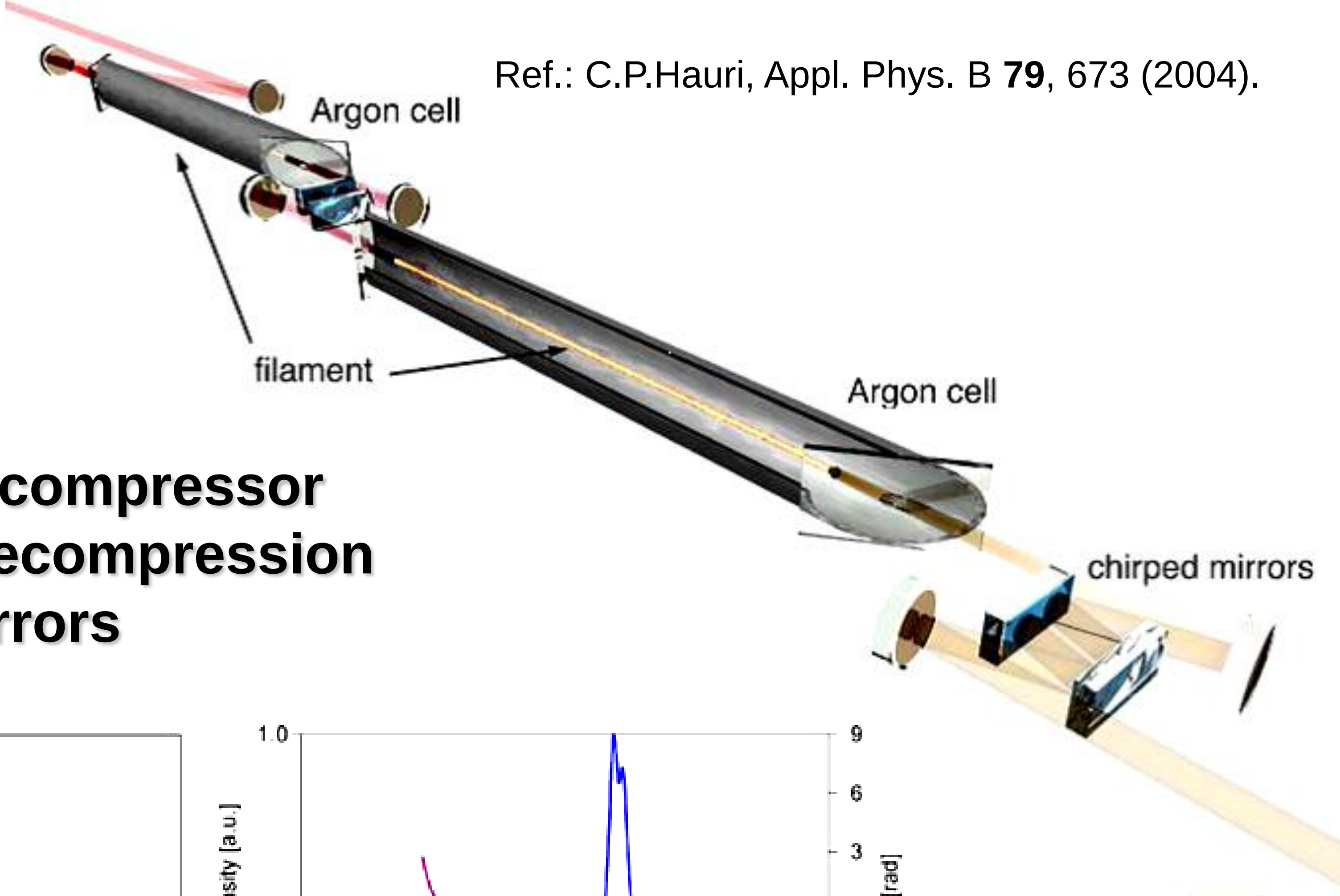
1 cycle = 2.35 fs

FWHM=1.6 cycles

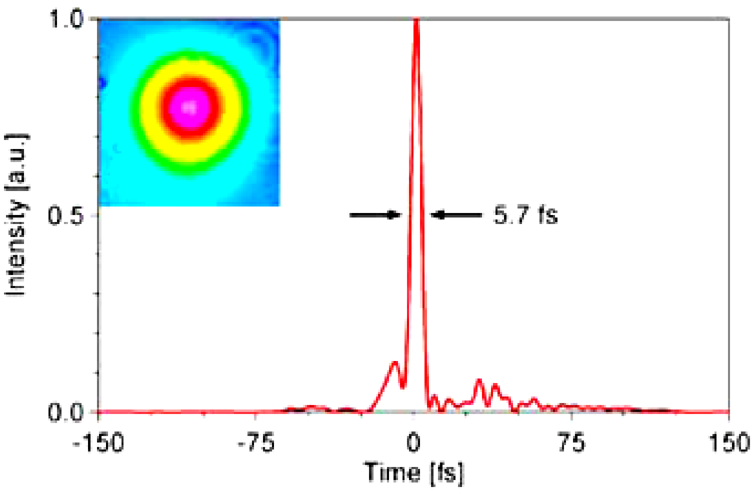


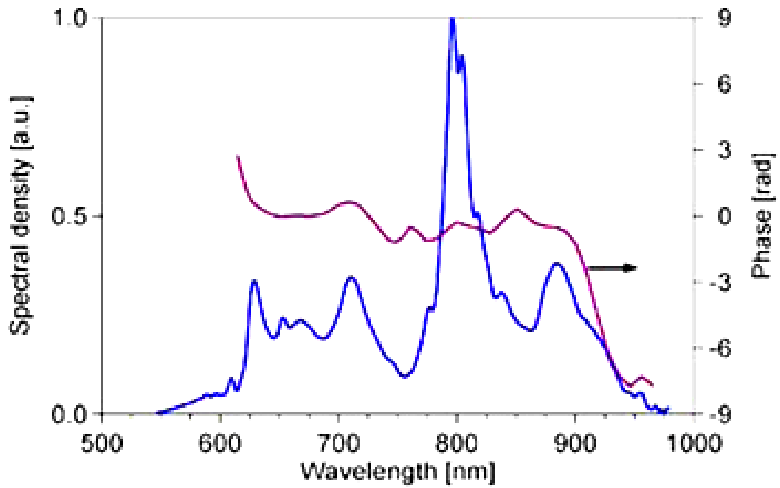
The filament compressor

Ref.: C.P.Hauri, Appl. Phys. B **79**, 673 (2004).

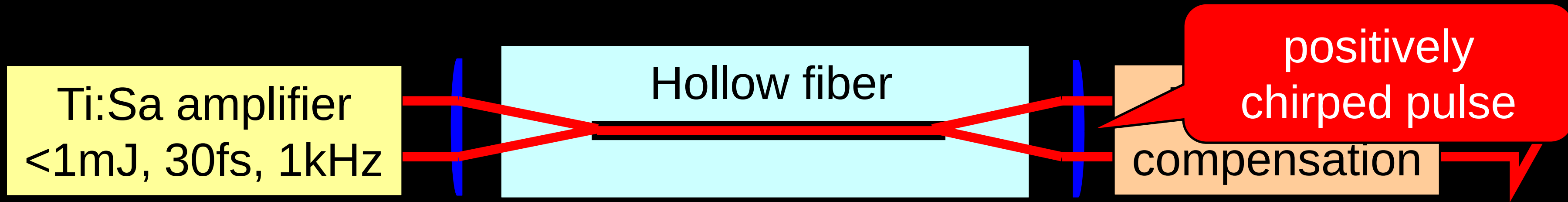


Double stage compressor
Subsequent recompression w/ chirped mirrors

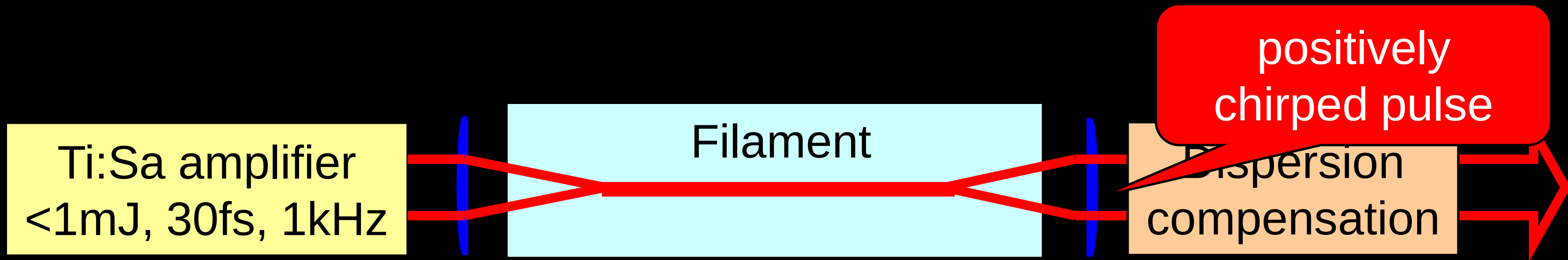




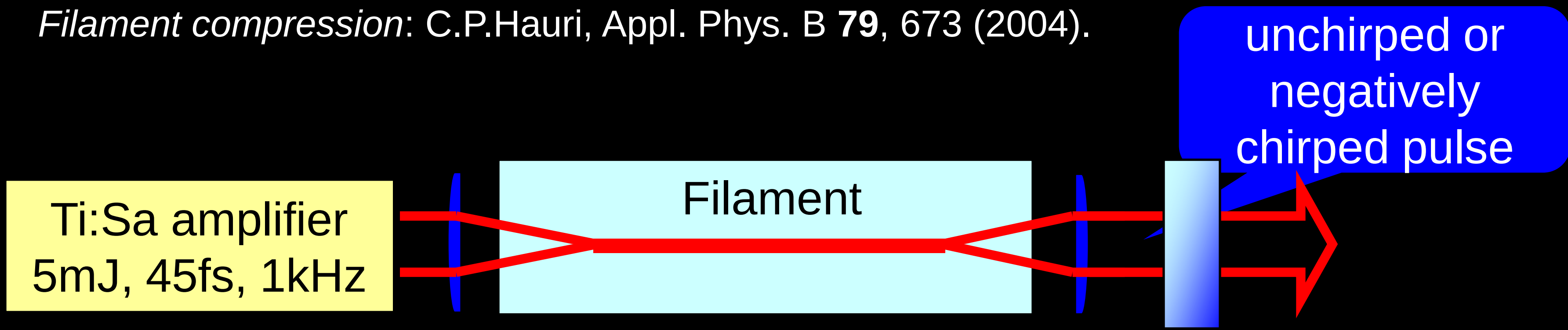
Compressing amplified pulses



Hollow fiber compressor: M.Nisoli et al., Opt. Lett. **22**, 522 (1997).



Filament compression: C.P.Hauri, Appl. Phys. B **79**, 673 (2004).



Self-compression: G. Stibenz et al., Opt. Lett. **31**, 274 (2006).

Dispersion compensation by 1.3 mm fused silica

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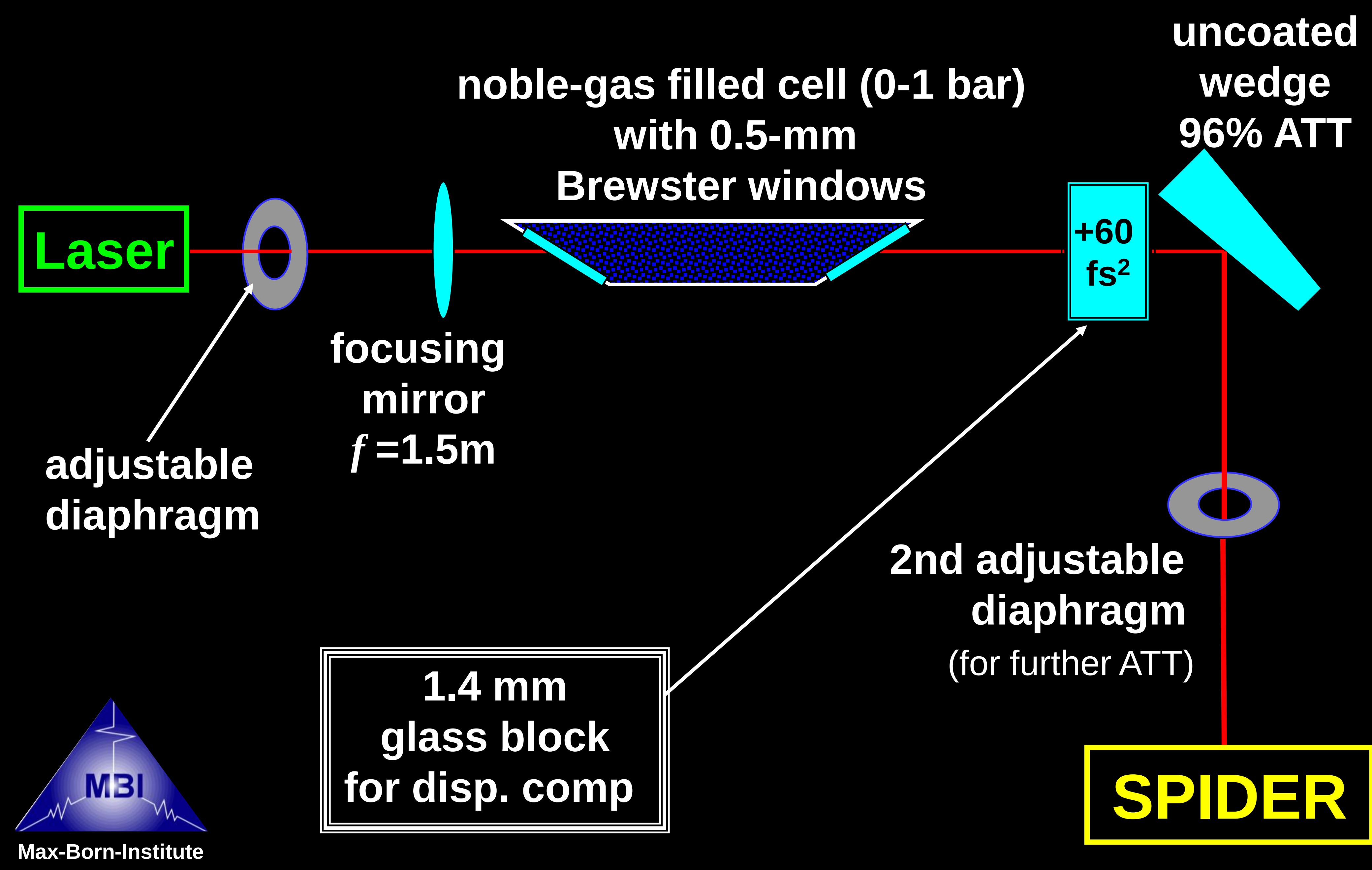
The set-up


uncoated
wedge
96% ATT
noble-gas filled cell (0-1 bar)
with 0.5-mm
Brewster windows
Laser
+60
fs^2
focusing
mirror
f =1.5m
adjustable
diaphragm
2nd adjustable
diaphragm
(for further ATT)
1.4 mm
glass block
for disp. comp
SPIDER
MBI
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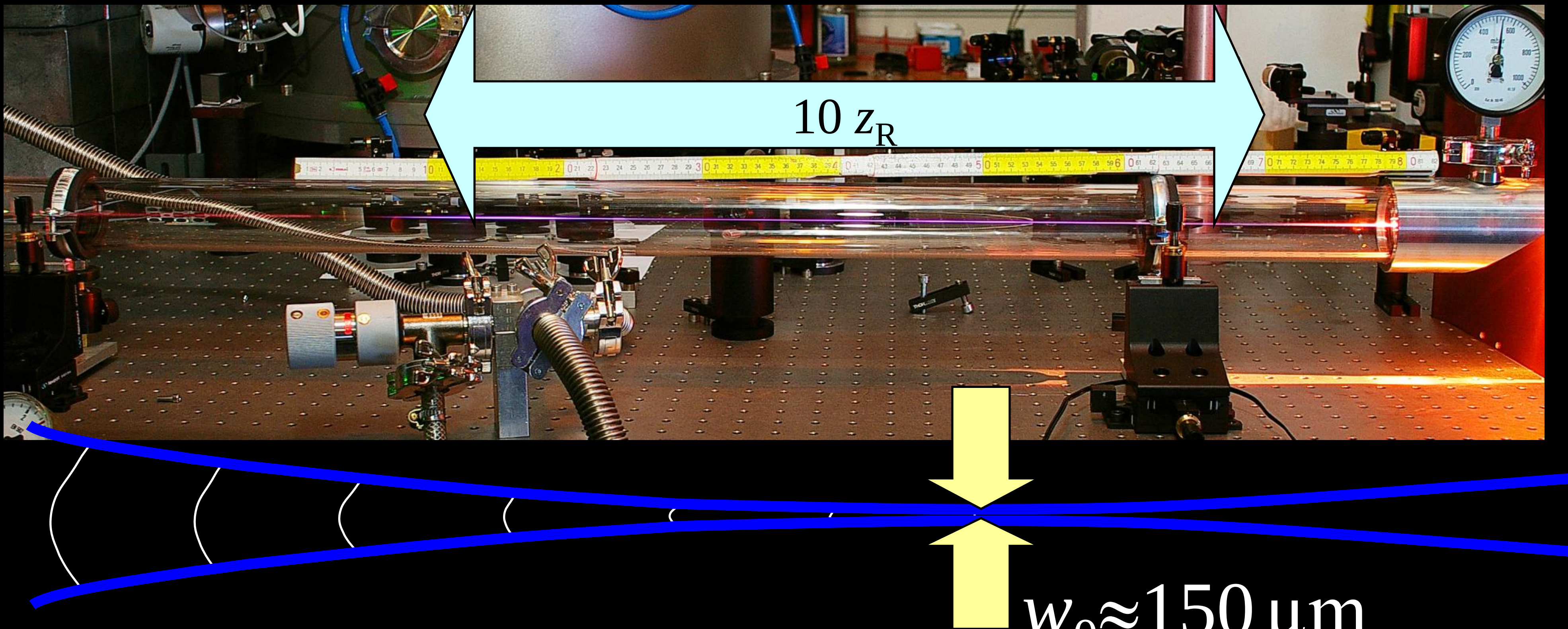
Parameters



$w_0 \approx \underline{150\,\mu m}$

$z_R = \pi w_0^2/\lambda \approx \underline{8\ cm}$

$P = 2\text{-}5\ P_{cr}$

$N_e = 0.1 \ldots 1\% \approx \underline{10^{16} - 10^{17}\ cm^{-3}}$

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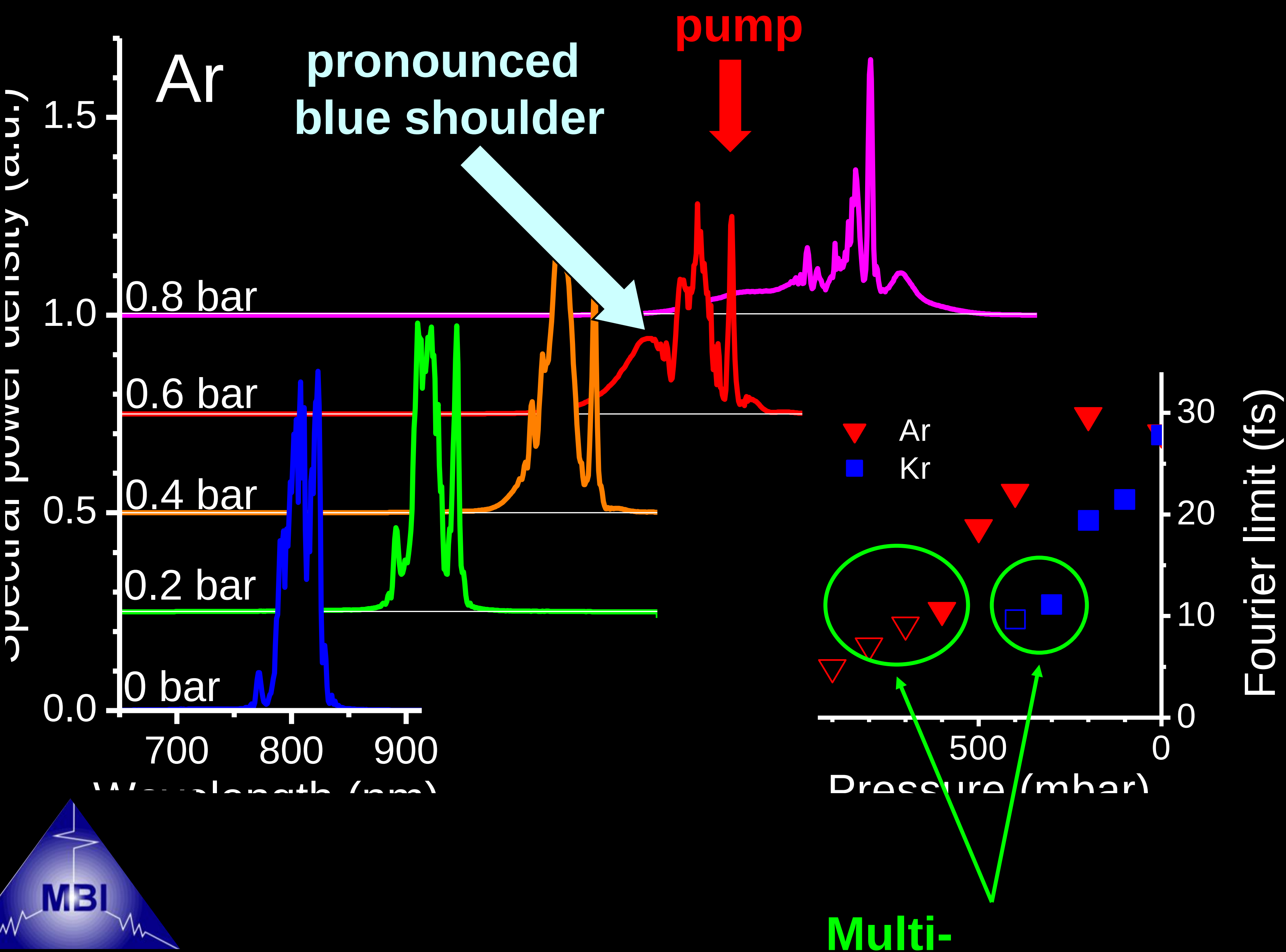
Spectral broadening vs. pressure



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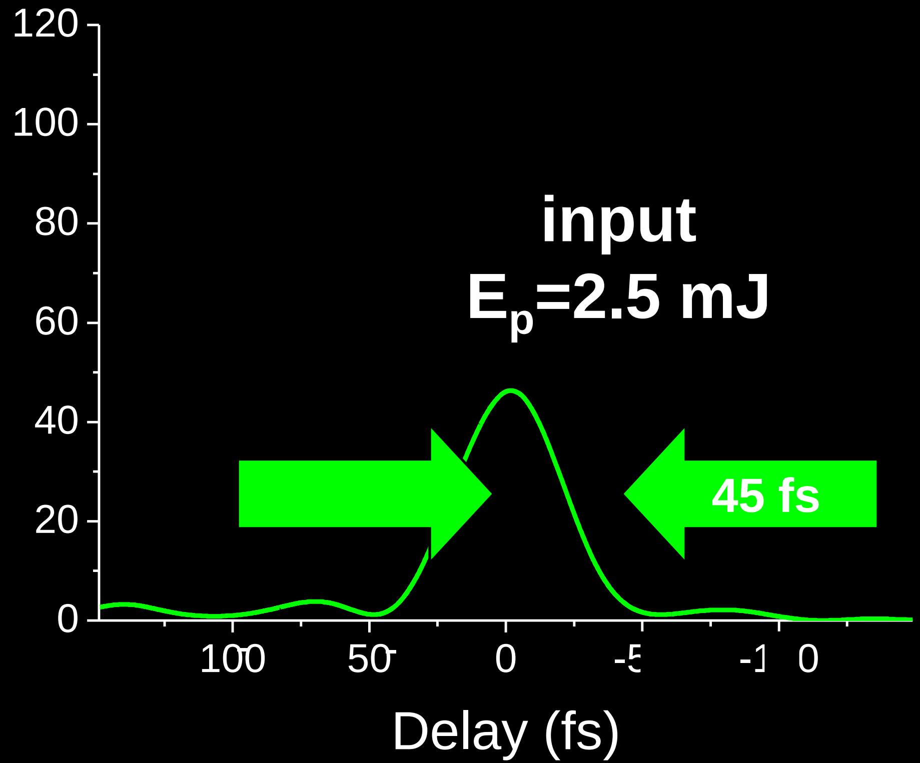
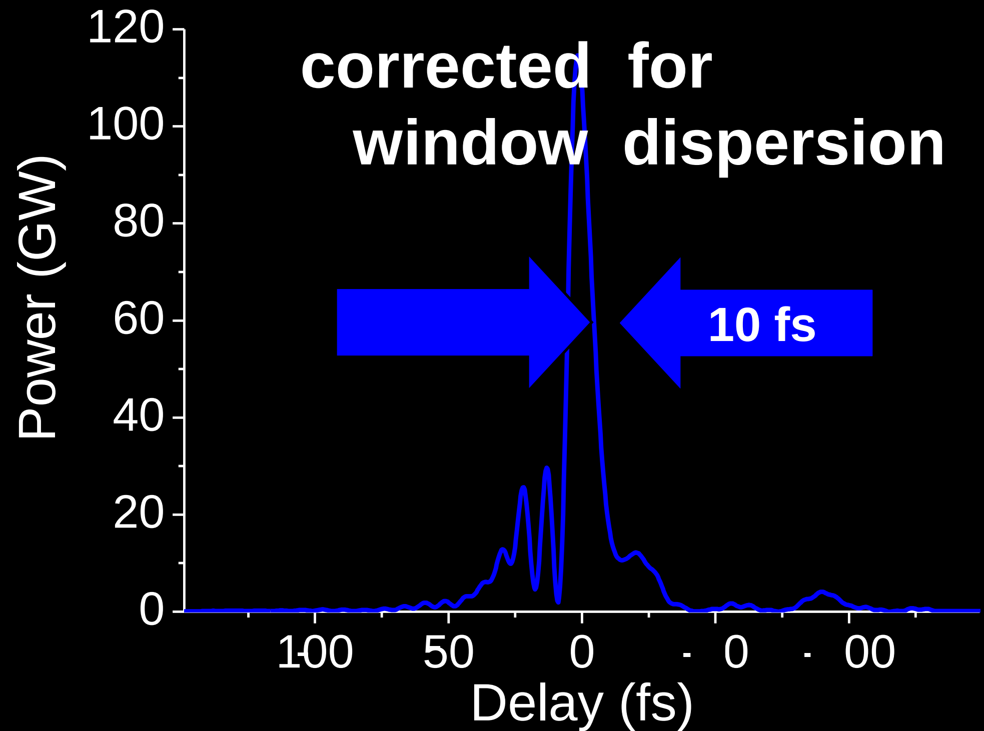
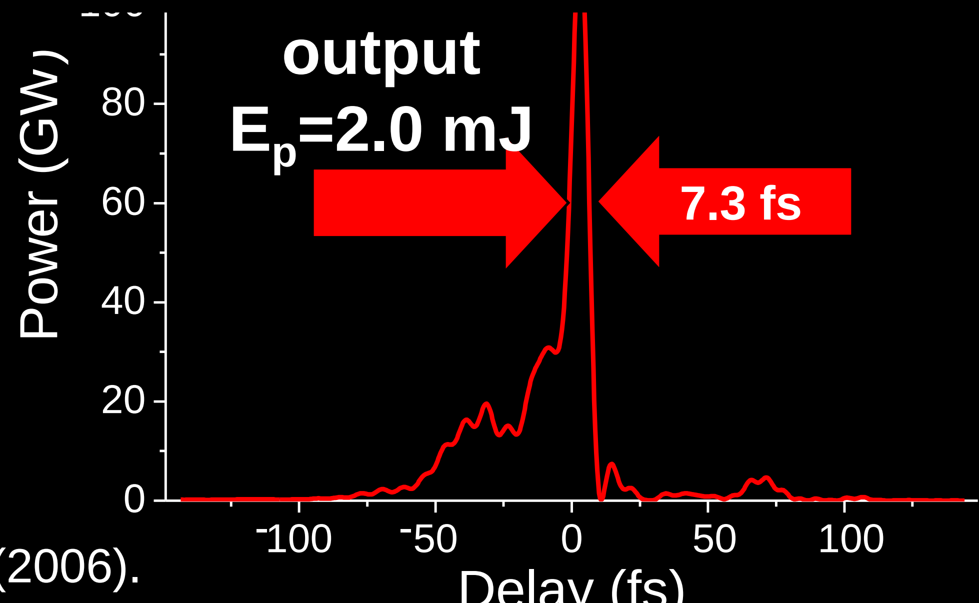
Stibenz & Steinmeyer, OL 31, 274 (2006).

Self compression results







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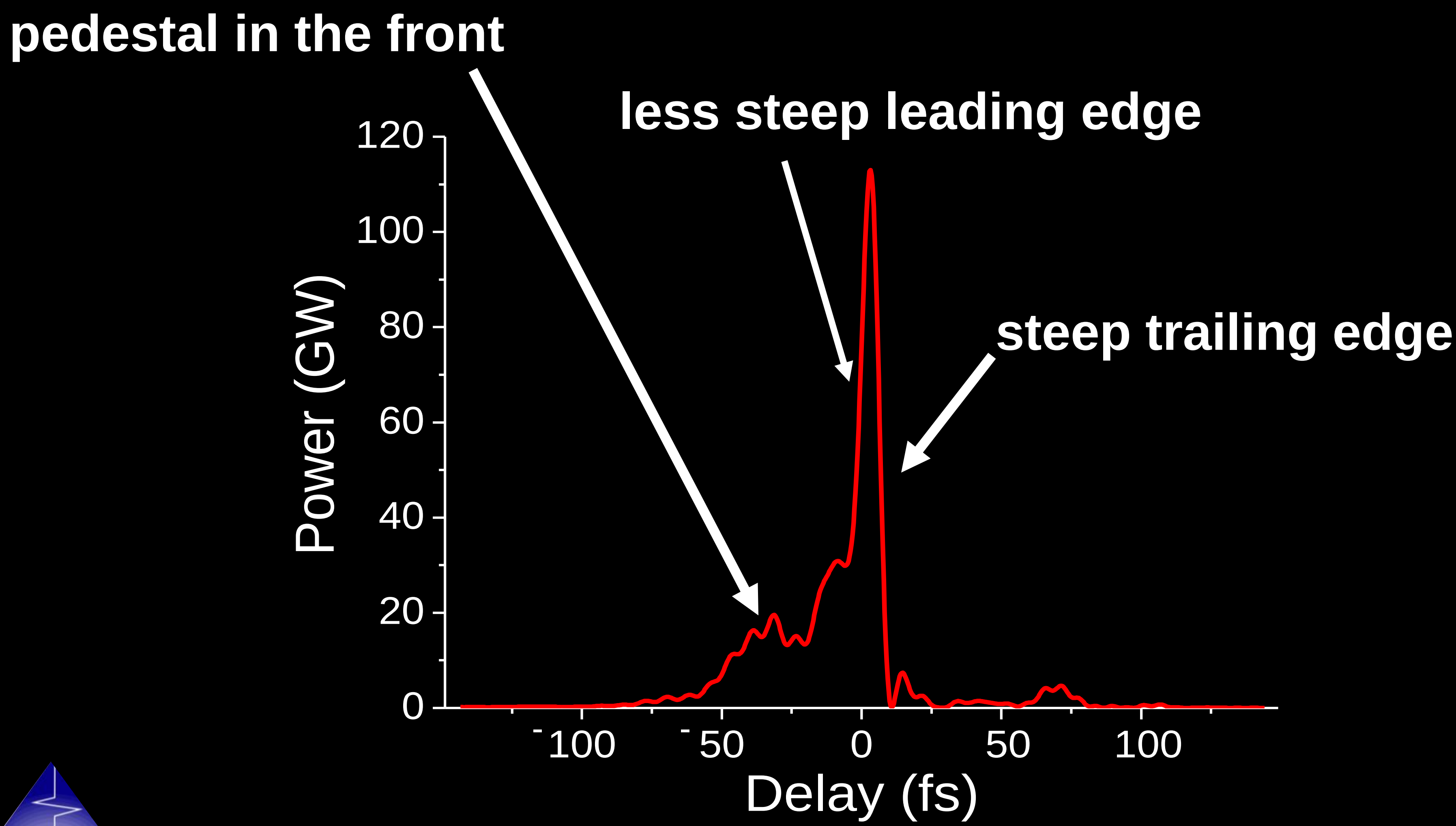
measured with SPIDER:
Stibenz & Steinmeyer,
Rev. Sci. Instrum. **77**, 073105 (2006).

Characteristic shape of 7.4 fs pulse



krypton

G. Stibenz et al., Opt. Lett. **31**, 274 (2006).

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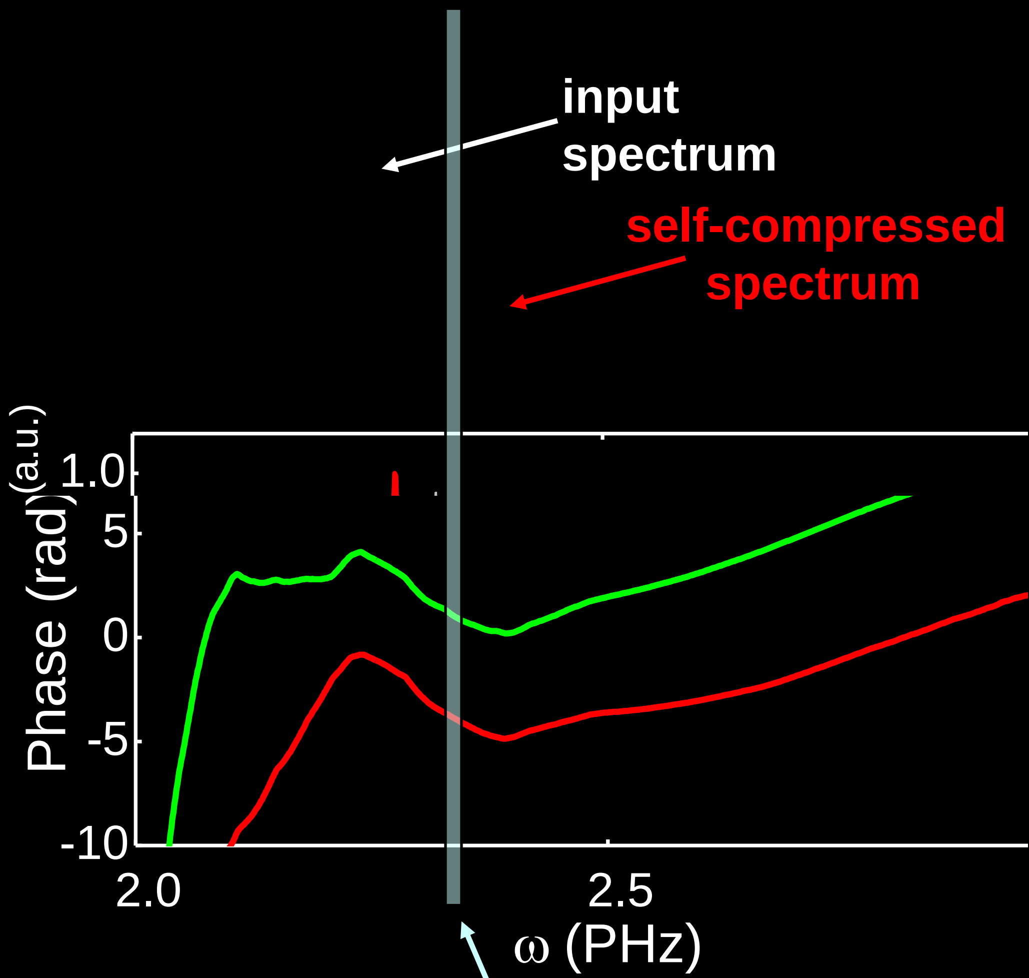
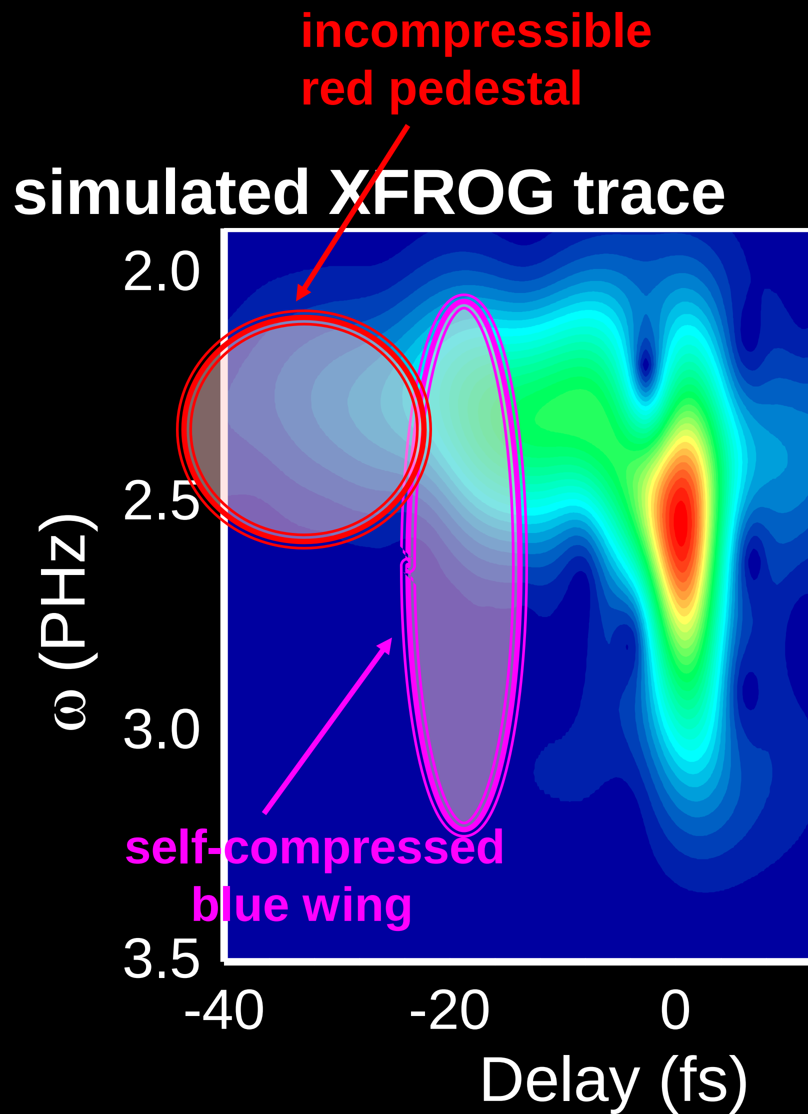
Experimental data on temporal structure



„phase
kink“



krypton

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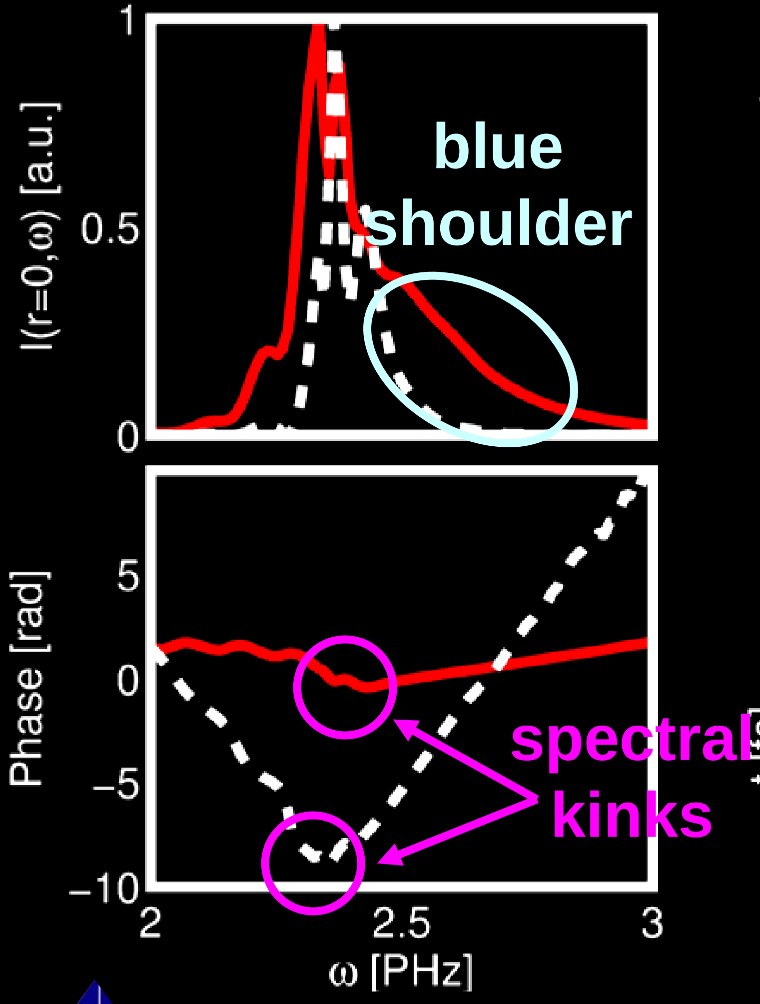
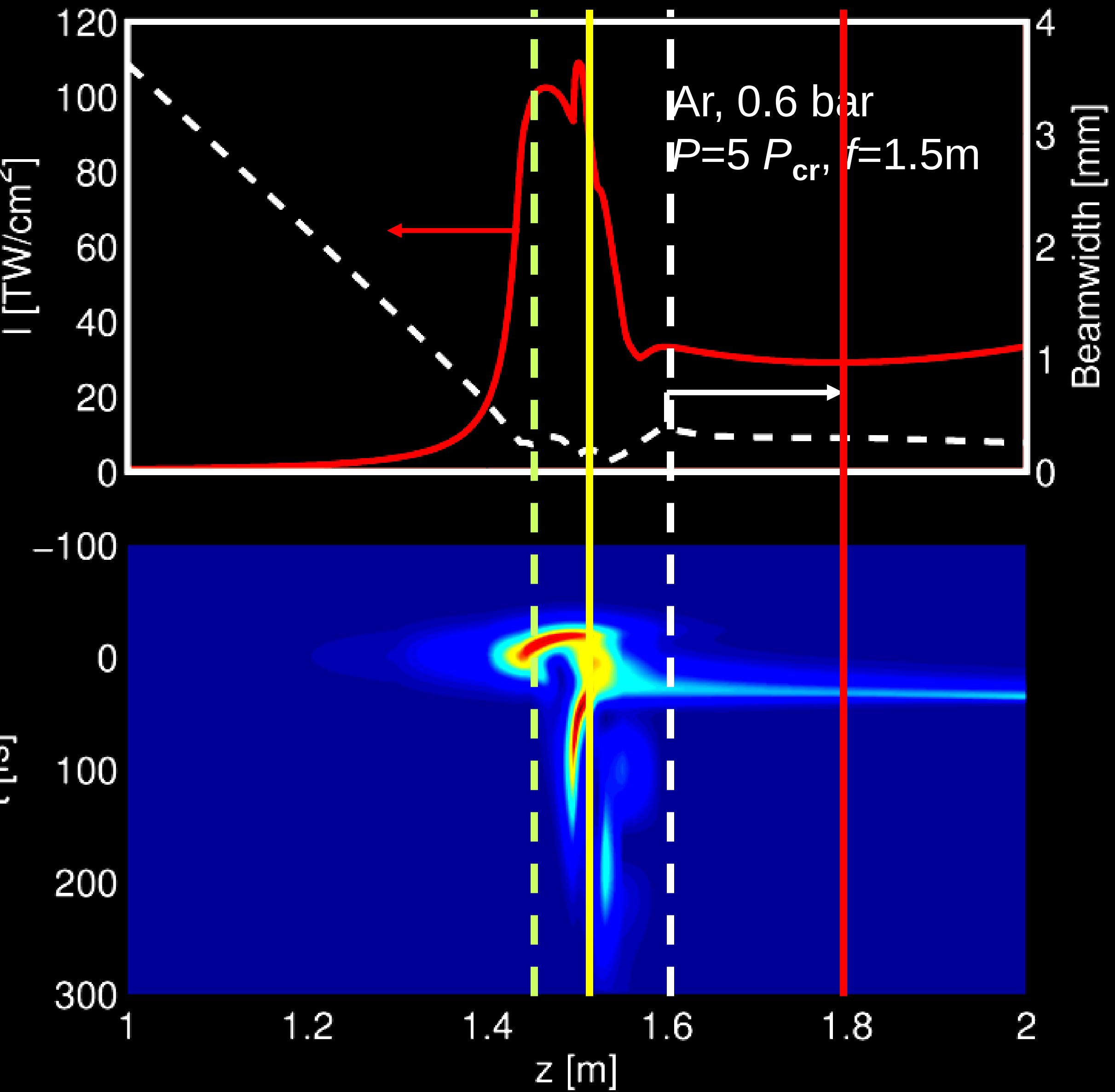
Numerical modeling





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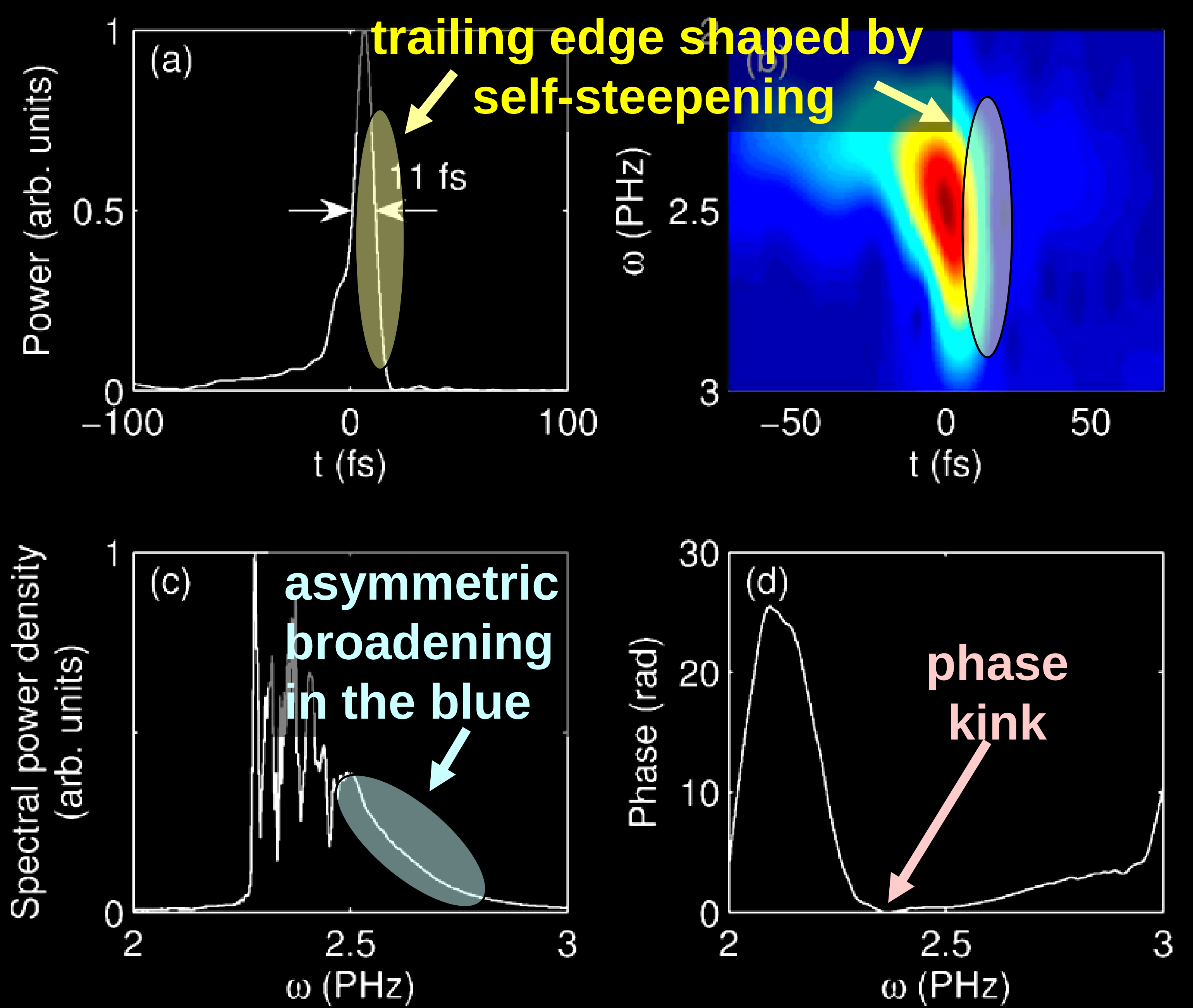
S. Skupin et al., Phys. Rev. E **74**, 056604 (2006).

Experimental data on temporal structure



argon

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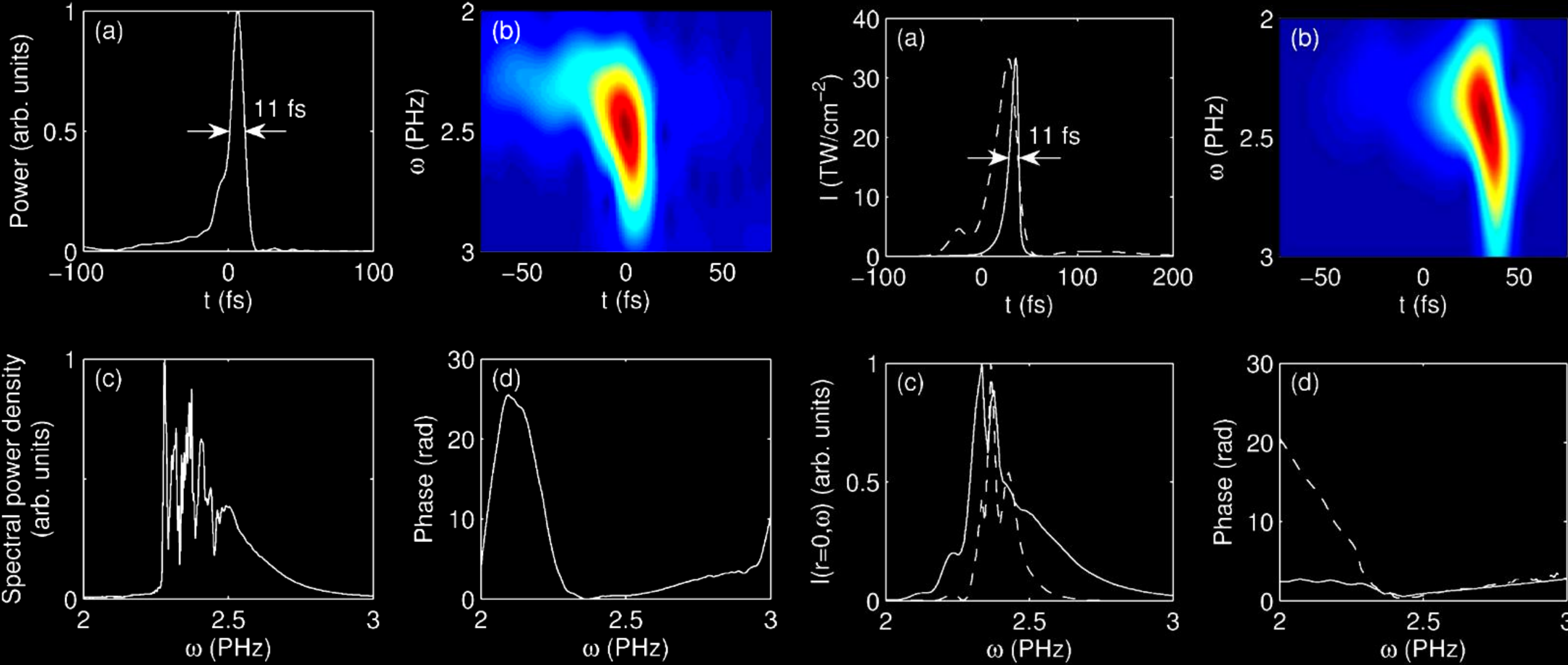
Agreement b/t simulation & experiment



Experiment **Theory**

Parameters: argon

***p*=500 mbar (600 mbar theo.)**

***z*=2m**

***E*=4 mJ, 45 fs input, 4 P_{crit}**

S. Skupin et al., Phys. Rev. E **74**, 056604 (2006).

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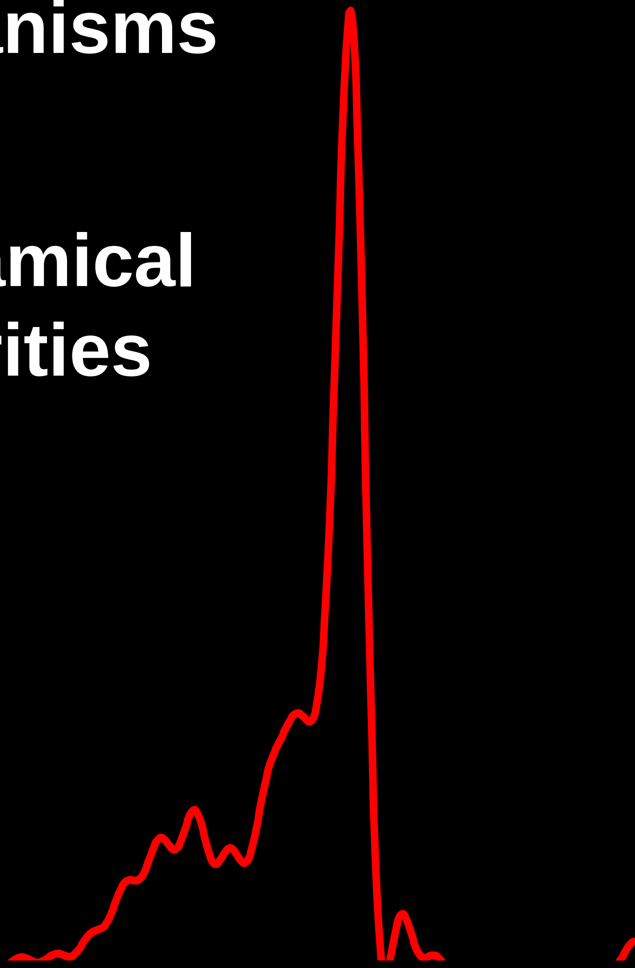
What is behind self-compression?

- **Numerics indicate a very complex interplay of many linear and nonlinear mechanisms**
- **Are they all equally important?**
- **Can we try to reduce to, e.g., a dynamical balance of two competing nonlinearities**
- **Asympotic pulse shape: Is there something like a**

soliton

behind?



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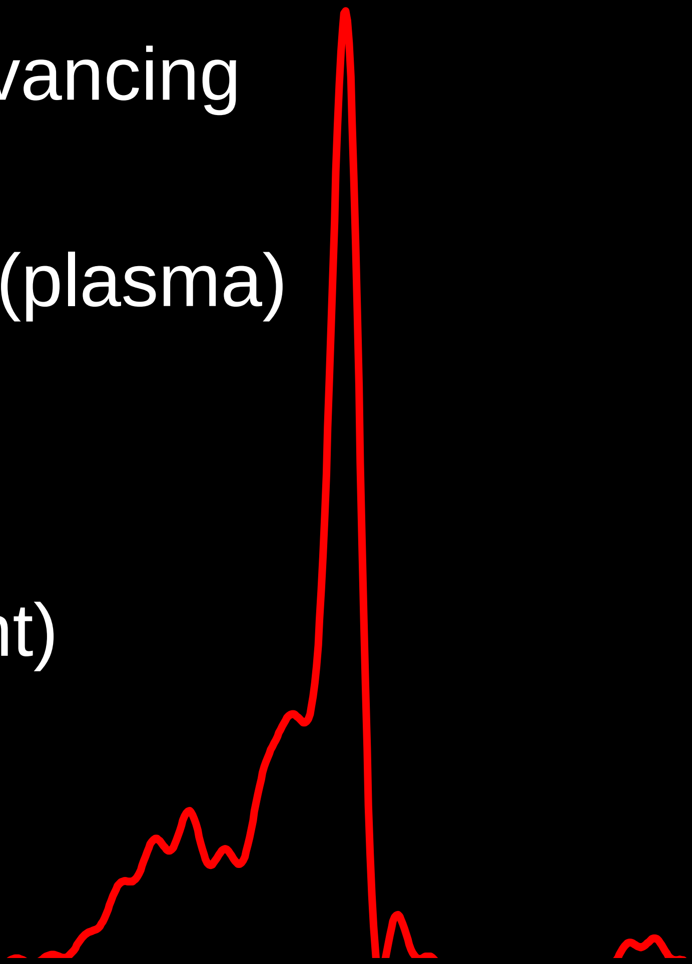
Pulse shaping effects inside filament ?

- **Kerr nonlinearity**
  - bound electrons, instantaneous, retarding
- **Plasma nonlinearity**
  - free electrons, persistent, advancing
- **Self-steepening**
  - blue-shift in the trailing edge (plasma)
- **Spatio-temporal effects:**
  - plasma defocusing in the tail
- Dispersion (probably less important)



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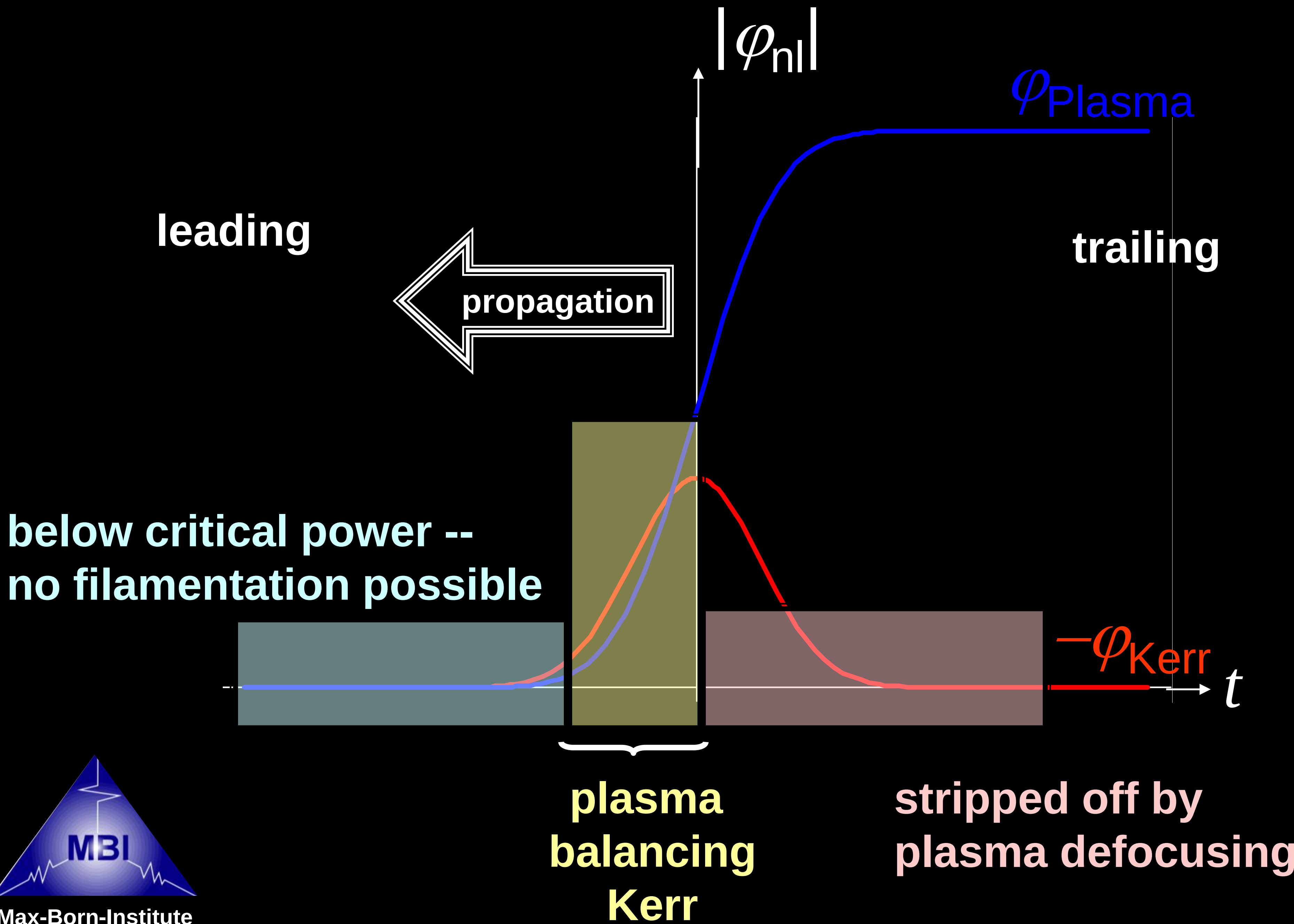
Instantaneous vs. Persistent Nonlinearity



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ADK / PPT solitons ?

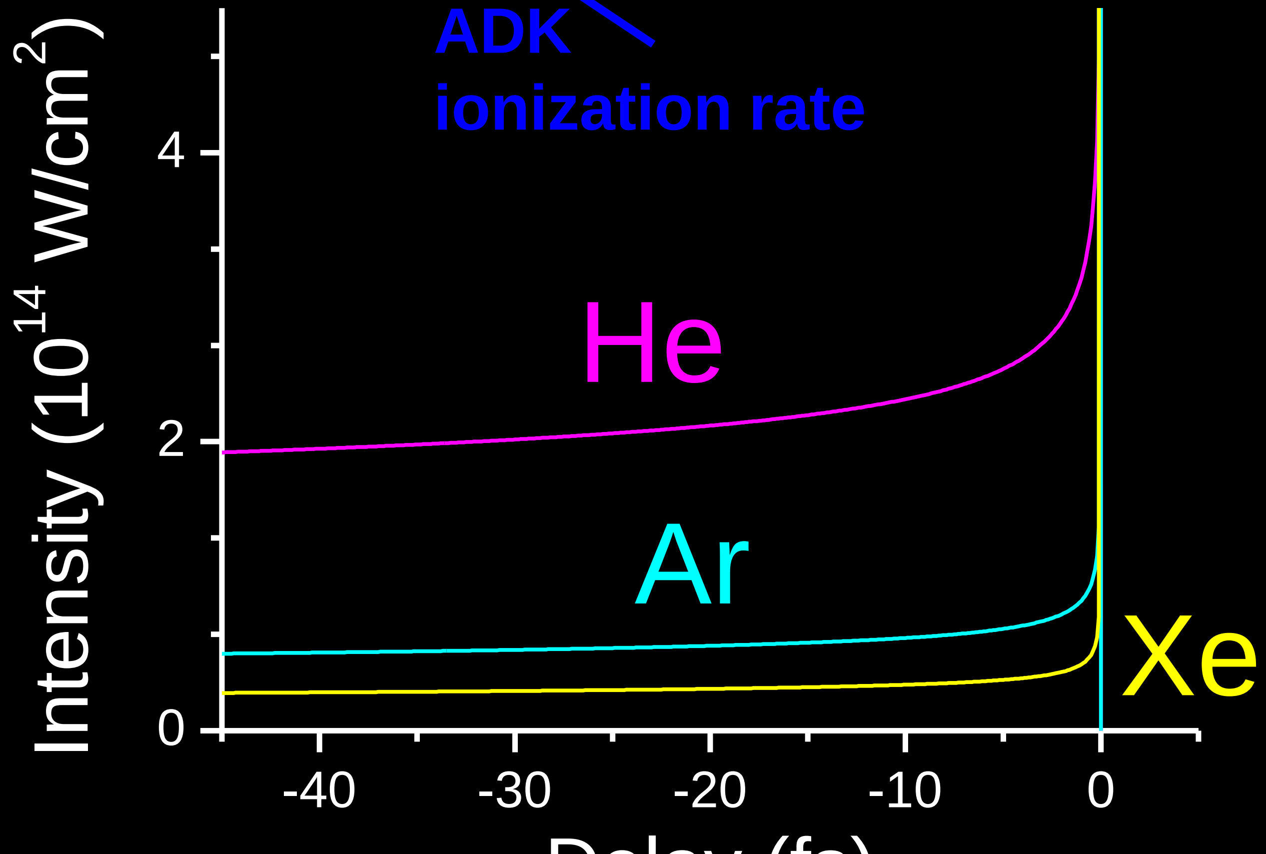
Self-consistent pulse shape arising from balance of Kerr effect and plasma induced n_2?

$$\frac{N q_e^2}{2 m_e \epsilon_0 \omega^2}\int_{-\infty}^{t} w(I)\, dt = n_2 I(t)$$

number density

Drude factor

ADK ionization rate



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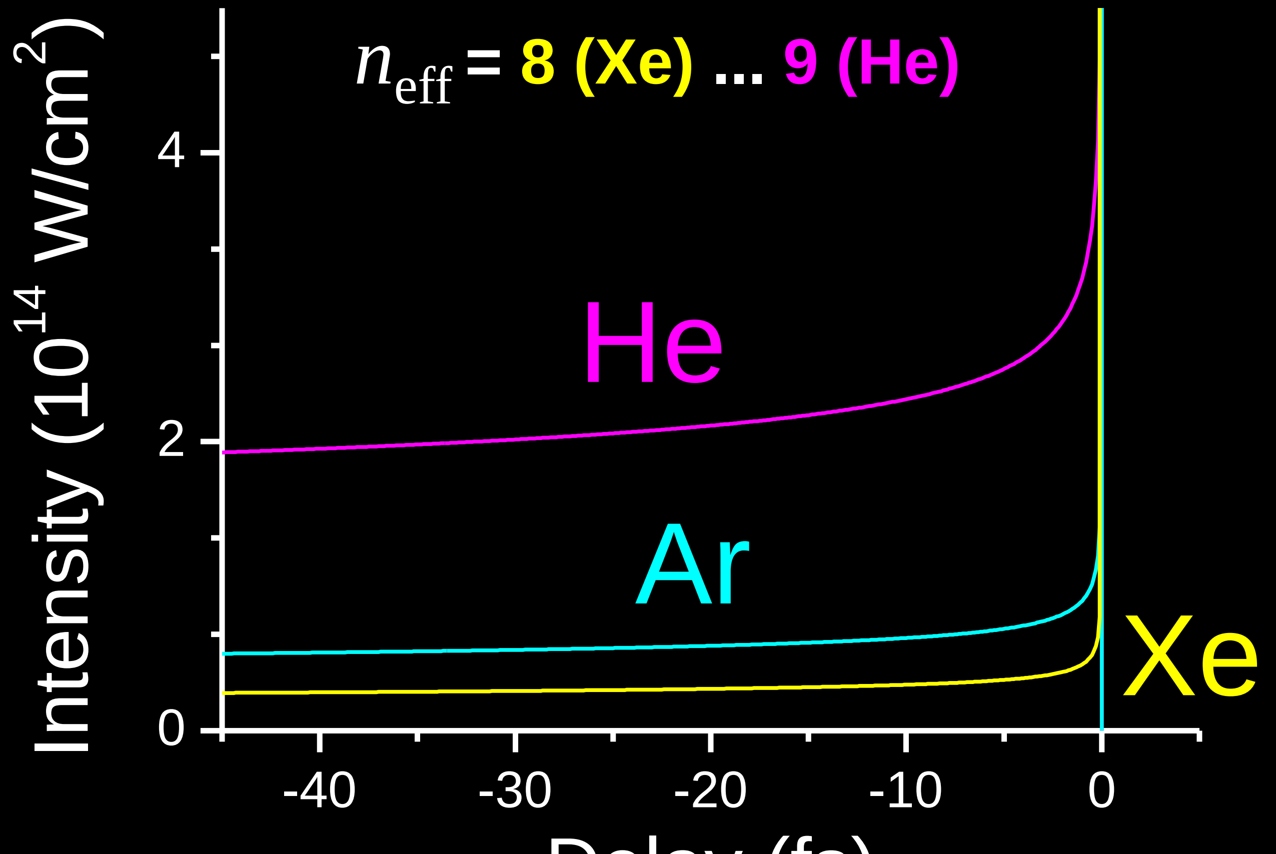
ADK solitons

$$\frac{N\,q_e^2}{2\,m_e\,\epsilon_0\,\omega^2}\int_{-\infty}^{t} w(I)\,dt \;=\; n_2 I(t)$$

Approximate solutions: $$I(t) \propto (-t)^{-\frac{1}{n_{\mathrm{eff}}-1}}$$



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Inclusion of Diffraction

$$\frac{Nq_e^2}{2\,m_e\,\epsilon_0\,\omega^2}\int_{-\infty}^{t} w(I)\,dt' + \frac{n_2\,P_{\mathrm{cr}}}{\pi\,w_0^2} = n_2\,I$$

Self-guiding model, cf. A. Couairon & A. Mysyrowicz, Phys. Rep. **441**, 47 (2007).

BUT: $w_0(t)$ **is now considered as time-dependent**

This enforces a boundary condition via $\frac{P(t)}{\pi w_0^2(t)} = I(t)$

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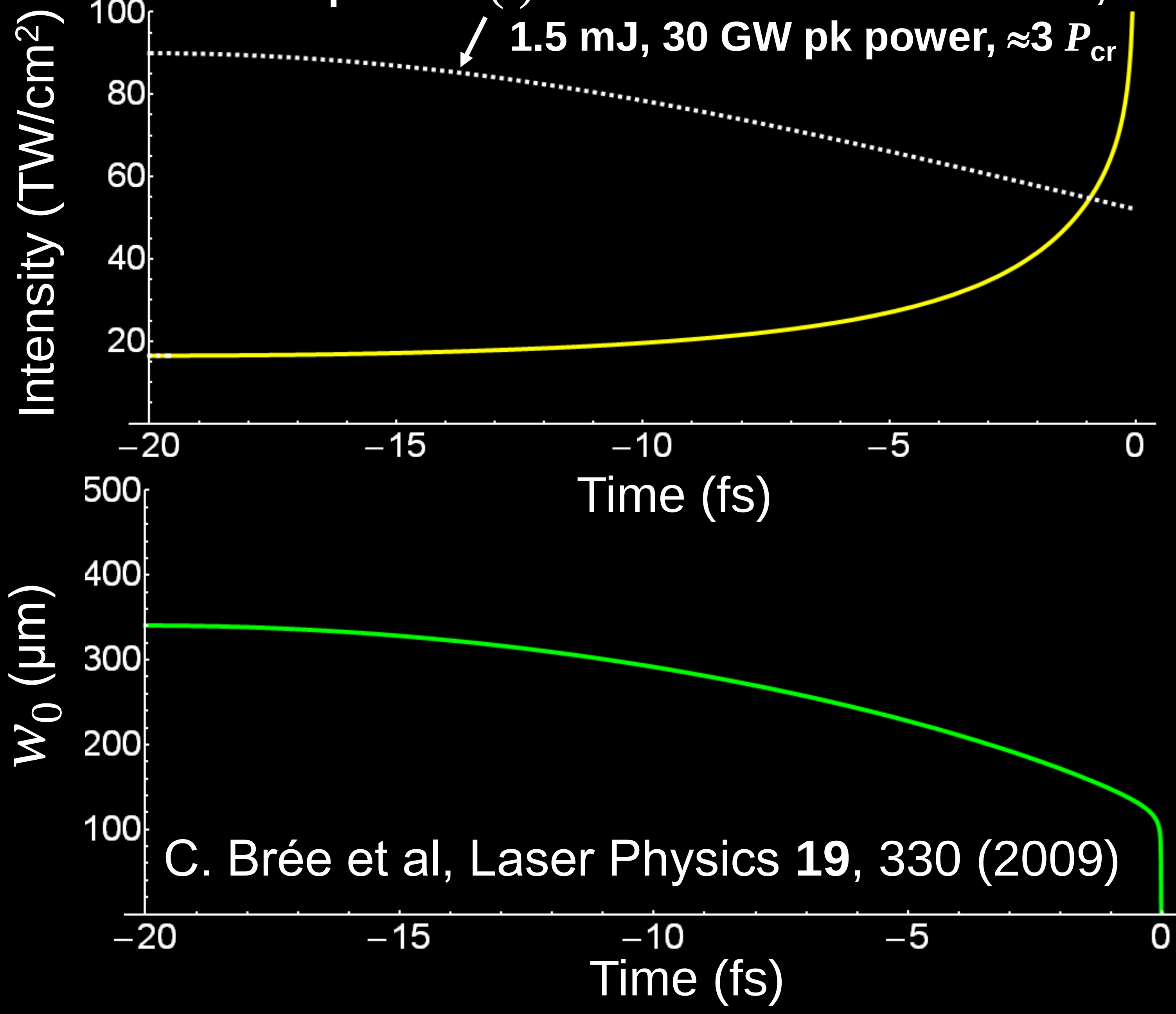
C. Brée et al, Laser Physics **19**, 330 (2009)

WIAS

Solutions of the full equations

Assumption: $P(t)$ **is Gaussian w/ 40 fs FWHM, 1.5 mJ, 30 GW pk power, ≈3** P_{cr}



C. Brée et al, Laser Physics **19**, 330 (2009)

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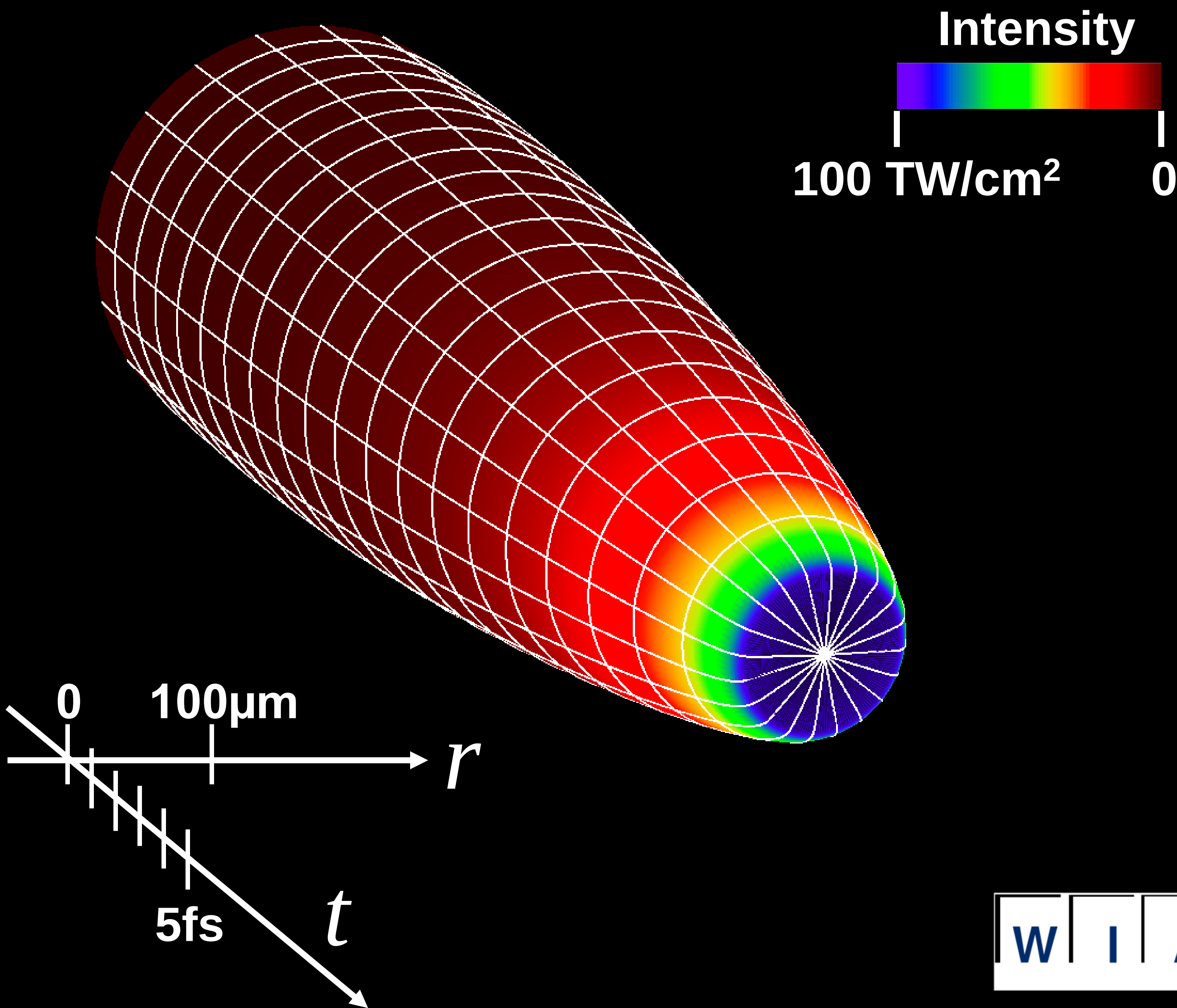
Space-time structure of the light bullet



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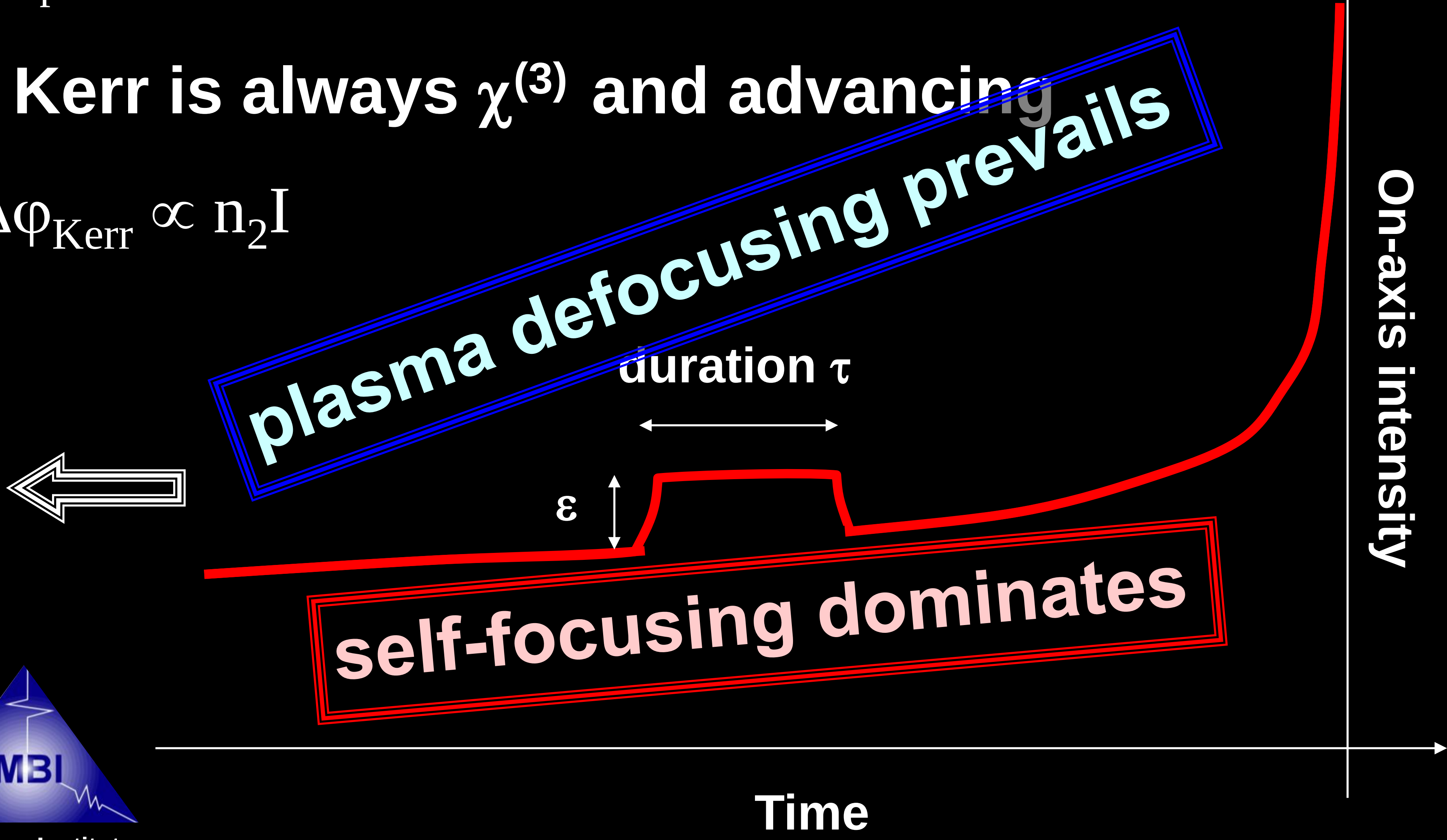
„Stability“ analysis of ADK solitons

- **plasma generation follows ADK and is retarding**

$\Delta\varphi_{plasma} \propto -\sigma_{n*} I^{n*}$ with $n^* \approx 8 \ldots 9$

- **Kerr is always $\chi^{(3)}$ and advancing**

$\Delta\varphi_{Kerr} \propto n_2 I$



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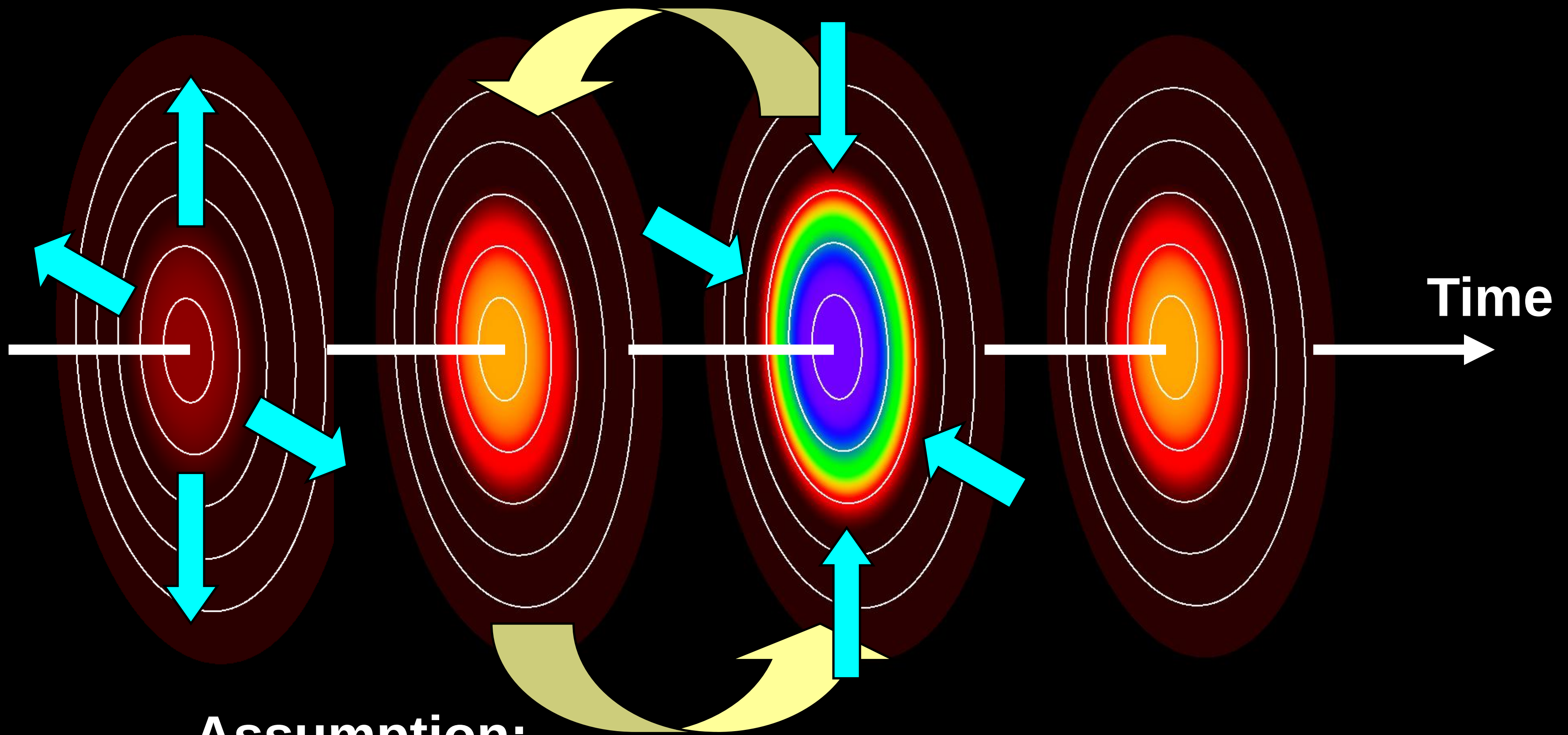
Self-channeling model



Assumption:
negligible energy flow between temporal slices

consider only radial energy flow

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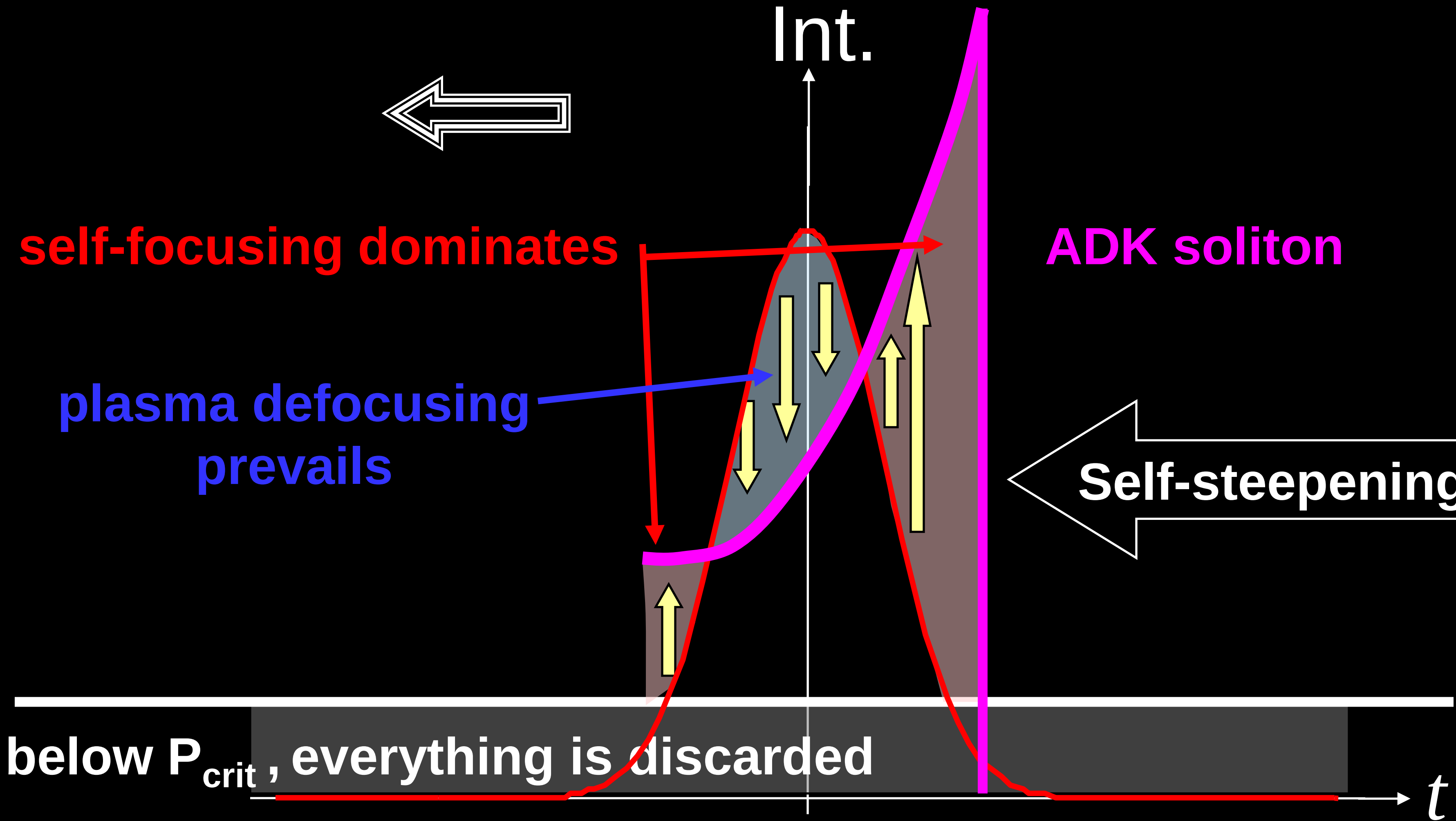
A simple model of self-compression



C. Brée et al, Laser Physics **19**, 330 (2009)

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Light bullet structure in numerical simulations

Several simplifying assumptions...

1. **Gaussian pulse profile assumed**
2. **Only leading edge considered**
3. **Does self-channeling occur in the presence of Kerr, plasma, and diffraction only?**
4. **What happens when we include space-time focusing, self-steepening and the rest of the zoo?**

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Reduced model of filamentary propagation

Reduced model equation (paraxial wave equation with nonlinear refractive index):

$$\partial_z \mathcal{E} = \frac{i}{2k_0}\Delta\mathcal{E} + i\frac{\omega_0}{c}n_2|\mathcal{E}|^2\mathcal{E} - i\frac{1}{2n_0\rho_c}\frac{\omega_0}{c}\rho(I)\mathcal{E}$$

$$\rho(I) = \rho_{\rm nt}\left(1 - \exp\left(-\int_{-\infty}^{t} dt' W[I(t')]\right)\right)$$

$$P(z,t) = \int |\mathcal{E}(z,r,t)|^2 d\sigma_\perp$$

$$\partial_z P(z,t) \equiv 0$$

Surface element in the diffraction plane for the radially symmetric case:

$$d\sigma_\perp = 2\pi r dr$$

Optical power conserved along propagation path, no flow of energy between temporal slices

Quasi-hydrodynamic spatio-temporal shaping in filamentary propagation of fs pulses

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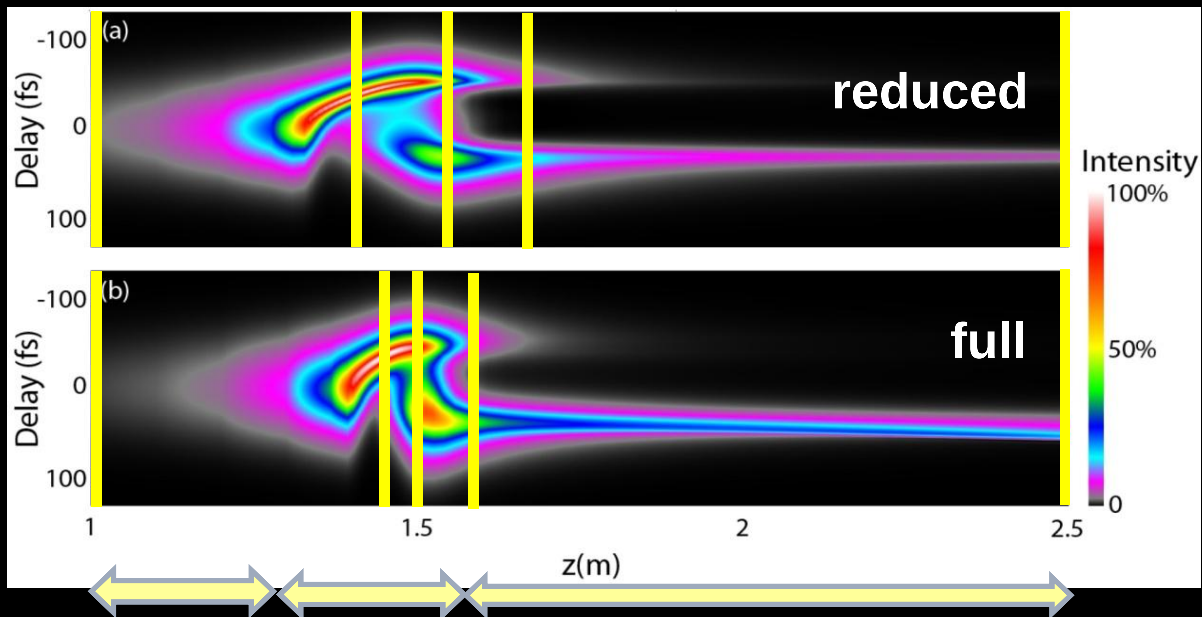
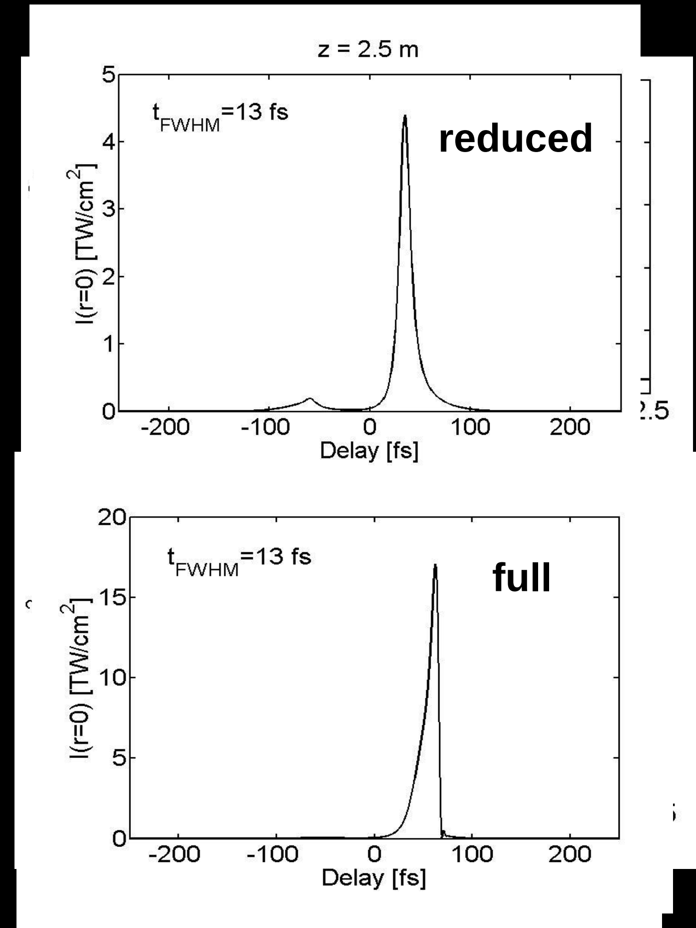
Three-stage compression scheme

(a) Evolution of ***on-axis*** temporal intensity profile along *z*



linear | **Kerr & plasma**: $\rho_{max}=10^{16}$ cm^{-3} | **weakly ionized, Kerr only:** $\rho_{max}<10^{13}$ cm^{-3}



Three stages of filament self-compression:

- **I. Temporal break-up**
- **II. Leading sub-pulse diffracts out**
- **III. Additional compression in the weakly ionized channel**

→ Dynamics dominated by spatial effects

C. Brée et al., Opt. Exp. **17**, 16429 (2009)

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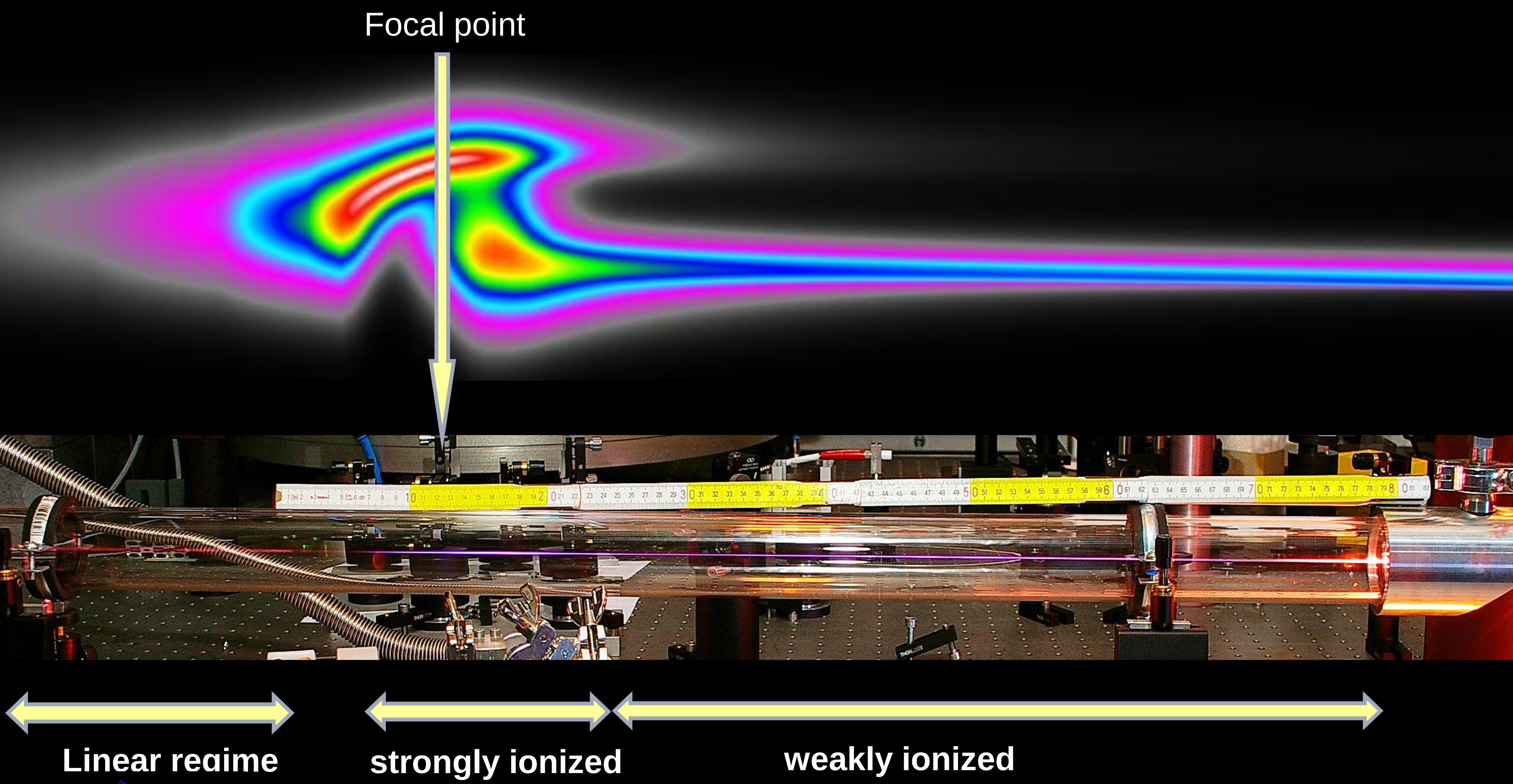
W I A S

Comparison with experiments



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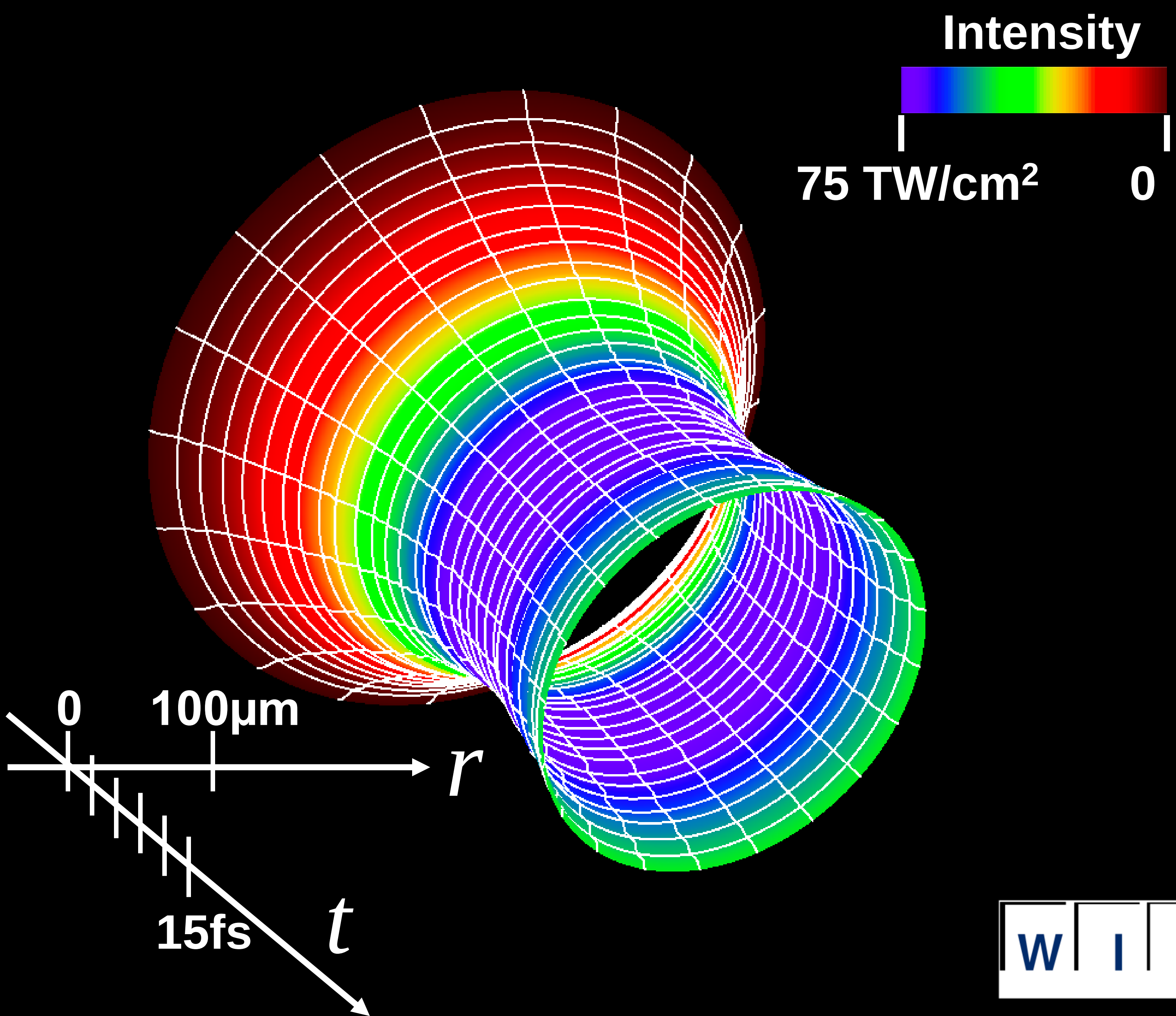
G. Stibenz et al., *Opt. Lett.,* **31**, 274 (2006)
Measured with SPIDER: Stibenz & Steinmeyer, Rev. Sci. Instrum. **77**,

Space-time structure



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