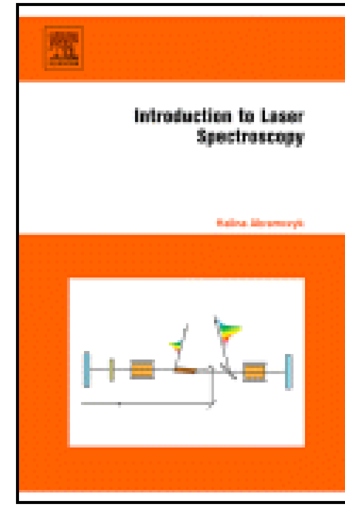

SPH 618
Optical and Laser Physics
University of Nairobi, Kenya
Lecture 1
Overview of wave propagation phenomena in various media

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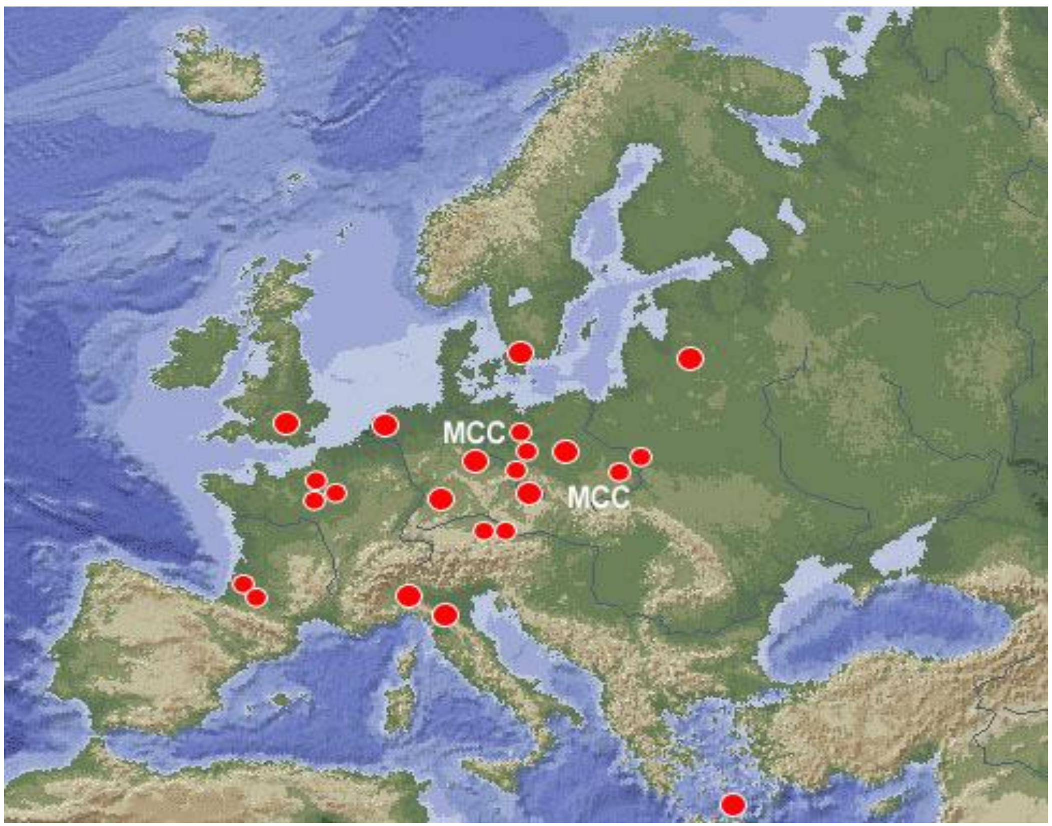
Max Born Institute for Nonlinear Optics and Ultrashort Laser Pulses, Berlin, Germany,
Technical University of Lodz, Lodz, Poland



http://mitr.p.lodz.pl/evu


P O L S K A


SUPER MARKET

MAX BORN-INSTITUTE

What does OLIVIA NEWTON JOHN have in common with Max Born Institute in Berlin?

V Solvay Conference Brussels, October 1927

http://216.120.242.82/~greensp/video.html 17 Noblistów na 29 uczestników
Home movie Irving Langmuir (nagroda Nobla z chemii 1932)

Max Born (1882 - 1970)

Statistical interpretation of the vawe function- square of the vawe function (ψ*ψ) in Schrodinger equation describes the density of probability to find a particle in a given position

$$\hat{H}\psi(\mathbf{r},t) = i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t)$$

$$dP(\vec{r},t) = C|\psi(\vec{r},t)|^2 d^3r$$

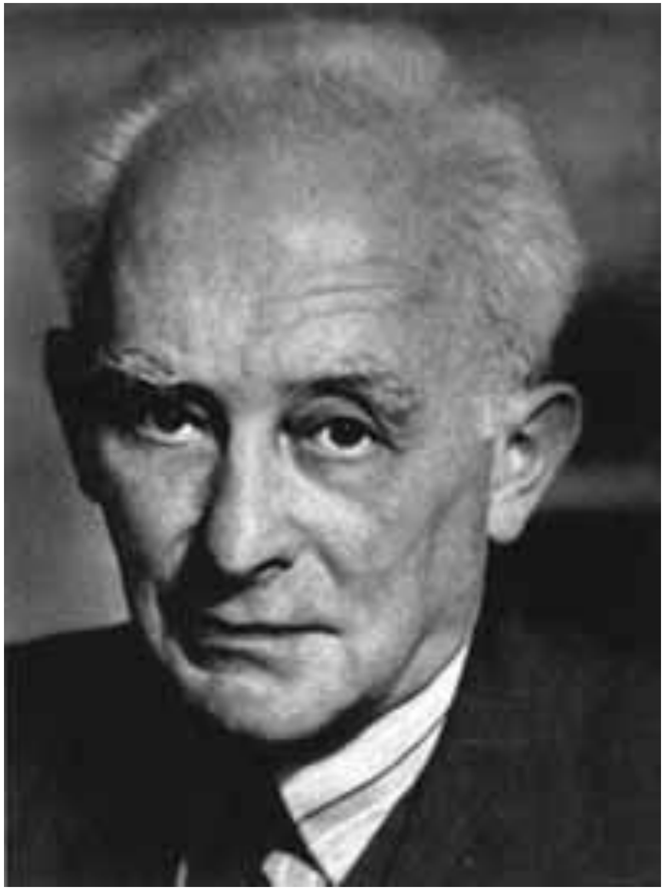
Einstein - God *does* not play dice –

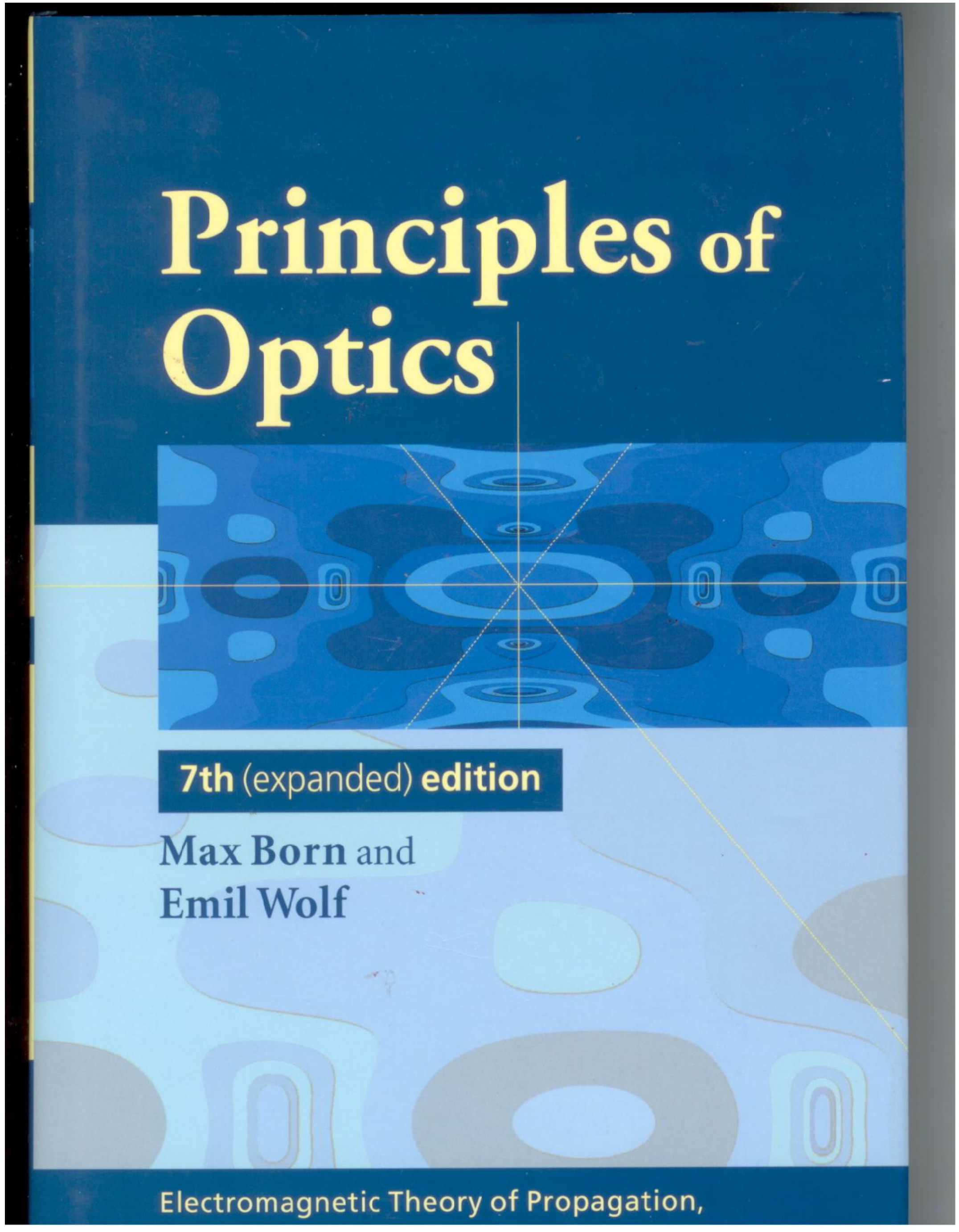
Born- we indeed live in a world of uncertainty

„The world is not ruled by reason; even less by love”-1921 z listu do A. Einsteina

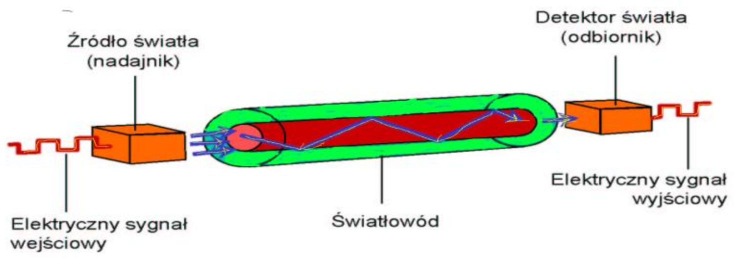
Wnuczka Maxa Borna Olivia Newton Jones, circa 1988.

Max Born, circa 1920. Photo courtesy AIP Emilio Segrè Visual Archives, Born Collection




Principles of Optics
7th (expanded) edition
Max Born and
Emil Wolf
Electromagnetic Theory of Propagation,

PHYSICAL PRINCIPLES OF OPTICAL COMMUNICATION



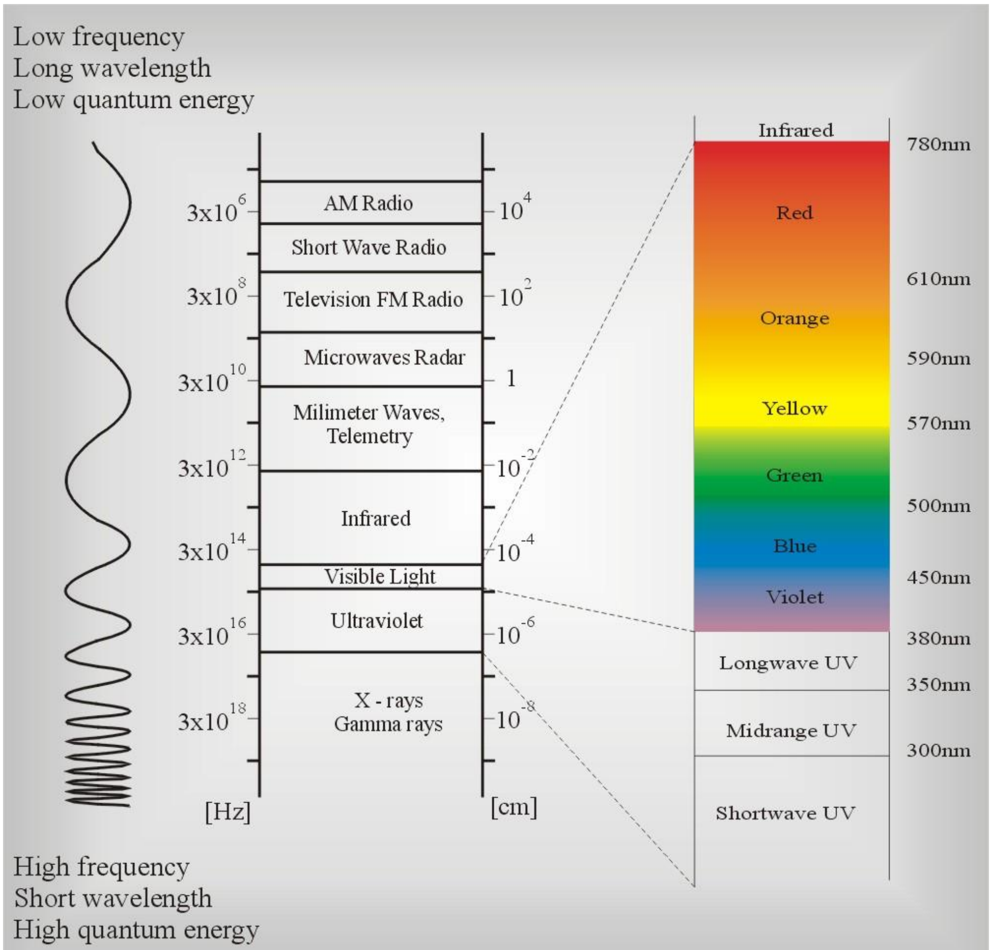
- Fast optical networks utilizing the wavelength multiplexing technologies WDM, DWDM, UWDM are one of the greatest beneficient of the modern laser technology in building of complete optical platforms that includes optical fibers
- lasers, modulators, reflectometers
- optical amplifiers
- multiplexers and demultiplexers
- switches and teracommutators

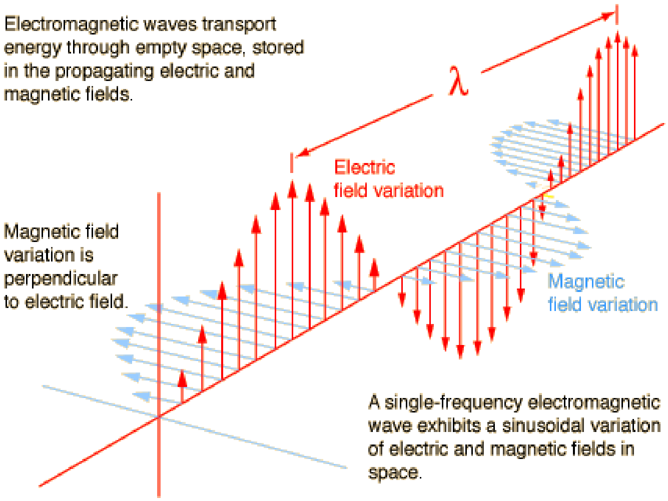
Propagation of light in various media (vacum, dielectrics like optical fibers) is described by Maxwell equations, similarly to all other electromagnetic phenomena. The wave equation derived from the Maxwell equations describes propagation of light in variuos media.

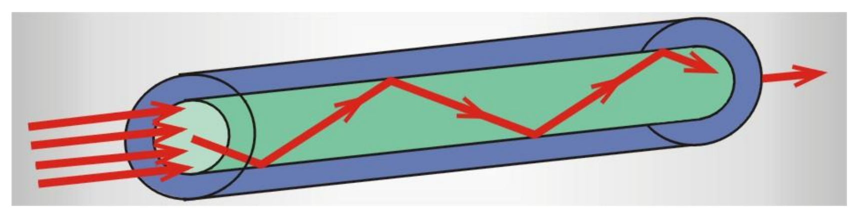
$c = \nu\lambda$ Also commonly written $v = f\lambda$

velocity = frequency x wavelength

Speed of light $c = 3 \times 10^8$ m/s

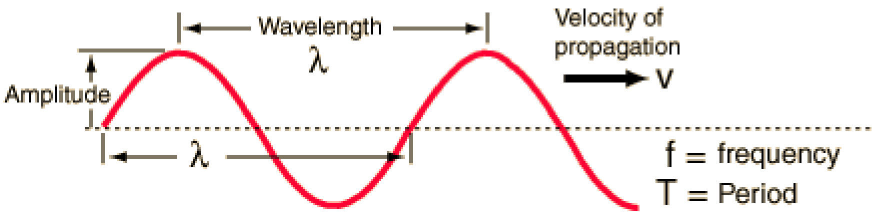






Traveling Wave Relationship

A single frequency traveling wave will take the form of a sine wave. A snapshot of the wave in space at an instant of time can be used to show the relationship of the wave properties frequency, wavelength and propagation velocity.



The motion relationship "distance = velocity x time" is the key to the basic wave relationship. With the wavelength as distance, this relationship becomes $\lambda = vT$. Then using $f=1/T$ gives the standard wave relationship

$$v = f\lambda$$

This is a general wave relationship which applies to sound and light waves, other electromagnetic waves, and waves in mechanical media.

Maxwell's Equations

http://hyperphysics.phy-astr.gsu.edu/Hbase/electric/maxeq.html

Maxwell's equations represent one of the most elegant and concise ways to state the fundamentals of electricity and magnetism. From them one can develop most of the working relationships in the field. Because of their concise statement, they embody a high level of mathematical sophistication and are therefore not generally introduced in an introductory treatment of the subject, except perhaps as summary relationships. These basic equations of electricity and magnetism can be used as a starting point for advanced courses, but are usually first encountered as unifying equations after the study of electrical and magnetic phenomena.

- Symbols Used
- E = Electric field, ρ = charge density
- i = electric current, B = Magnetic field
- ε_0 = permittivity, J = current density
- D = Electric displacement, $D = \varepsilon_0 E + P,$
- μ_0 = permeability
- c = speed of light, H = Magnetic field strength
- M = Magnetization, P = Polarization

Maxwell's Equations

- Differential form in the absence of magnetic or polarizable media:
- I. Gauss' law for electricity $\nabla \cdot E = \frac{\rho}{\varepsilon_0} = 4\pi k \rho$
-
- II. Gauss' law for magnetism $\nabla \cdot B = 0$
-
- III. Faraday's law of induction $\nabla x\, E = -\frac{\partial B}{\partial t}$
-
- IV. Ampere's law

$$\nabla\ x\ B = \frac{4\pi k}{c^2} J + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$= \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$k = \frac{1}{4\pi\varepsilon_0} =$ *Coulomb's constant* $\quad c^2 = \frac{1}{\mu_0 \varepsilon_0}$

Divergence

- **Application in Cartesian coordinates**
- Let *x, y, z* be a system of Cartesian coordinates on a 3-dimensional Euclidean space, and let **i**, **j**, **k** be the corresponding basis of unit vectors.
- The divergence of a continuously differentiable vector field **F** = *U* **i** + *V* **j** + *W* **k** is defined to be the scalar-valued function:

Nabla symbol

$$\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}.$$

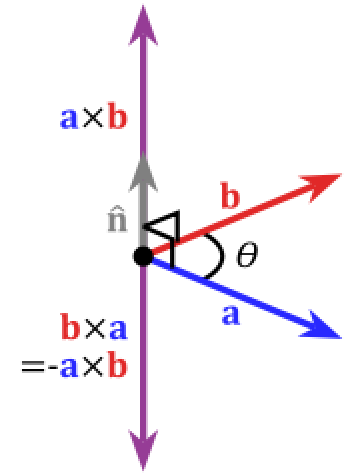
- Although expressed in terms of coordinates, the result is invariant under orthogonal transformations, as the physical interpretation suggests.
- The common notation for the divergence **∇·F** is a convenient mnemonic, where the dot denotes an operation reminiscent of the dot product: take the components of ∇ (see del), apply them to the components of **F**, and sum the results. As a result, this is considered an abuse of notation**.**
- IT IS NOT A GRADIENT!!!!

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) \qquad \frac{\partial f}{\partial x}\hat{\mathbf{i}} + \frac{\partial f}{\partial y}\hat{\mathbf{j}} + \frac{\partial f}{\partial z}\hat{\mathbf{k}}$$

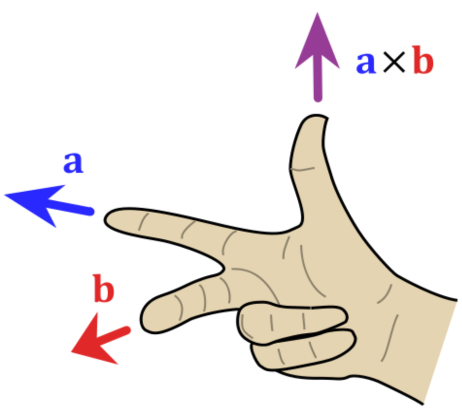
$\hat{\mathbf{n}}$

the cross product of two vectors

$$\nabla x\, E = -\frac{\partial B}{\partial t}$$



the **cross product** is a binary operation on two vectors in a three-dimensional Euclidean space that results in another vector which is perpendicular to the plane containing the two input vectors. The algebra defined by the cross product is neither commutative nor associative. It contrasts with the dot product which produces a scalar result



The cross product is defined by the formula[2]

$$\mathbf{a} \times \mathbf{b} = ab \sin\theta \ \hat{\mathbf{n}}$$

where *θ* is the measure of the smaller angle between **a** and **b** (0° ≤ *θ* ≤ 180°), *a* and *b* are the magnitudes of vectors **a** and **b**, and n is a unit vector perpendicular to the plane containing **a** and **b** in the direction given by the right-hand rule as illustrated. If the vectors **a** and **b** are collinear (i.e., the angle *θ* between them is either 0° or 180°), by the above formula, the cross product of **a** and **b** is the zero vector **0**.

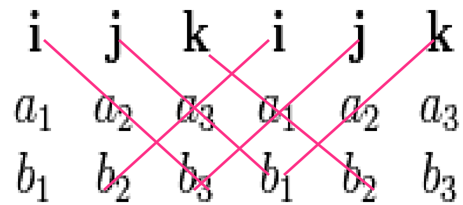
The cross-product in respect to a right-handed coordinate system

Matrix notation

- The definition of the cross product can also be represented by the determinant of a matrix:

$$\mathbf{a} \times \mathbf{b} = \det \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}.$$

- This determinant can be computed using Sarrus' rule. Consider the table



- From the first three elements on the first row draw three diagonals sloping downward to the right (for example, the first diagonal would contain **i**, a_2, and b_3), and from the last three elements on the first row draw three diagonals sloping downward to the left (for example, the first diagonal would contain **i**, a_3, and b_2). Then multiply the elements on each of these six diagonals, and negate the last three products. The cross product would be defined by the sum of these products:

$$\mathbf{i}a_2b_3 + \mathbf{j}a_3b_1 + \mathbf{k}a_1b_2 - \mathbf{i}a_3b_2 - \mathbf{j}a_1b_3 - \mathbf{k}a_2b_1.$$

Gauss' Law for Electricity

- The electric flux out of any closed surface is proportional to the total charge enclosed within the surface.
- The integral form of Gauss' Law finds application in calculating electric fields around charged objects.
- In applying Gauss' law to the electric field of a point charge, one can show that it is consistent with Coulomb's law.
- While the area integral of the electric field gives a measure of the net charge enclosed, the divergence of the electric field gives a measure of the density of sources. It also has implications for the conservation of charge.

Integral form

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} = 4\pi k q$$

Differential form

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0} = 4\pi k \rho$$

Gauss' Law for Magnetism

- The net magnetic flux out of any closed surface is zero. This amounts to a statement about the sources of magnetic field. For a magnetic dipole, any closed surface the magnetic flux directed inward toward the south pole will equal the flux outward from the north pole. The net flux will always be zero for dipole sources. If there were a magnetic monopole source, this would give a non-zero area integral. The divergence of a vector field is proportional to the point source density, so the form of Gauss' law for magnetic fields is then a statement that there are no magnetic monopoles.

Integral form

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Differential form

$$\nabla \cdot B = 0$$

Ampere's Law

- In the case of static electric field, the line integral of the magnetic field around a closed loop is proportional to the electric current flowing through the loop. This is useful for the calculation of magnetic field for simple geometries.

Integral form

$$\oint B \cdot ds = \mu_0 i + \frac{1}{c^2}\frac{\partial}{\partial t}\int E \cdot dA$$

Differential form

$$\nabla x B = \frac{4\pi k}{c^2} J + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$\nabla x B = \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

Faraday's Law of Induction

- The line integral of the electric field around a closed loop is equal to the negative of the rate of change of the magnetic flux through the area enclosed by the loop.

Integral form

$$\oint \vec{E} \cdot \vec{ds} = -\frac{d\Phi_B}{dt}$$

Differential form

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

- This line integral is equal to the generated voltage or emf in the loop, so Faraday's law is the basis for electric generators. It also forms the basis for inductors and transformers.

Maxwell's Equations

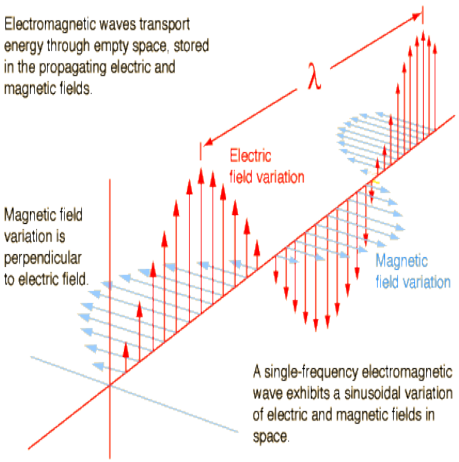
Differential form with magnetic and/or polarizable media:

- I. Gauss' law for electricity $\nabla \cdot D = \rho$
- $D = \varepsilon_0 E + P$ *General case* $D = \varepsilon_0 E$ *Free space* $D = \varepsilon E$ *Isotropic linear dielectric* ε_0 = permittivity
- II. Gauss' law for magnetism
- $\nabla \cdot B = 0$
- III. Faraday's law of induction
- $\nabla x\, E = -\frac{\partial B}{\partial t}$
- IV. Ampere's law

$B = \mu_0 (H + M)$ *General case* $B = \mu_0 H$ *Free space* $B = \mu H$ *Isotropic linear magnetic medium*

μ_0 = permeability

Electromagnetic Wave Equation



The wave equation for a plane electric wave traveling in the x direction in space is

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

vacuum

$$\left(\frac{\epsilon}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\boldsymbol{E} = 0,$$

Dielectric medium

with the same form applying to the magnetic field wave in a plane perpendicular the electric field.
Both the electric field and the magnetic field are perpendicular to the direction of travel x.
The symbol c represents the speed of light or other electromagnetic waves.
The wave equation for electromagnetic waves arises from Maxwell's equations.
The form of a plane wave solution for the electric field is

$$E = E_m \sin(kx - \omega t)$$

and that for the magnetic field

$$B = B_m \sin(kx - \omega t)$$

In Cartesian coordinate system Laplace operator is expressed as follows:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \frac{\partial^2}{\partial z^2}$$

Helmholtz equation (when we can neglect the imaginary component of dielectric constant)

- the wave equation takes form of equation known as Helmholtz equation
- $$\nabla^2 \tilde{\boldsymbol{E}} + n^2(\omega) k_0^2 \tilde{\boldsymbol{E}} = 0$$
- In Cartesian coordinate system Laplace operator is expressed as follows:
- $$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \frac{\partial^2}{\partial z^2}$$
- where $k_0 = \frac{\omega}{c} = \omega\sqrt{\mu_0 \varepsilon_0}$

is the wave vector length (wave number), μ_0 and ε_0 denote magnetic and dielectric permeability of free space, $\tilde{\boldsymbol{E}} = \tilde{\boldsymbol{E}}(\boldsymbol{r}, \omega)$

is Fourier transform of electric field

- $$\tilde{\boldsymbol{E}}(\boldsymbol{r}, \omega) = \int_{-\infty}^{+\infty} \boldsymbol{E}(\boldsymbol{r}, t)\, exp(i\omega t) dt$$

Phase velocity

- in a conventional (*i.e.*, real and greater than unity) dielectric medium an electromagnetic wave propagates with a phase velocity which is slower than the velocity of light
- The phase velocity of the wave is given by

$$v = \frac{\omega}{k} = \frac{c}{n},$$

$$n = \sqrt{\epsilon}$$

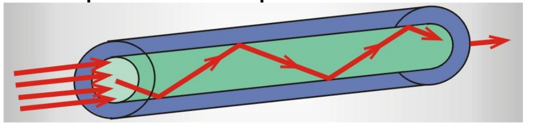
is called the *refractive index* of the medium

Dielectric constant is a complex

$$\varepsilon(\omega) = (n(\omega) + i\alpha c / 2\omega)^2$$

- $n(\omega)$ refractive index
- $\alpha(\omega)$ losses (absorption)

- For example, in some dielectric media we can neglect the imaginary component of dielectric constant , because the loss in optical fiber is low in spectrum range of interest for fiber optics techniques



COMPLEX DIELECTRIC CONSTANT

- In some dielectric media is complex. This leads to a complex wave vector . For a wave propagating in the –x direction we obtain

$$\boldsymbol{E} = \boldsymbol{E}_0 \exp[\mathrm{i}(\mathrm{Re}(k)x - \omega t)]\exp[-\mathrm{Im}(k)x].$$

Thus, a complex dielectric constant leads to the attenuation (or amplification) of the wave as it propagates through the medium in question.

Thus, the phase velocity of the wave is determined by the real part of the refractive index via

$$v = \frac{c}{\mathrm{Re}(n)}.$$

DISPERSION RELATION

Let us investigate an electromagnetic wave propagating through a transparent, isotropic, non-conducting, medium. The electric displacement inside the medium is given by

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P},$$

- where P is the electric polarization. Since electrons are much lighter than ions (or atomic nuclei), we would expect the former to displace further than the latter under the influence of an electric field. Thus, to a first approximation the polarization is determined by the electron response to the electric field .
- Suppose that the electrons displace a s distance from their rest positions in the presence of the wave. It follows that

$$\boldsymbol{P} = -N\,e\,\boldsymbol{s},$$

where N is the number density of electrons.

Let us assume that the electrons are bound ``quasi-elastically'' to their rest positions, so that they seek to return to these positions when displaced from them by a field E, It follows that s satisfies the differential equation of the form

$$m\,\ddot{\boldsymbol{s}} + f\,\boldsymbol{s} = -e\,\boldsymbol{E},$$

- where m is the electron mass, -fs is the restoring force, and $g\,\omega_0\,\dot{s}$ denotes a partial derivative with respect to time. The above equation can also be written

$$\ddot{s} + g\,\omega_0\,\dot{s} + \omega_0^{2} s = -\frac{e}{m} E,$$

- Let us assume that the electrons oscillate in sympathy with the wave at the wave frequency, where

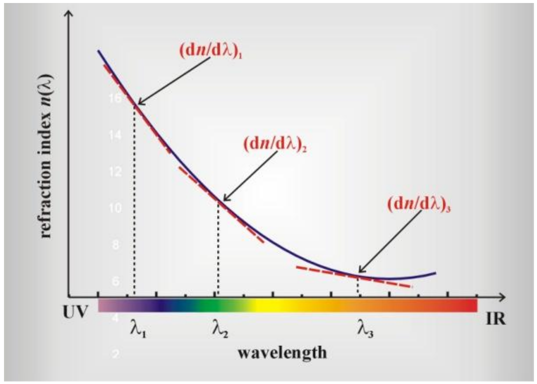
$$\omega_0^{2} = \frac{f}{m}$$

In order to take into account the fact that an electron excited by an impulsive electric field does not oscillate for ever. We add the damping term

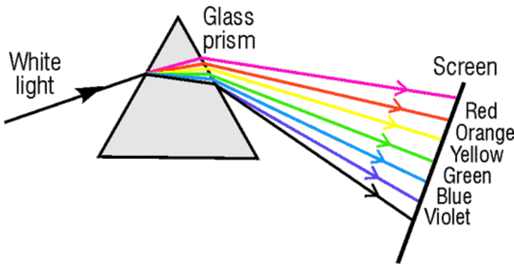
$$g\,\omega_0\,\dot{s}$$

$$s = -\frac{(e/m)\,E}{\omega_0^{2} - \omega^{2} - \mathrm{i}\,g\,\omega\,\omega_0}.$$

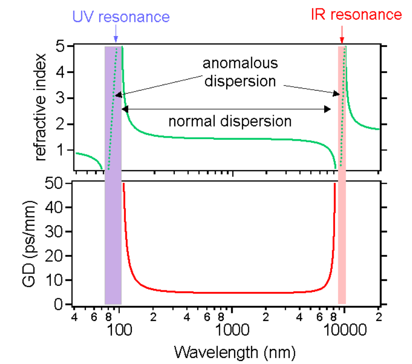
$$P = \frac{(N\,e^{2}/m)\,E}{\omega_0^{2} - \omega^{2} - \mathrm{i}\,g\,\omega\,\omega_0}.$$



Since by definition



$$D = \epsilon_0 \epsilon E = \epsilon_0 E + P,$$



it follows that

$$\epsilon(\omega) \equiv n^2(\omega) = 1 + \frac{(N e^2/\epsilon_0 m)}{\omega_0^2 - \omega^2 - i g \omega \omega_0}.$$

Thus, the index of refraction is frequency dependent. Since ω_0 typically lies in the ultraviolet region of the spectrum (and since $g \ll 1$ it is clear that the denominator

$$\omega_0^2 - \omega^2 - i g \omega \omega_0 \simeq \omega_0^2 - \omega^2$$

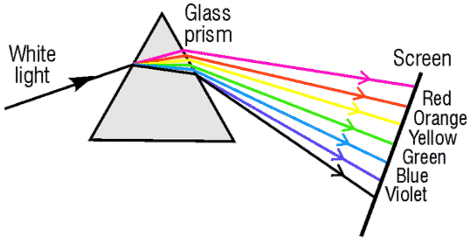
is positive in the entire visible spectrum, and is larger at the red end than at the blue end. This implies that *blue light is refracted more than red light*. This is normal dispersion. Incidentally, an expression, like the above, which specifies the dispersion of waves propagating through some dielectric medium is usually called a *dispersion relation*.

- Let us now suppose that there are N molecules per unit volume with Z electrons per molecule, and that instead of a single oscillation frequency for all electrons, there are fi electrons per molecule with oscillation frequency ωi and damping constant gi. It is easily demonstrated that

$$n^2(\omega) = 1 + \frac{N e^2}{\epsilon_0 m} \sum_i \frac{f_i}{\omega_i^2 - \omega^2 - i\, g_i\, \omega\, \omega_i},$$

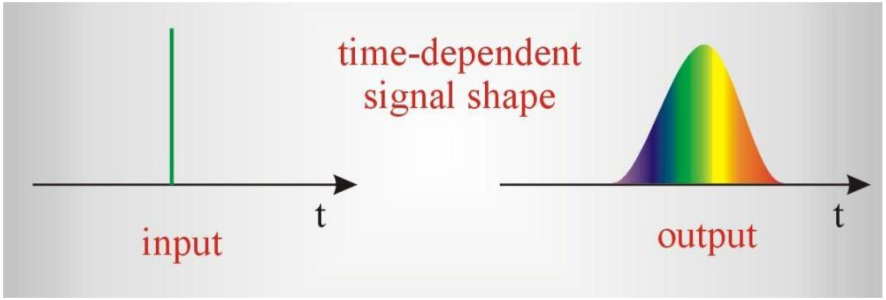
where the *oscillator strengths* fi satisfy the sum rule,

$$\sum_i f_i = Z.$$

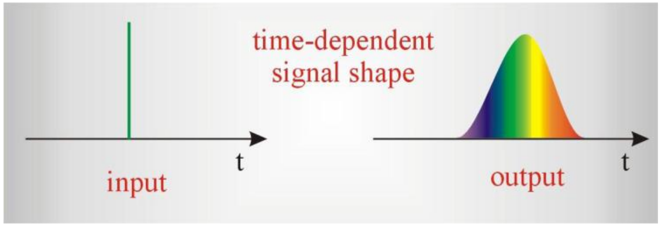


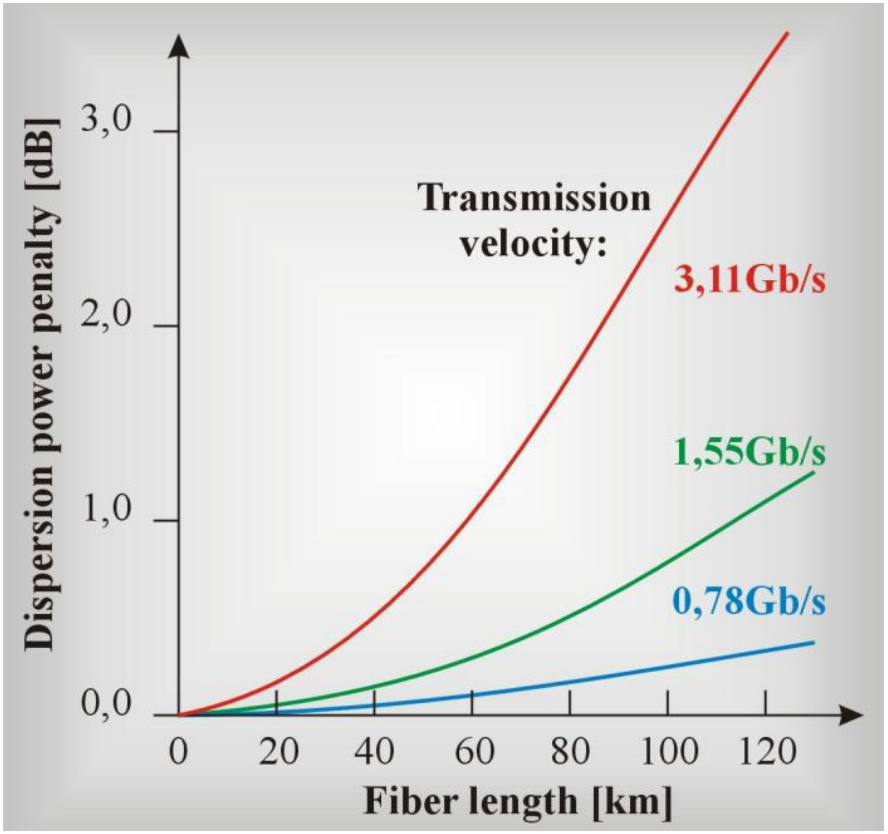
DISPERSION

- Up to now, we have tacitly assumed that n is the same for waves of all frequencies. In practice, this is not the case. In dielectric media is, in general, complex, and varies (in some cases, strongly) with the wave frequency. Thus, waves of different frequencies propagate through a dielectric medium with different phase velocities. This phenomenon is known as *dispersion*. Moreover, there may exist frequency bands in which the waves are attenuated (*i.e.*, absorbed). All of this makes the problem of determining the behaviour of a wave packet as it propagates through a dielectric medium far from straightforward. Recall, that the solution to this problem for a wave packet traveling through a vacuum is fairly trivial. The packet propagates at the velocity without changing its shape. What is the equivalent result for the case of a dielectric medium? This is an important question, since nearly all of our information regarding the universe is obtained from the study of electromagnetic waves emitted by distant objects. All of these waves have to propagate through dispersive media (*e.g.*, the interstellar medium, the ionosphere, the atmosphere) before reaching us. It is, therefore, vitally important that we understand which aspects of these wave signals are predominantly determined by the wave sources, and which are strongly modified by the dispersive media through which they have propagated in order to reach us.
- The study of wave propagation through dispersive media was pioneered by two scientists, Arnold Sommerfeld and Léon Brillouin, during the first half of this century



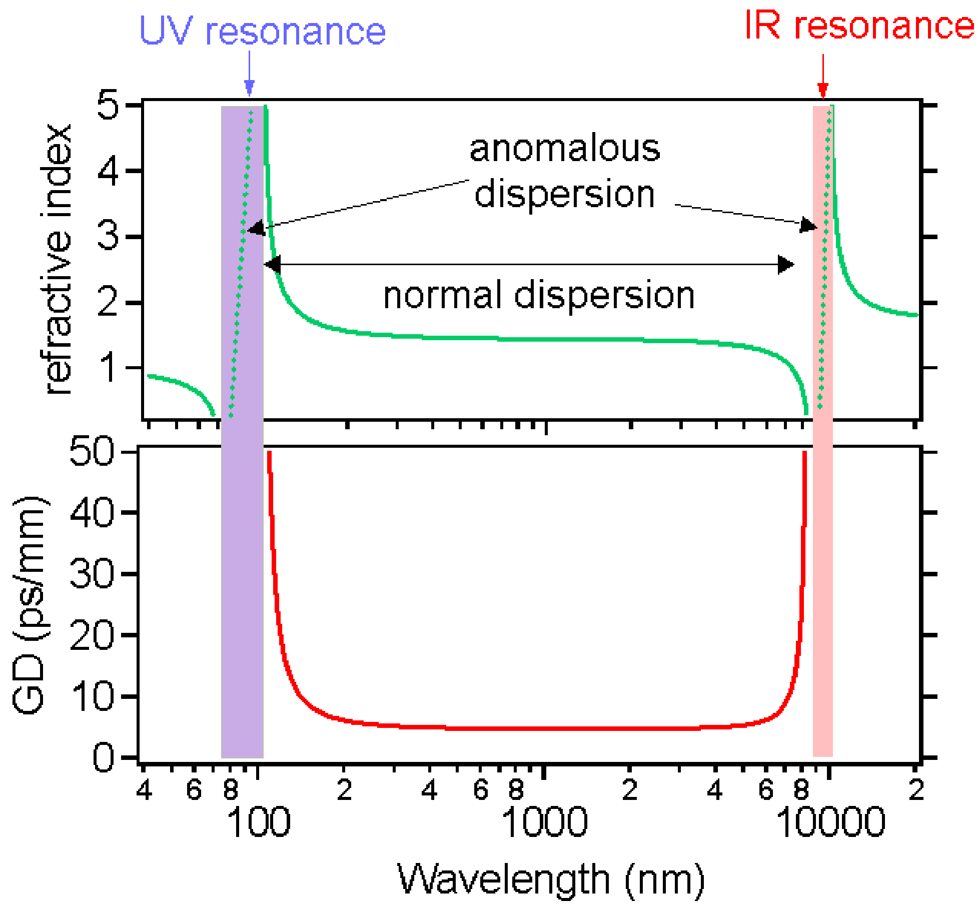
Dispersion





Attenuation caused by dispersion at transmission speed a) 0.78 Gb/s, b) 1.33 Gb/s, c) 3.11 Gb/s for the optical fiber characterized by the chromatic dispersion of 17 ps/nm/km and propagating the light from the single-mode laser DFB at spectral width of 0.1 nm

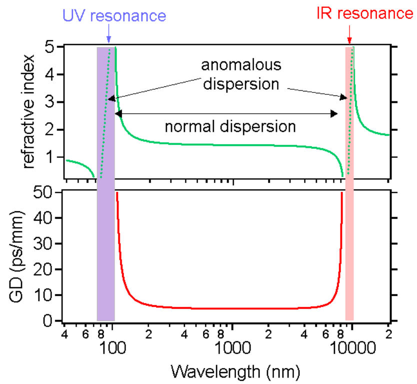
Normal dispersion
Anomalous dispersion and resonant absorption



$$\varepsilon(\omega) = (n(\omega) + i\alpha c / 2\omega)^2$$

$\mathrm{Im}(n)$

$\mathrm{Re}(n)$



- These curves are indicative of the variation of $\mathrm{Re}(n)$ and $\mathrm{Im}(n)$, respectively, with frequency in the vicinity of the resonant frequency ω_i . Recall that normal dispersion is associated with an increase in $\mathrm{Re}(n)$
- with increasing ω . The reverse situation is termed *anomalous dispersion*. It is clear from the figure that normal dispersion occurs everywhere except in the immediate neighbourhood of the resonant frequency ω_i . It is also clear that the imaginary part of the refractive index is only appreciable in those regions of the electromagnetic spectrum where anomalous dispersion takes place. A positive imaginary component of the refractive index implies that the wave is absorbed as it propagates through the medium, so the regions of the spectrum where $\mathrm{Im}(n)$ is appreciable are called regions of *resonant absorption*. Anomalous dispersion and resonant absorption take place in the vicinity of the resonance when $|\omega - \omega_i| \lesssim O(g_i)$. Since the damping constants g_i are, in practice, very small compared to unity, the regions of the spectrum in which resonant absorption takes place are strongly localized in the vicinity of the various resonant frequencies.
- The dispersion relation only takes electron resonances into account. Of course, there are also resonances associated with displacements of the ions (or atomic nuclei). The off-resonance contributions to the right-hand side of Eq. (4.18) from the ions are smaller than those from the electrons by a factor of order m/M (where is a typical ion mass). Nevertheless, the ion contributions are important because they give rise to anomalous dispersion and resonant absorption close to the ion resonant frequencies. The ion resonances associated with the stretching and bending of molecular bonds typically lie in the infrared region of the electromagnetic spectrum. Those associated with molecular rotation (these resonances only affect the dispersion relation if the molecule is polar) occur in the microwave region of the spectrum. Thus, both air and water exhibit strong resonant absorption of electromagnetic waves in both the ultraviolet and infrared regions of the spectrum. In the first case this is due to electron resonances, and in the second to ion resonances. The visible region of the spectrum exists as a narrow window lying between these two regions in which there is comparatively little attenuation of electromagnetic waves.

ENERGY TRANSPORT

- **Energy in Electromagnetic Waves**
- Electromagnetic waves carry energy as they travel through empty space. There is an energy density associated with both the electric and magnetic fields. The rate of energy transport per unit area is described by the vector
- $$\vec{S} = \frac{1}{\mu_0} \vec{E} \ x \ \vec{B}$$
- which is called the Poynting vector. This expression is a vector product, and since the magnetic field is perpendicular to the electric field, the magnitude can be written

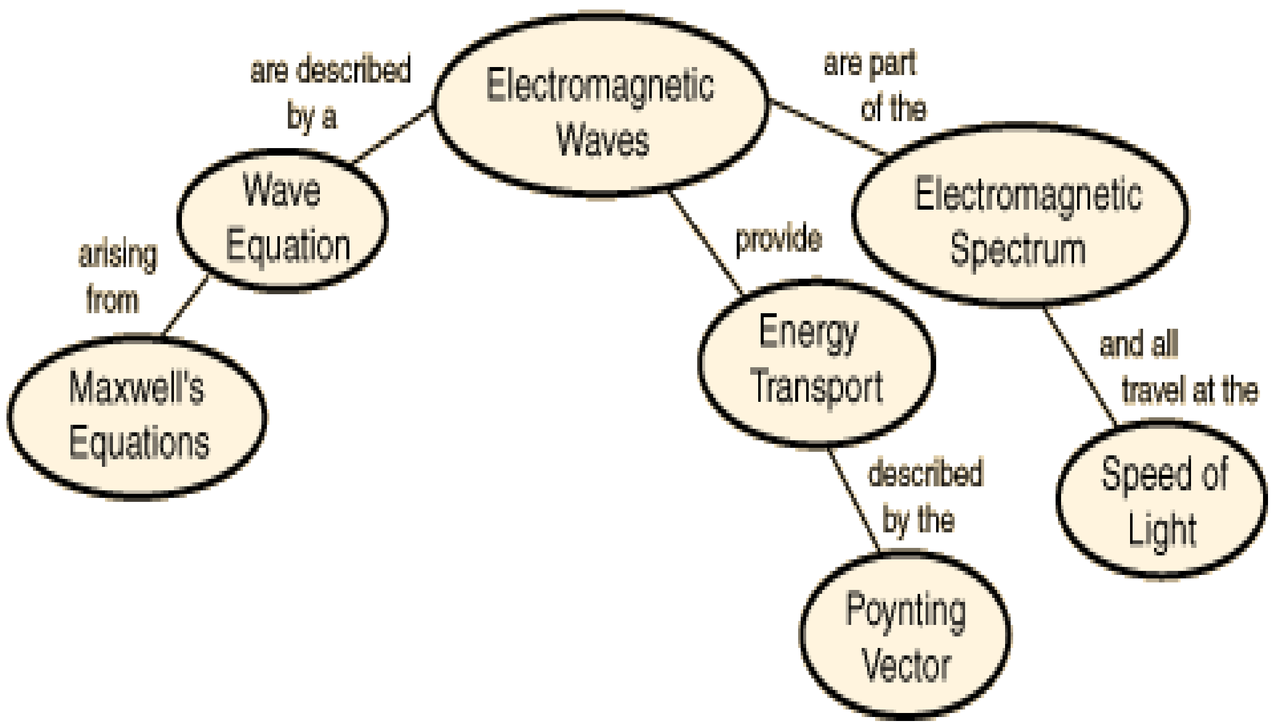
$$S = \frac{1}{\mu_0} EB$$

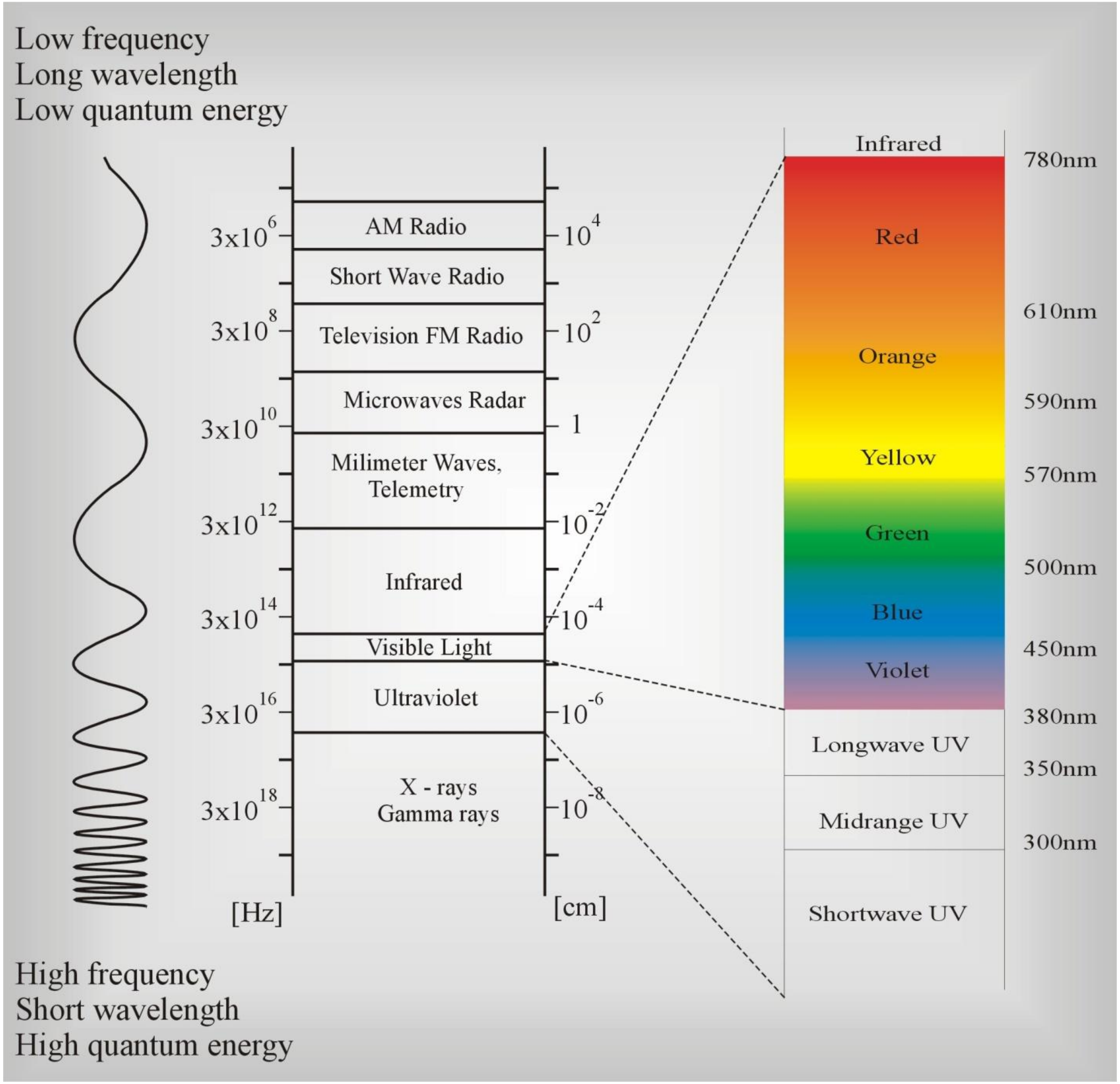
- The rate of energy transport S is perpendicular to both E and B and in the direction of propagation of the wave. A condition of the wave solution for a plane wave is $B_m = E_m/c$ so that the average intensity for a plane wave can be written

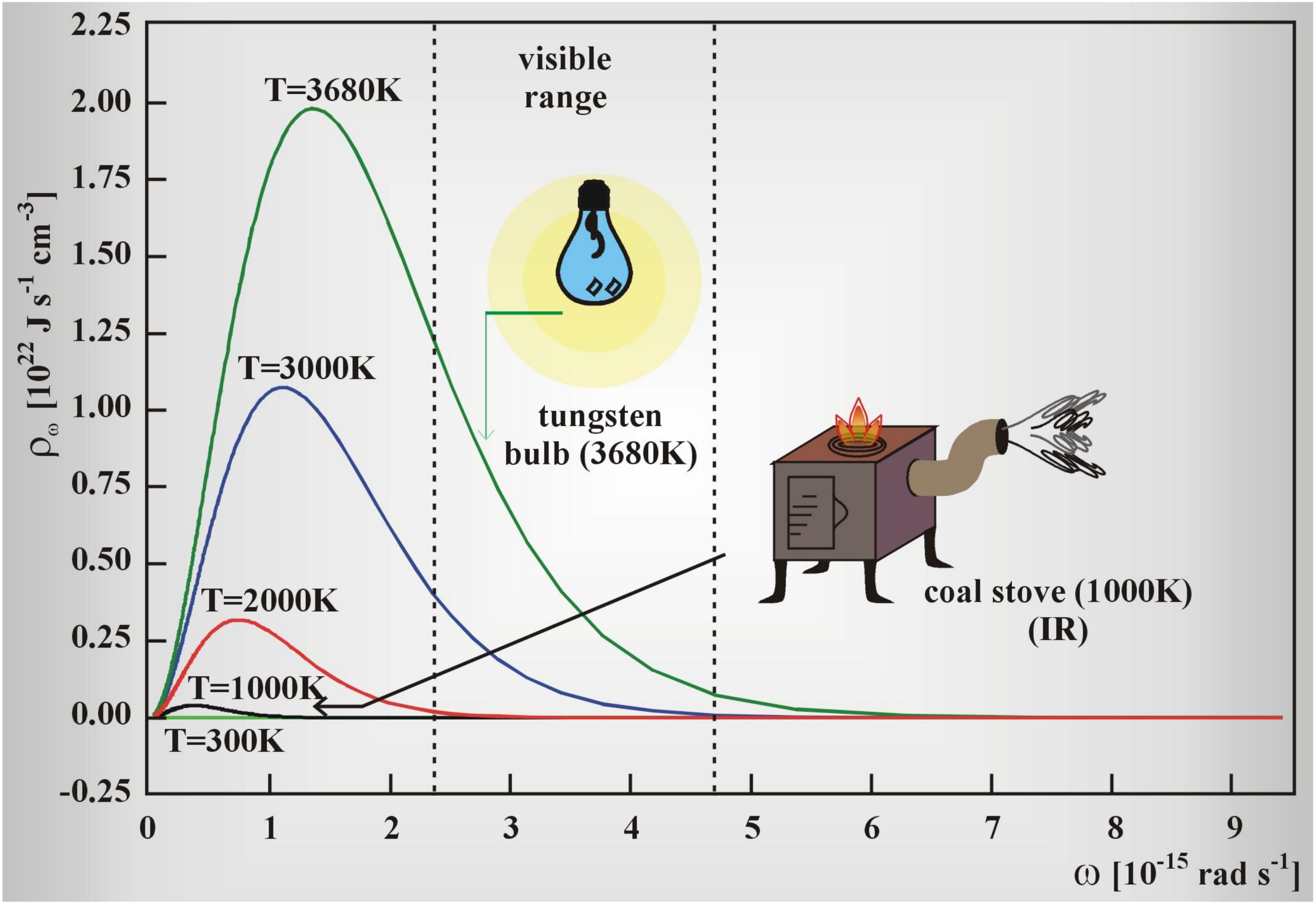
$$S = \frac{1}{c\mu_0} E_m^2 \overline{\sin^2(kx - \omega t)} = \frac{1}{c\mu_0} \frac{E_m^2}{2}$$

- This makes use of the fact that the average of the square of a sinusoidal function over a whole number of periods is just 1/2.

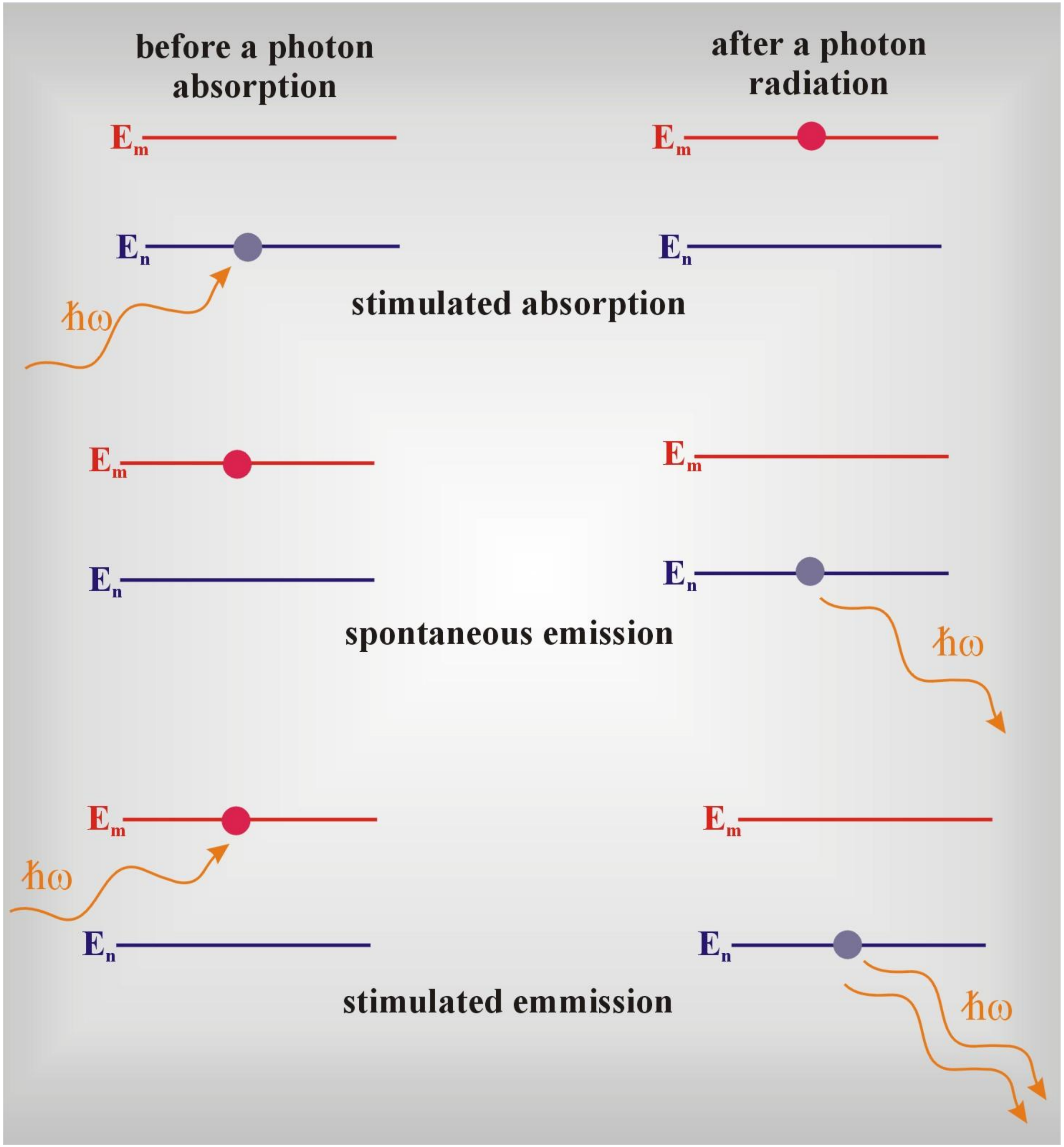
CONCLUSIONS-1

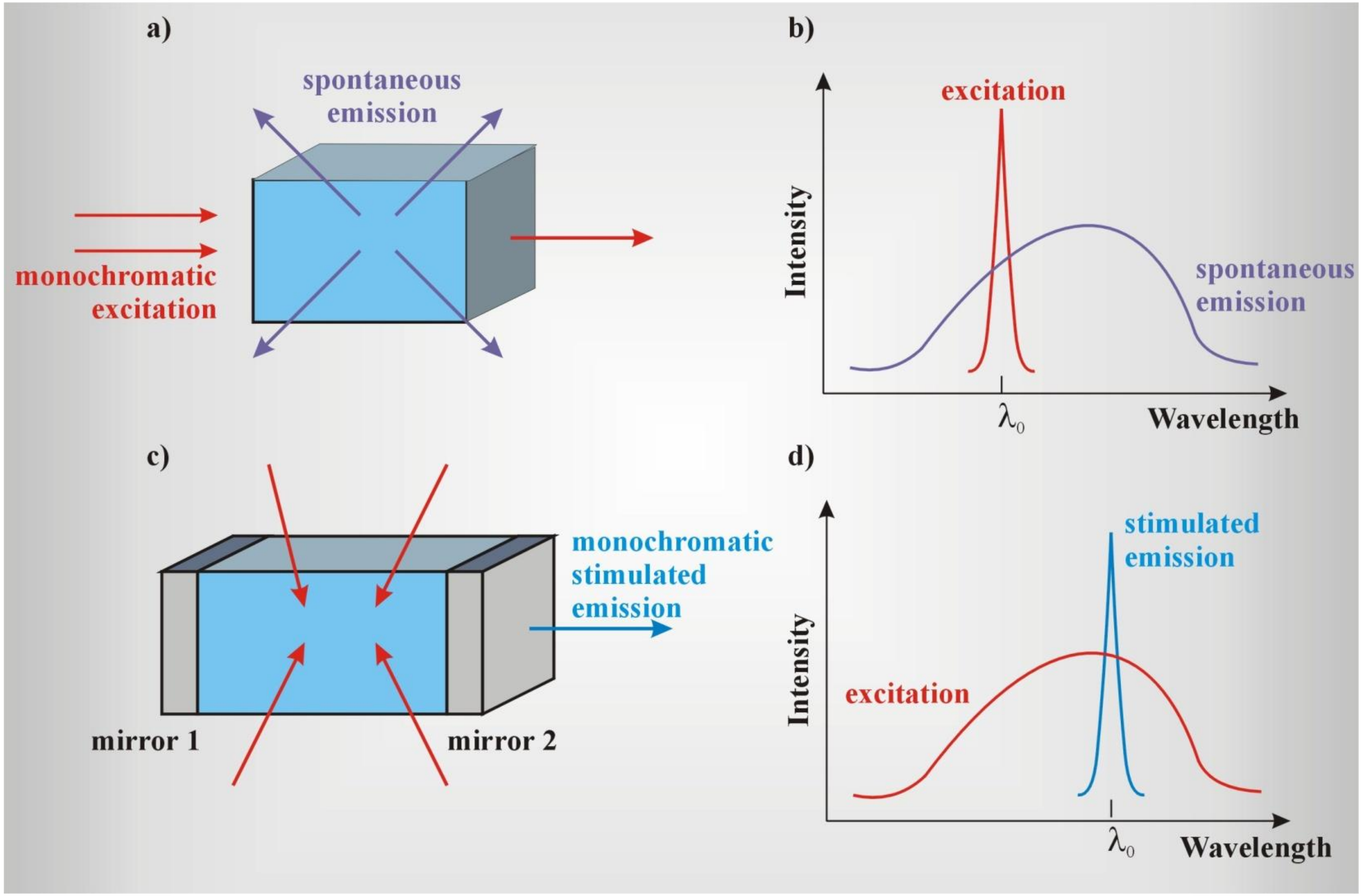


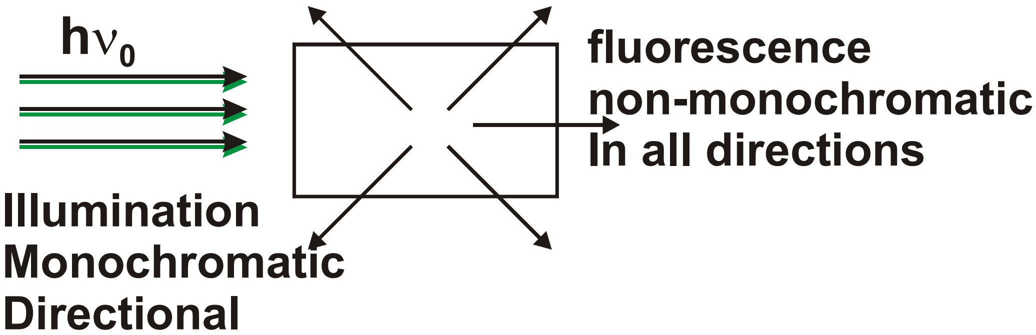

Low frequency
Long wavelength
Low quantum energy
AM Radio
Short Wave Radio
Television FM Radio
Microwaves Radar
Milimeter Waves, Telemetry
Infrared
Visible Light
Ultraviolet
X - rays
Gamma rays
3x10^6
3x10^8
3x10^10
3x10^12
3x10^14
3x10^16
3x10^18
[Hz]
10^4
10^2
1
10^-2
10^-4
10^-6
10^-8
[cm]
Infrared
Red
Orange
Yellow
Green
Blue
Violet
Longwave UV
Midrange UV
Shortwave UV
780nm
610nm
590nm
570nm
500nm
450nm
380nm
350nm
300nm
High frequency
Short wavelength
High quantum energy

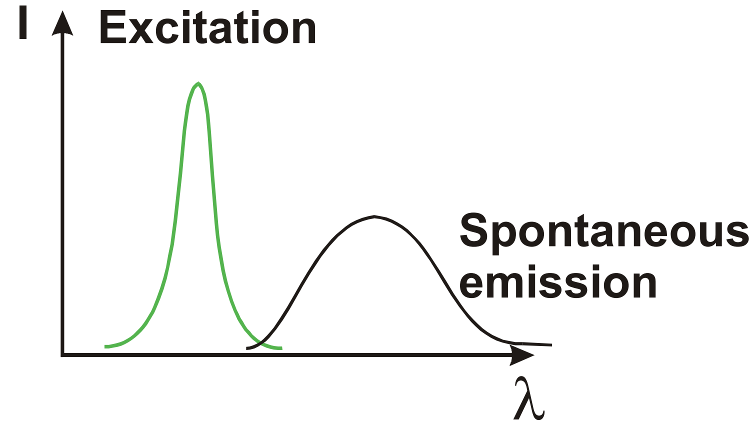


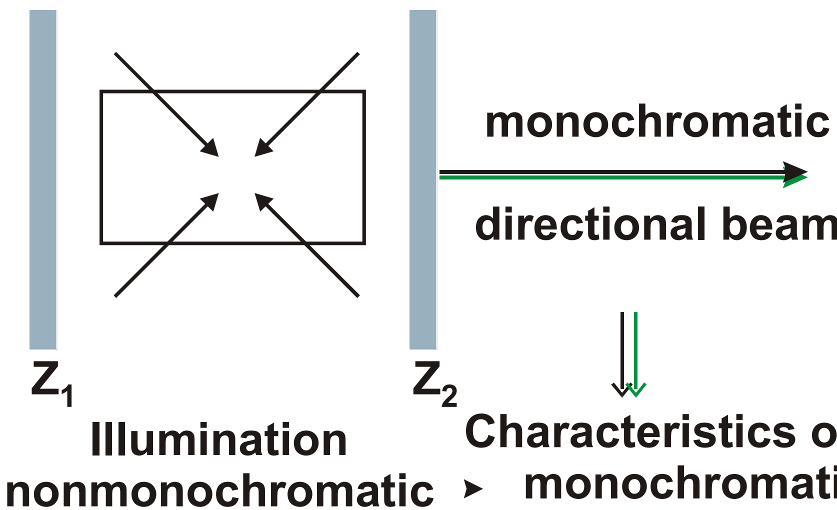
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
ρ_ω [10^{22} J s^{-1} cm^{-3}]
visible range
T=3680K
T=3000K
T=2000K
T=1000K
T=300K
tungsten bulb (3680K)
coal stove (1000K) (IR)
0
1
2
3
4
5
6
7
8
9
ω [10^{-15} rad s^{-1}]

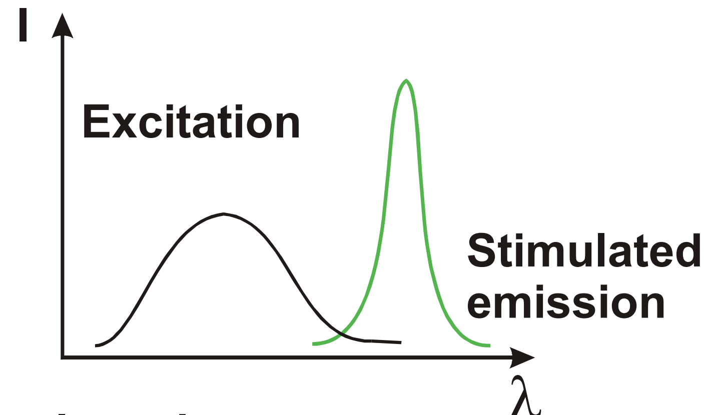

before a photon absorption
after a photon radiation
E_m
E_n
$\hbar\omega$
stimulated absorption
E_m
E_n
spontaneous emission
$\hbar\omega$
E_m
E_n
$\hbar\omega$
stimulated emmission
$\hbar\omega$


a)
spontaneous
emission
monochromatic
excitation
b)
excitation
Intensity
spontaneous
emission
λ_0
Wavelength
c)
monochromatic
stimulated
emission
mirror 1
mirror 2
d)
stimulated
emission
Intensity
excitation
λ_0
Wavelength



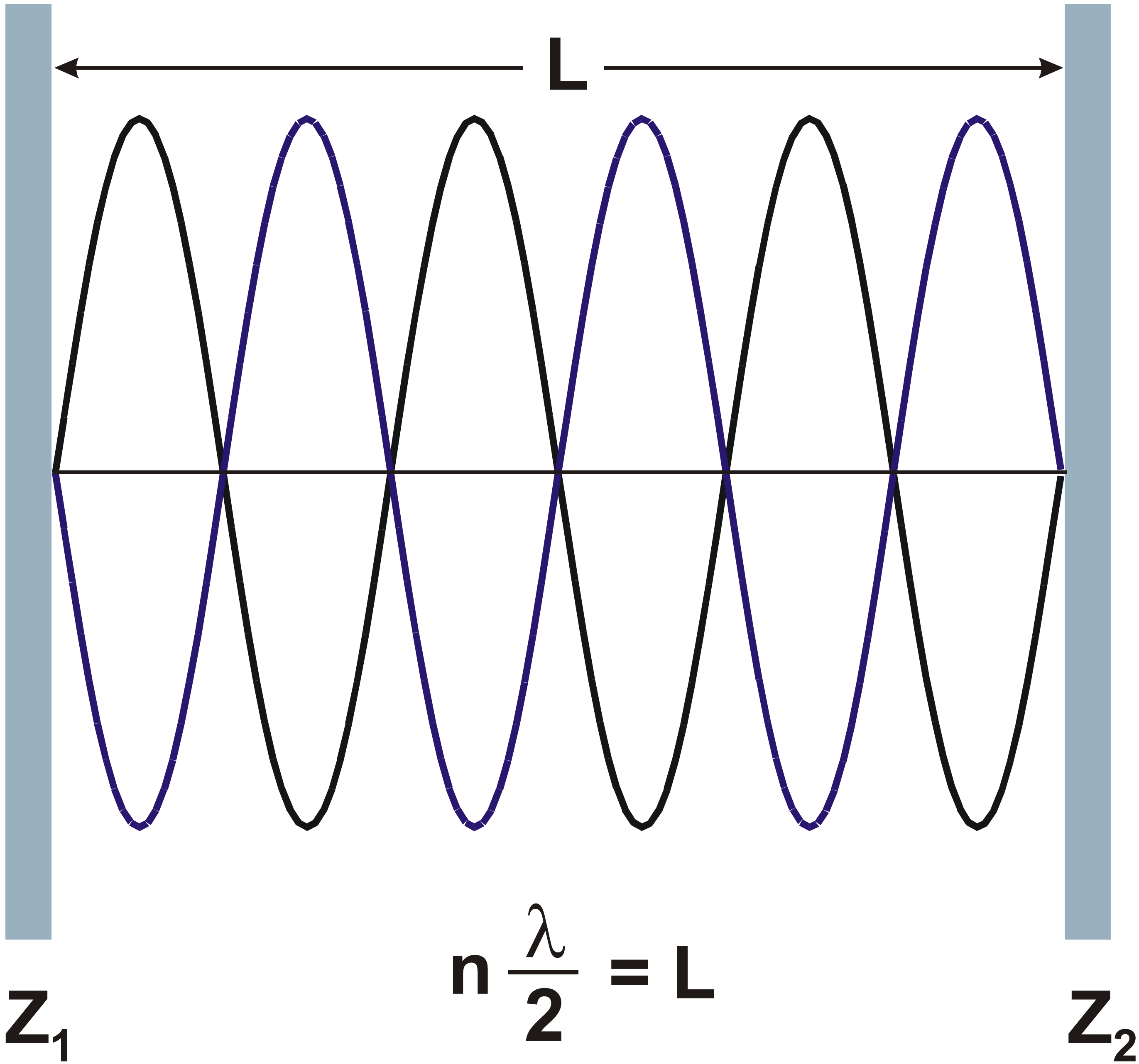


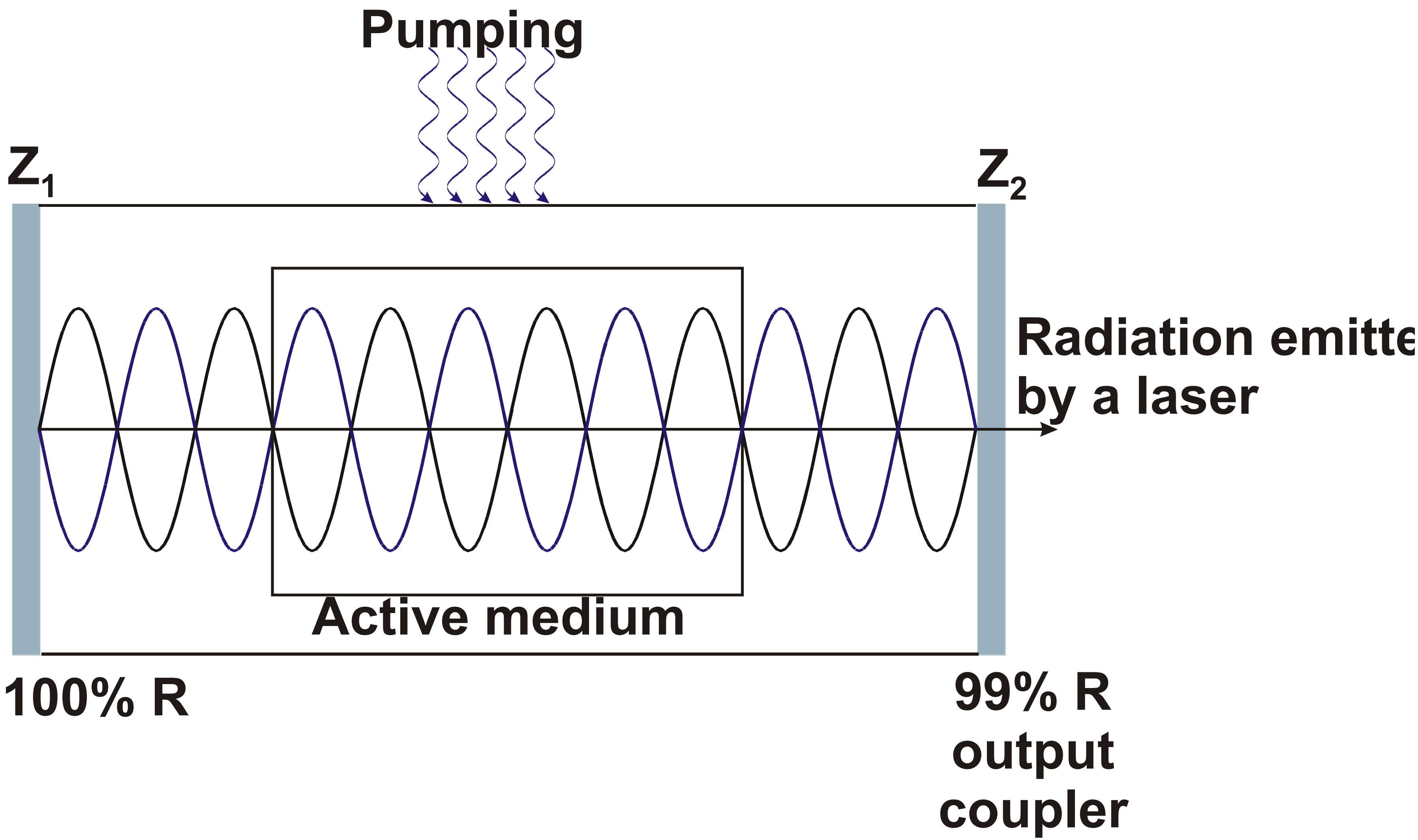




Characteristics of the laser beam:

- monochromatic
- directional
- polarized
- coherent


L
$n \frac{\lambda}{2} = L$
Z_1
Z_2


Pumping
Z_1
Z_2
Radiation emitte
by a laser
Active medium
100% R
99% R
output
coupler

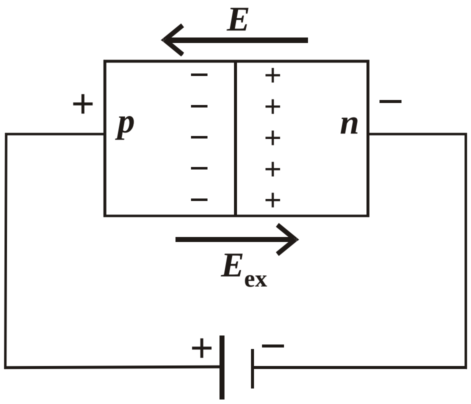
REGENERATIVE FEEDBACK – amplification related to many reflections from the mirrors in the optical

$$G^{2n} = \frac{I_0^{2n}}{I_0} = \left(R_1 \times R_2\right)^n \exp\left[-2n\left(\overset{<0}{\beta} + \overset{>0}{\alpha_s}\right) l\right]$$

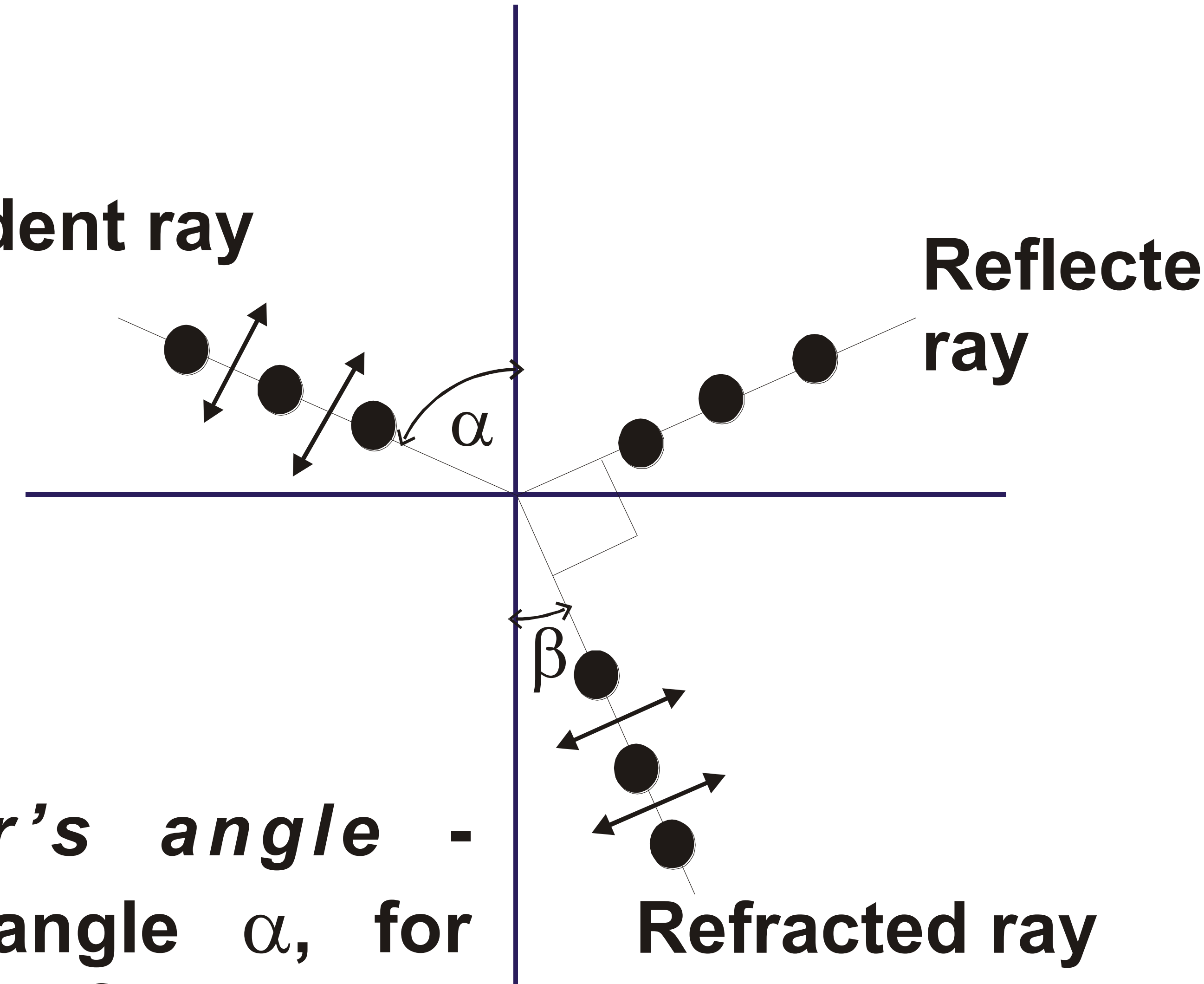
When the intensity reaches the stationary value – we speak about the cw (continuous wave) lasers. It happens, when the gains are equal to the losses after twice passage through the resonator. The value of β, for which $G^{(2)} = 1$ is called the threshold gain

METHODS OF *„PUMPING”*

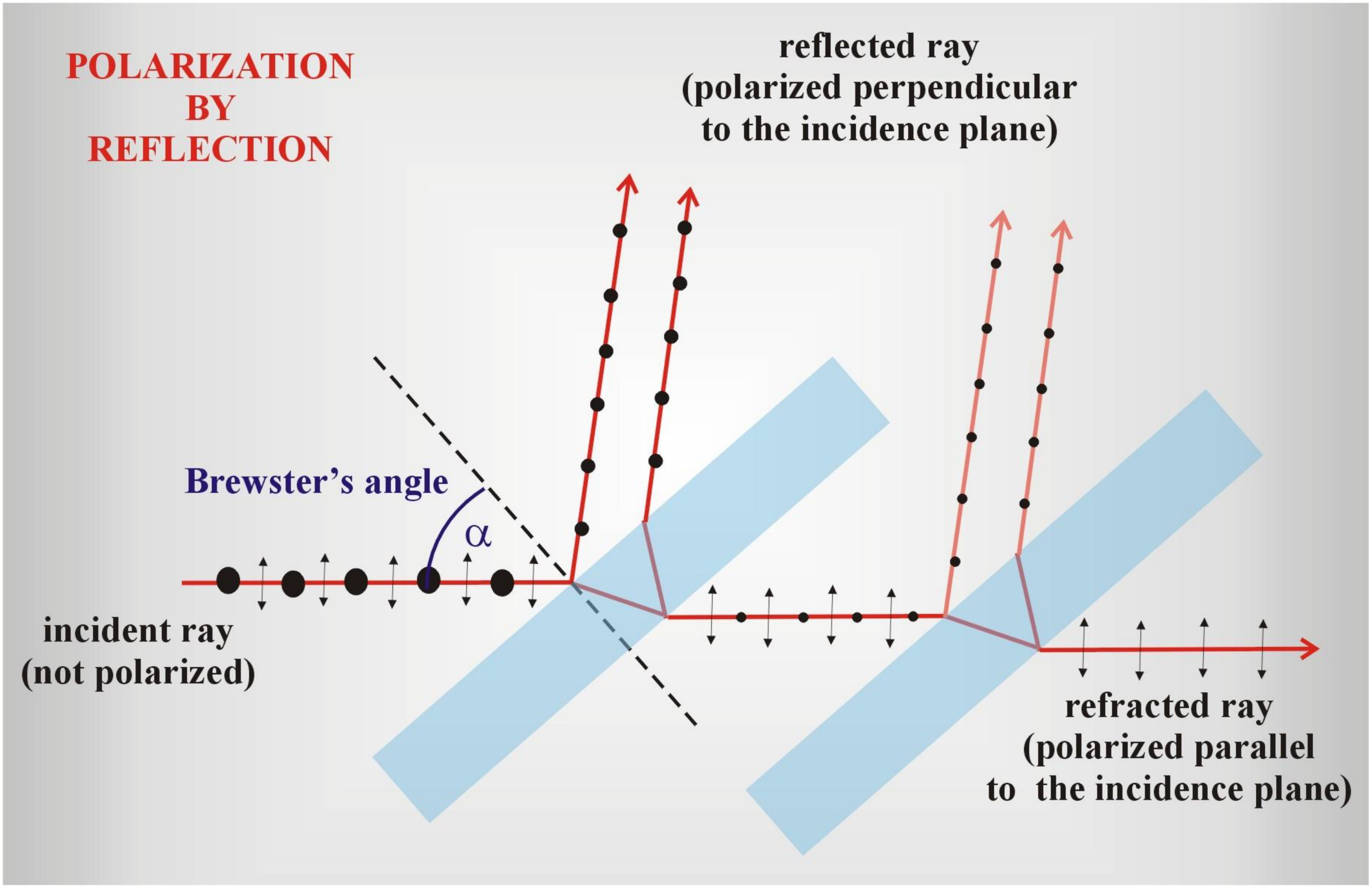
1. **OPTICAL - pumping by lasers (DYE LASERS)**
2. **OPTICAL- pumping by lamps(SOLID STATE LASERS)**
3. **ELECTRIC DISCHARGE (GAS LASERS)**
4. **external voltage supply to the p–n junction (DIODE LASERS)**



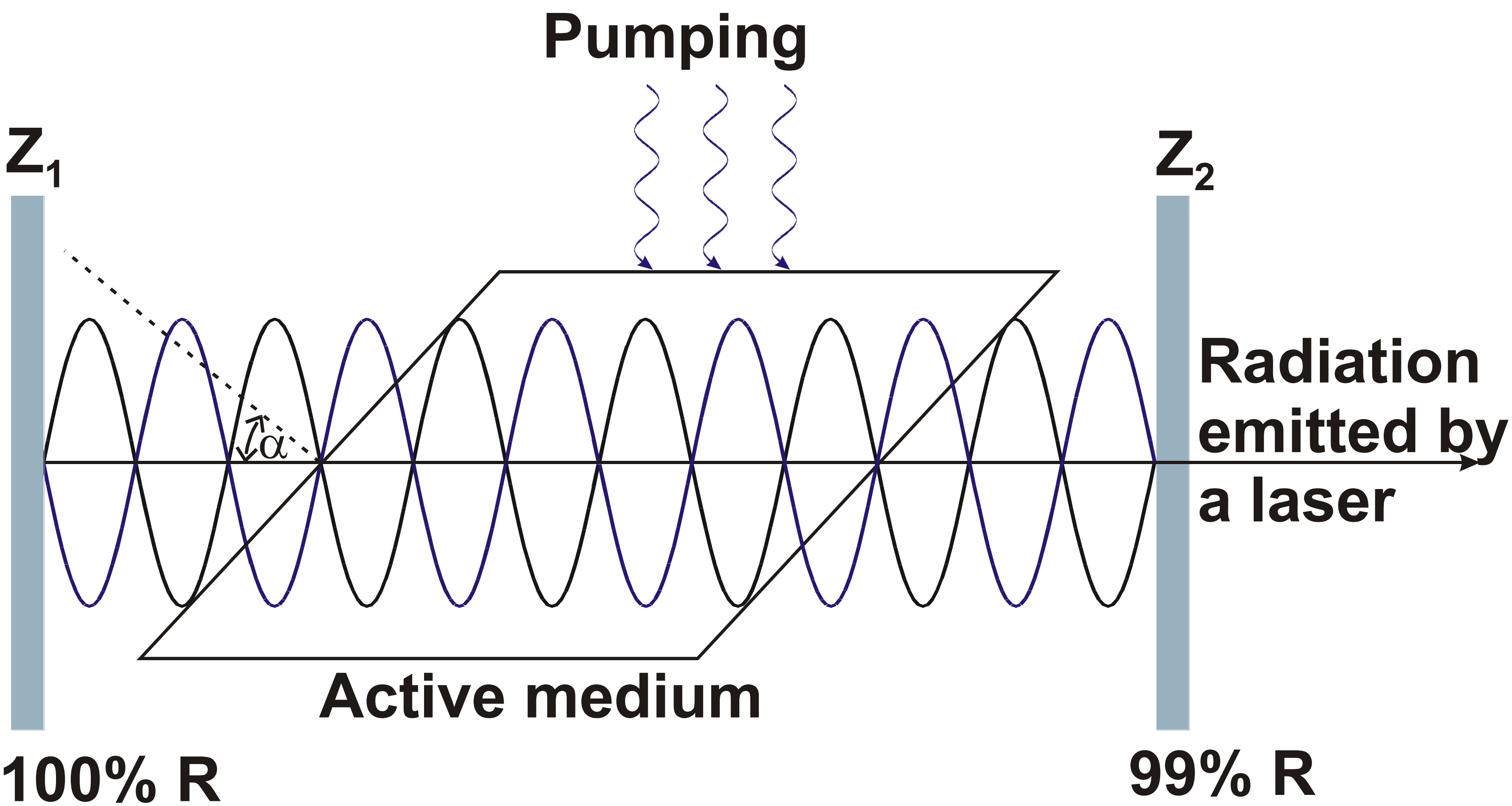
When a negative terminal of the external voltage source becomes connected to the *n* region, and the positive one to the *p* region, the *p-n* junction is said to be *forward biased*. This means that the charge carriers, both holes from the area *p* and electrons from the area *n*, flow towards the junction under the external field E_{ex} .
The external field E_{ex} is directed just opposite to the internal field *E*.
The semiconductor lasers, use the forward biased junction. On the other hand, when *the negative* terminal of the external voltage source is connected to the *p* region, and the positive one to the *n* region, the *p-n* junction is said to be *reverse biased* .

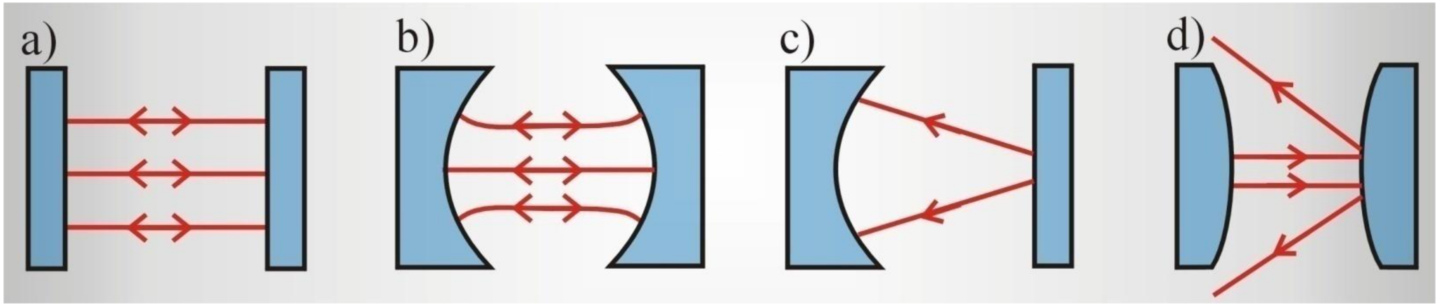


Brewster's angle **- incident angle α, for which the reflected and refracted angles form 90 degree**



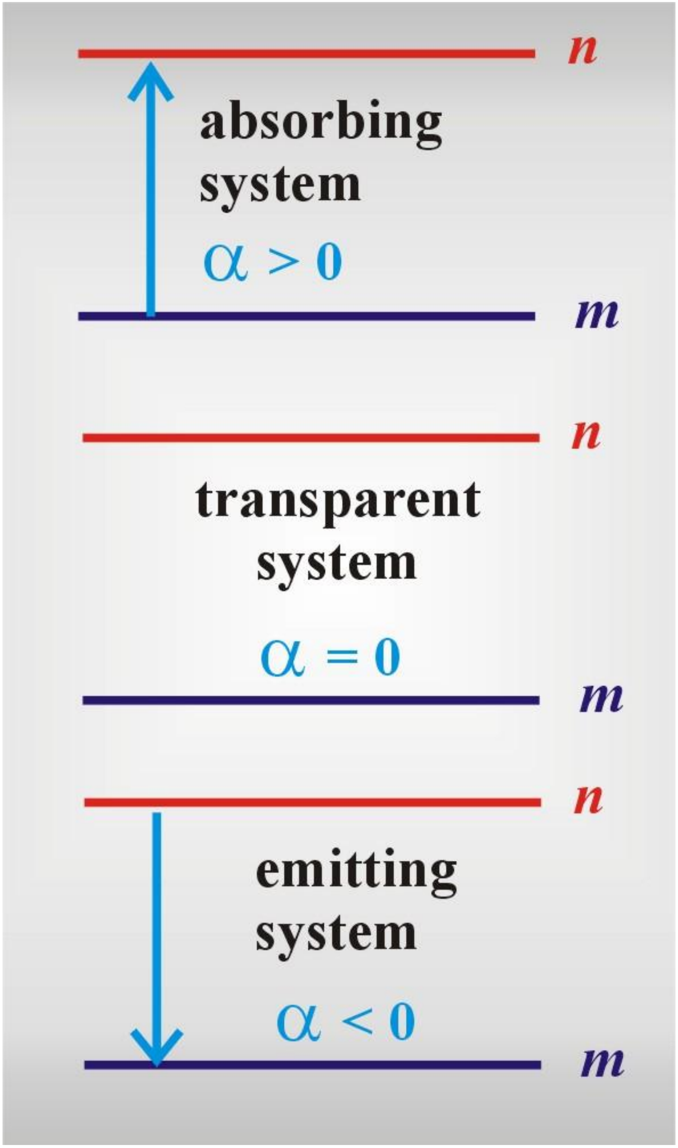
POLARIZATION
BY
REFLECTION
reflected ray
(polarized perpendicular
to the incidence plane)
Brewster's angle
α
incident ray
(not polarized)
refracted ray
(polarized parallel
to the incidence plane)


Pumping
Z1
Z2
α
Radiation emitted by a laser
Active medium
100% R
99% R



Resonators types: a) plane-parallel, b) confocal, c) hemispherical, d) unstable

Dependence of absorption coefficient on incident radiation intensity in nonlinear optics range

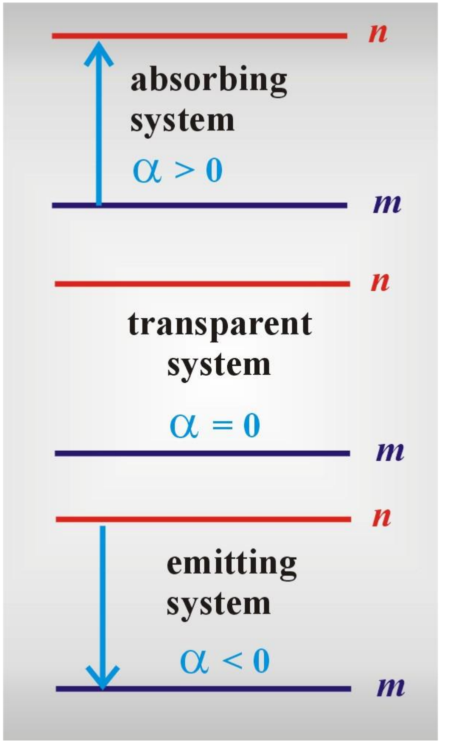


$I < I_0$ $\quad I = I_0 \exp(-\alpha l)$ $\quad \alpha > 0$

$I = I_0$ $\quad \alpha = 0$

$I > I_0$ $\quad I = I_0 \exp(-\beta l)$ $\quad \beta < 0$

β – small gain coefficient



Small signal gain equation The intensity (in watts per square meter) of the stimulated emission is governed by the following differential equation:

$$\frac{dI}{dz} = \sigma_{21}(\nu) \cdot \Delta N_{21} \cdot I(z)$$

as long as the intensity *I*(*z*) is small enough so that it does not have a significant effect on the magnitude of the population inversion. Grouping the first two factors together, this equation simplifies as

$$\frac{dI}{dz} = \gamma_0(\nu) \cdot I(z)$$

where

$$\gamma_0(\nu) = \sigma_{21}(\nu) \cdot \Delta N_{21}$$

the *small-signal gain coefficient* (in units of radians per meter). We can solve the differential equation using separation of variables:

$$\frac{dI}{I(z)} = \gamma_0(\nu) \cdot dz$$

Integrating, we find:

$$\ln\left(\frac{I(z)}{I_{in}}\right) = \gamma_0(\nu) \cdot z$$

$$I(z) = I_{in} e^{\gamma_0(\nu) z}$$

where

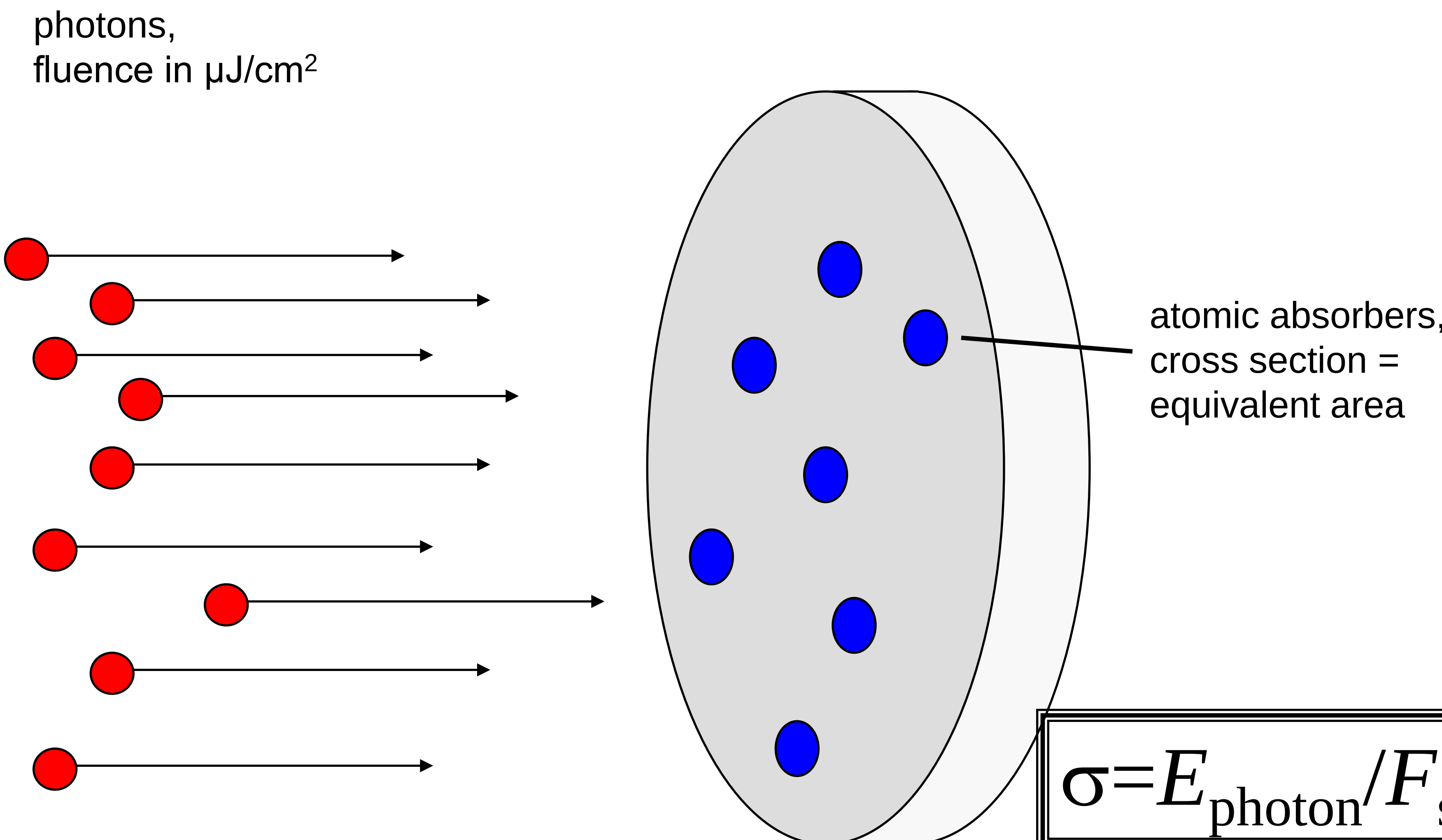
$$I_{in} = I(z = 0)$$

is the optical intensity of the input signal (in watts per square meter).

Laser gain

- Saturation fluence and cross section (most important parameter to model gain)
- Frantz-Nodvik equation
- Saturation fluence in absorbers

saturation fluence - microscopic picture



$$\sigma = E_{photon}/F_{sat}$$

Saturation fluence if on the average, one photon impinges on every atom

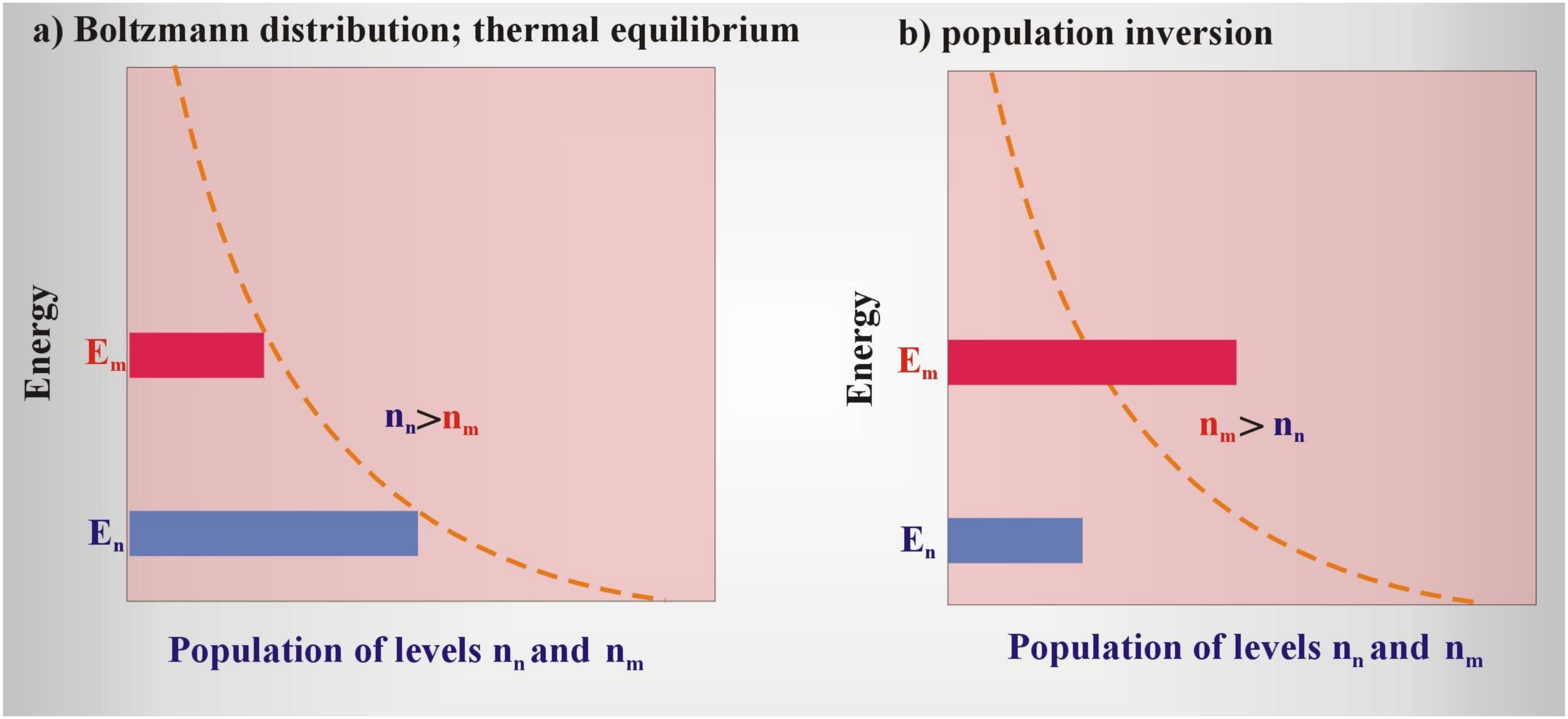
Frantz Nodvik equation

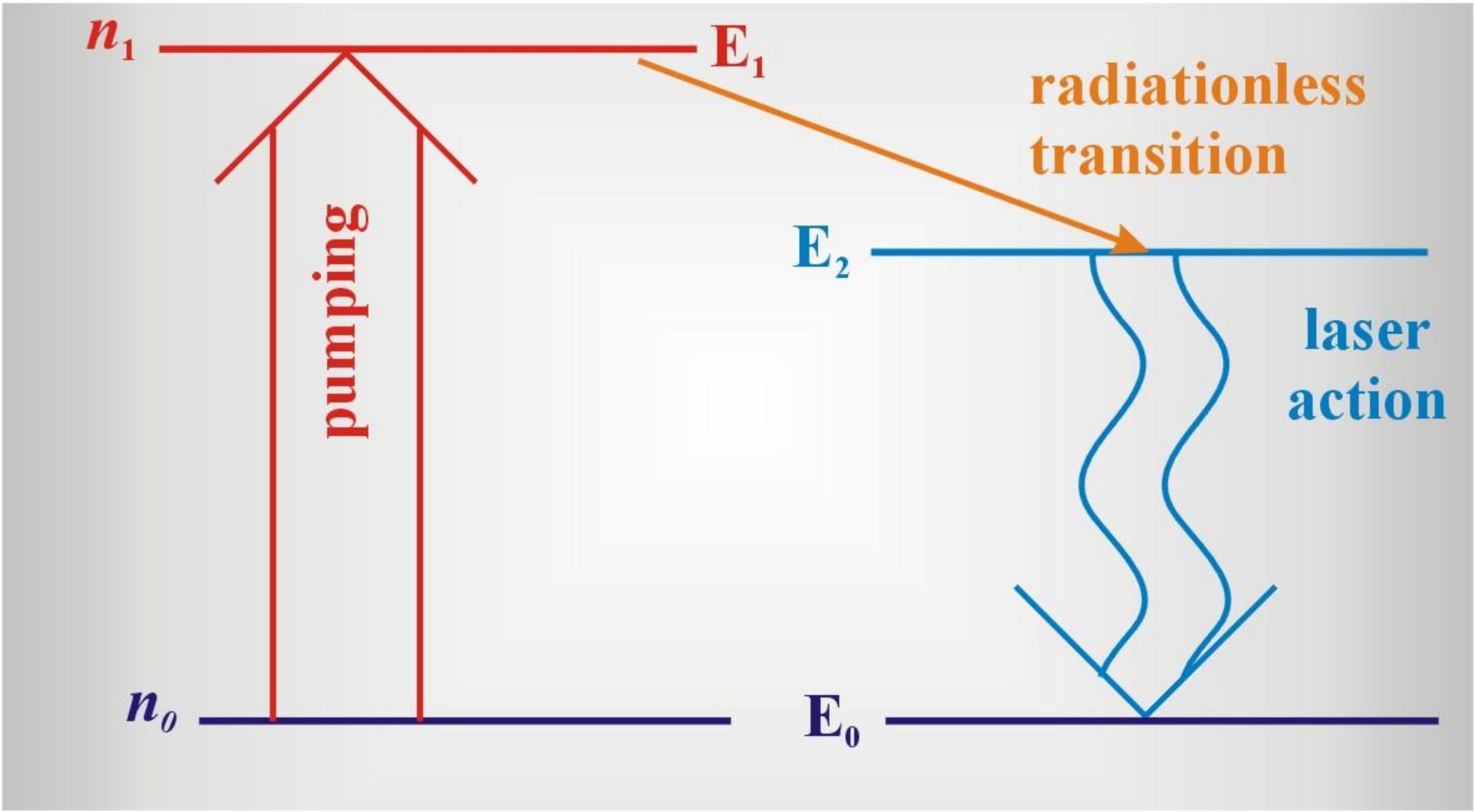
small-signal gain:

$$g_0 = \exp\left(\frac{F_{pump}}{F_{sat}}\right)$$

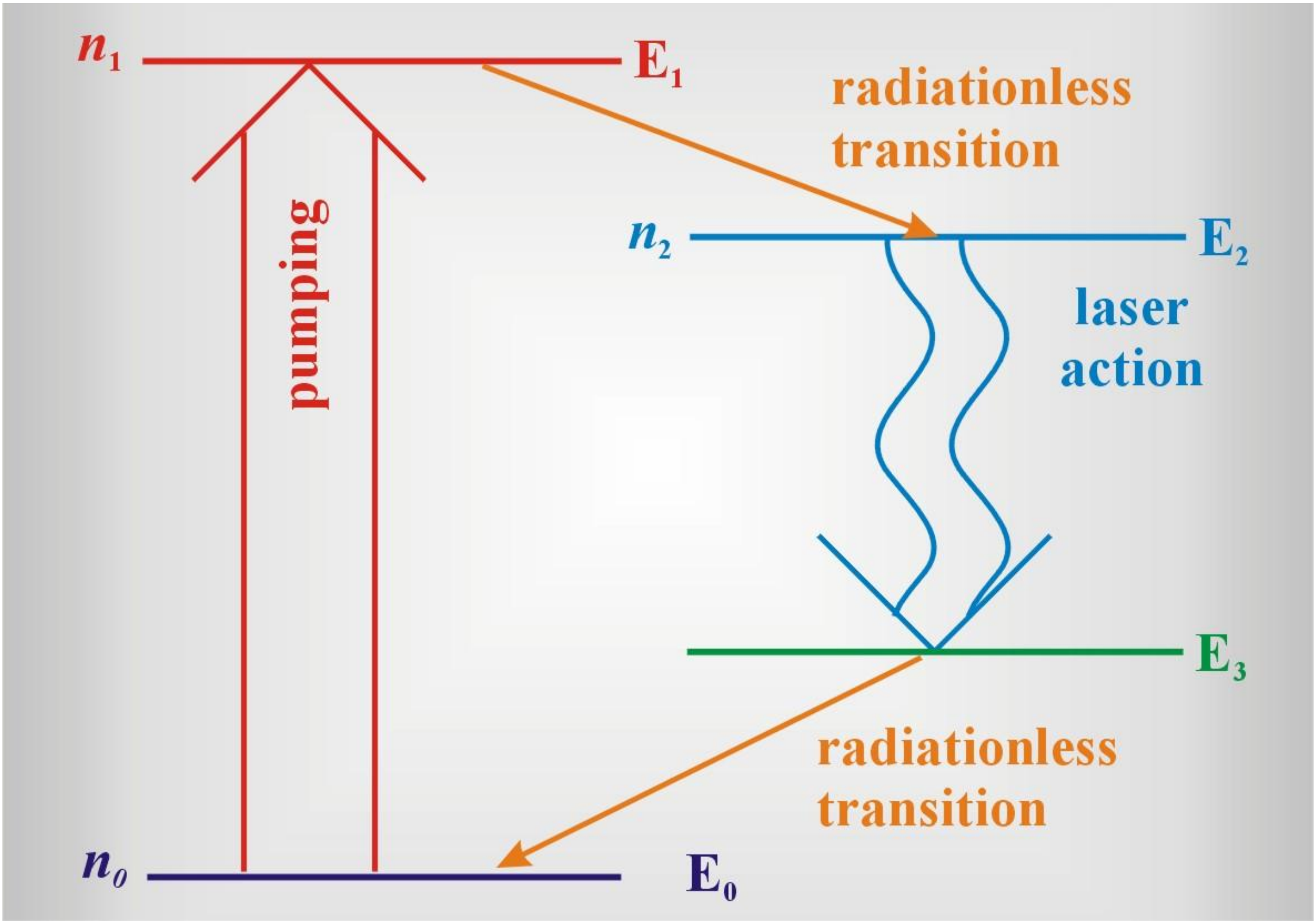
$$F_{out} = F_{sat} \ln\left[1 + \left(\exp\frac{F_{in}}{F_{sat}} - 1\right)\exp g_0\right]$$

L. M. Frantz and J. S. Nodvik, *J. Appl. Phys.*, 34, pp. 2346-2349, 1963.

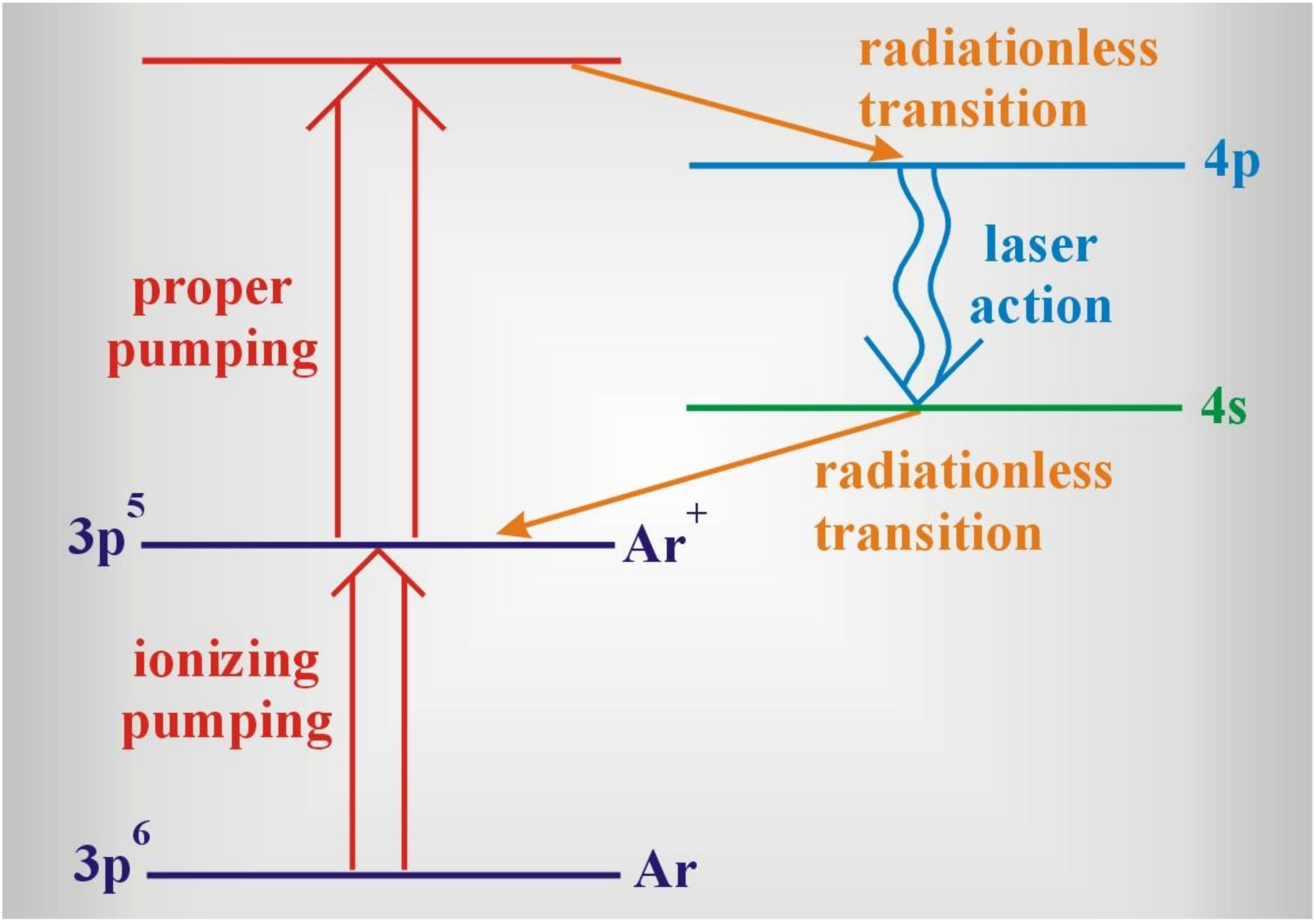

a) Boltzmann distribution; thermal equilibrium
Energy
E_m
E_n
$n_n > n_m$
Population of levels n_n and n_m
b) population inversion
Energy
E_m
E_n
$n_m > n_n$
Population of levels n_n and n_m



n1
E1
radiationless transition
pumping
E2
laser action
n0
E0



n1
E1
radiationless
transition
pumping
n2
E2
laser
action
E3
radiationless
transition
n0
E0


radiationless
transition
4p
laser
action
proper
pumping
4s
radiationless
transition
3p5
Ar+
ionizing
pumping
3p6
Ar