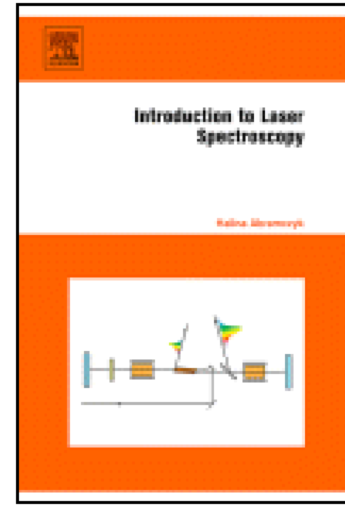
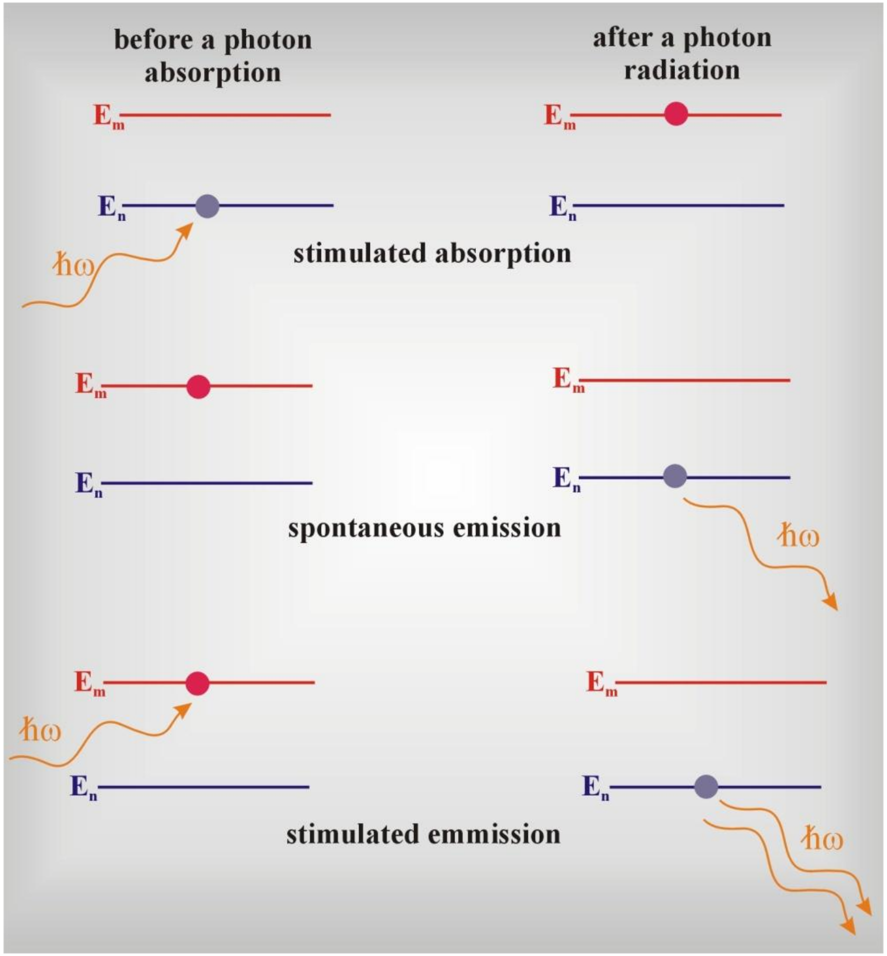


SPH 618
Optical and Laser Physics
University of Nairobi, Kenya
Lecture 2
LASER FUNDAMENTALS

Prof. Dr Halina Abramczyk
Max Born Institute for Nonlinear Optics and Ultrashort Laser Pulses, Berlin, Germany,
Technical University of Lodz, Lodz, Poland

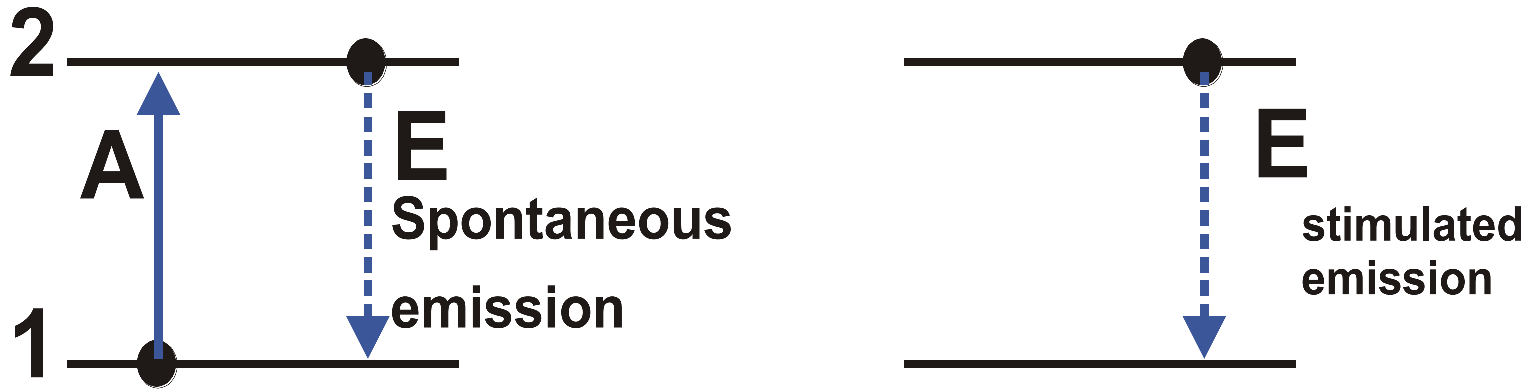
- Overview of wave propagation in various media (dielectrics, semiconductors, conductors)
- Normal and anomalous dispersion
- Emission and absorption of light
- Spontaneous and stimulated emission
- Population inversion
- Optical resonator



before a photon absorption
after a photon radiation
E_m
E_n
$\hbar\omega$
stimulated absorption
E_m
E_n
$\hbar\omega$
spontaneous emission
E_m
E_n
$\hbar\omega$
stimulated emmission

The stimulated transitions have several important properties:

- the probability of the stimulated transition between the states *m* and *n* is different from zero only for the external radiation field that is in the resonance with the transition, for which the photon energy $\hbar\omega$ of the incident radiation is equal to the energy difference between these states,
- the incident electromagnetic radiation and the radiation generated by the stimulated transitions have the same frequencies, phases, plane of polarization and direction of propagation. Thus, the stimulated emission is, in fact, completely indistinguishable from the stimulating external radiation field,
- the probability of the stimulated transitions per time unit is proportional to the energy density of the external field ρ_ω, that is the energy per unit of the circular frequency from the range between ω and $\omega + d\omega$ in the volume unit.



$$W_{1\to 2} = B_{12}\rho_\nu$$ **Absorption**

$$W_{2\to 1} = B_{21}\rho_\nu$$ **Stimulated emission**

W – probability of stimulated transitions in a time unit

ρ_ν – density of radiation field (spectral density in a volume unit)

THERMODYNAMIC EQUILIBRIUM

emsamble of quantum particles $\rightleftarrows$

radiation field

$\rightleftarrows$

$$N_{2\to1} = N_{1\to2}$$

$$W_{2\to1} \cdot n_2 = W_{1\to2} \cdot n_1$$

$$W_{2\to1} = W_{sp}^{em} + W_{wym}^{em}$$

$$W_{2\to1} = A_{21} + B_{21} \cdot \rho_\nu$$

$$W_{1\to2} = B_{12} \cdot \rho_\nu$$

$$n_2 = n_1 \exp\left(-\frac{E_2 - E_1}{kT}\right)$$

$$(A_{21} + B_{21}\rho_\nu)n_2 = B_{12}\rho_\nu n_1$$

$$(A_{21} + B_{21}\rho_\nu)n_1 \exp\left(-\frac{E_2 - E_1}{kT}\right) = B_{12}\rho_\nu n_1$$

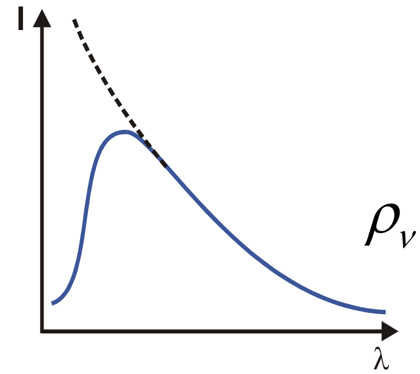
$$\rho_\nu = -\frac{A_{21}\exp\left(-\frac{E_2 - E_1}{kT}\right)}{\frac{B_{21}}{\exp\left(\frac{E_2 - E_1}{kT}\right)} - B_{12}} \Rightarrow$$

$$\rho_\nu = \frac{A_{21}}{B_{21}} \frac{1}{\left[\frac{B_{12}}{B_{21}}\exp\left(\frac{E_2 - E_1}{kT}\right) - 1\right]}$$

From the Planck equation we know that

$$\rho_\nu = -\frac{\frac{A_{21}\exp\left(-\frac{E_2-E_1}{kT}\right)}{B_{21}}}{\exp\left(\frac{E_2-E_1}{kT}\right)-B_{12}} \Rightarrow$$

$$\rho_\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{\exp\frac{h\nu}{kT}-1} \Rightarrow$$



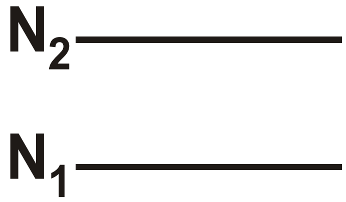
$$\rho_\nu = \frac{A_{21}}{B_{21}}\frac{1}{\left[\frac{B_{12}}{B_{21}}\exp\left(\frac{E_2-E_1}{kT}\right)-1\right]}$$

$$B_{12} = B_{21} \quad \text{i} \quad A_{21} = \frac{8\pi\nu^2}{c^3}h\nu B_{21}$$

$$\frac{A_{21}}{B_{21}} = 1{,}33\cdot 10^{-12}$$

$$\frac{A_{21}}{B_{21}\cdot\rho_\nu} = 10^{42}(T = 300K)$$

$$\sim 1 \ (T = 41500K)$$



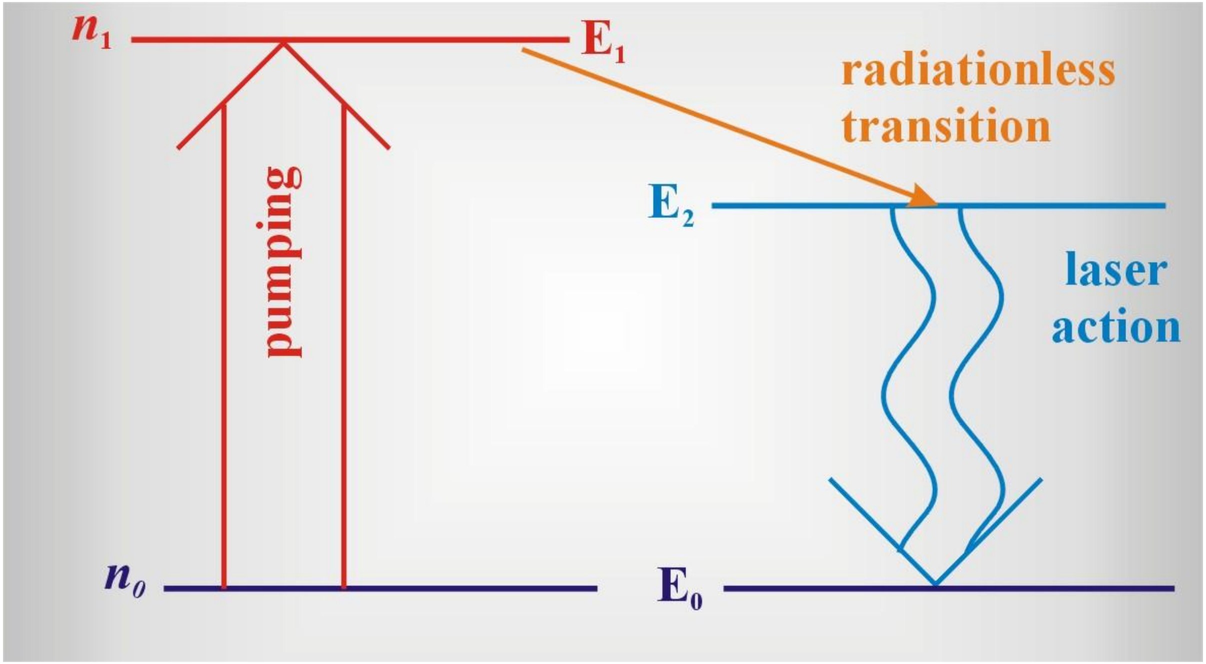
$$B_{12} = B_{21}$$

$$n_1 B_{12} \rho_\nu > n_2 B_{21} \rho_\nu$$

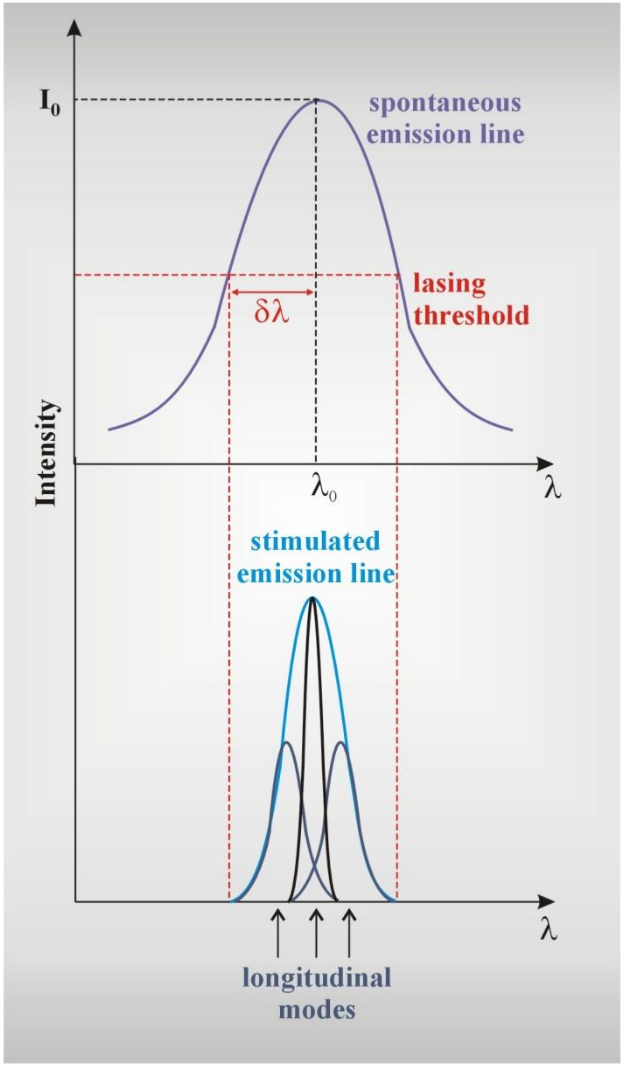
$$n_1 > n_2$$

In two level system the population inversion is not possible!!! (upper limit $N_1 = N_2$)

THREE LEVEL SYSTEM- YES



Longitudinal modes



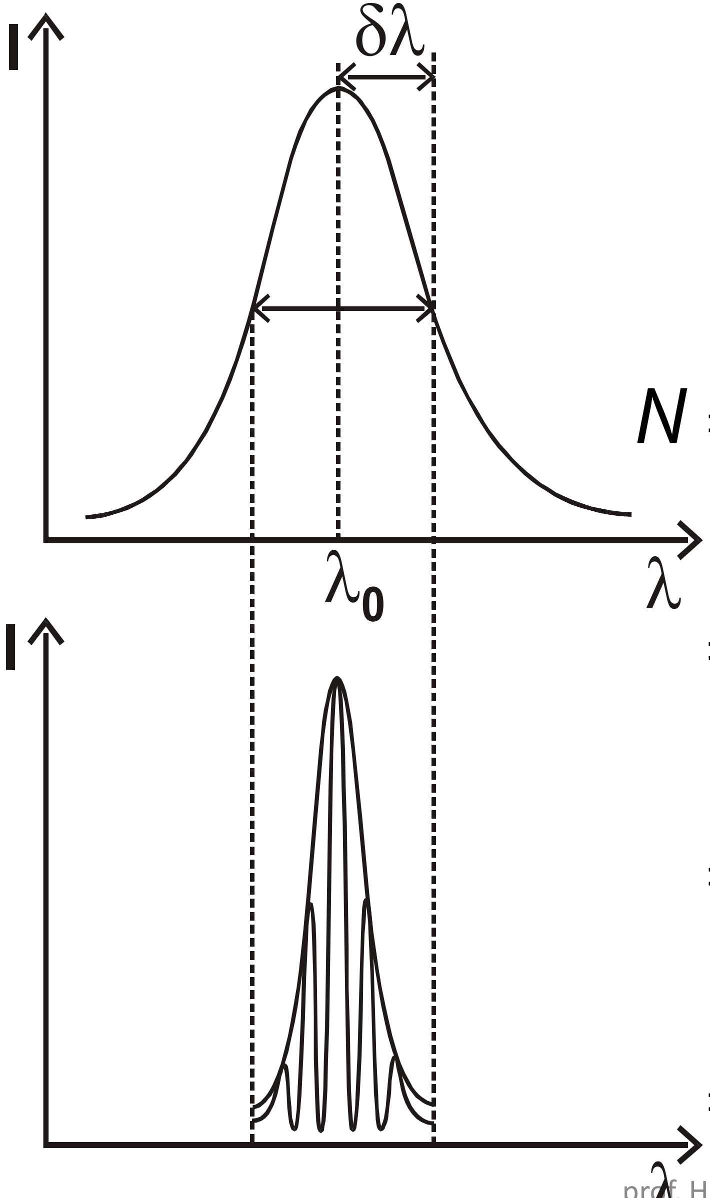
$$\nu_{m+1} - \nu_m = \frac{c}{2L}$$

$$m\lambda_m = 2L \qquad (m+1)\lambda_{m+1} = 2L$$

$$m\frac{c}{\nu_m} = 2L \qquad (m+1)\frac{c}{\nu_{m+1}} = 2L$$

$$\nu_{m+1} - \nu_m = (m+1)\frac{c}{2L} - m\frac{c}{2L} = \frac{c}{2L}$$

TOTAL NUMBER OF LONGITUDINAL MODES



$$n_{max}(\lambda_0 - \delta\lambda) = 2L$$

$$n_{min}(\lambda_0 + \delta\lambda) = 2L$$

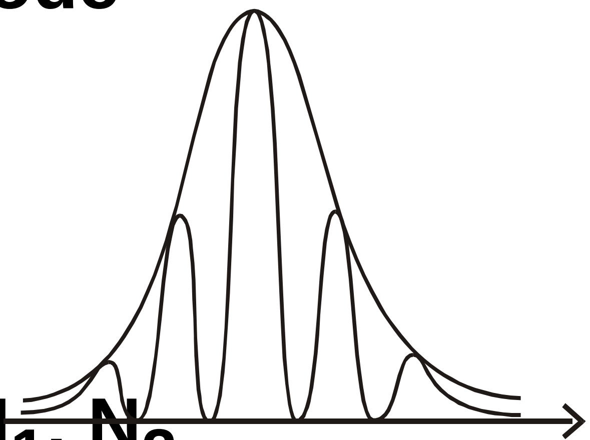
$$N = n_{max} - n_{min} = 2L\left(\frac{1}{\lambda_0 - \delta\lambda} - \frac{1}{\lambda_0 + \delta\lambda}\right) =$$

$$= \frac{2L}{\lambda_0^2 - \delta\lambda^2}\left[(\lambda_0 + \delta\lambda) - (\lambda_0 - \delta\lambda)\right]$$

$$= \frac{2L}{\lambda_0^2} 2 \cdot \delta\lambda =$$

$$= \frac{4L\delta\lambda}{\lambda_0^2}$$

Bandwith of a single mode



1. Resonator Quality Q
2. Degree of Population inversion N_1, N_2
3. Laser power P

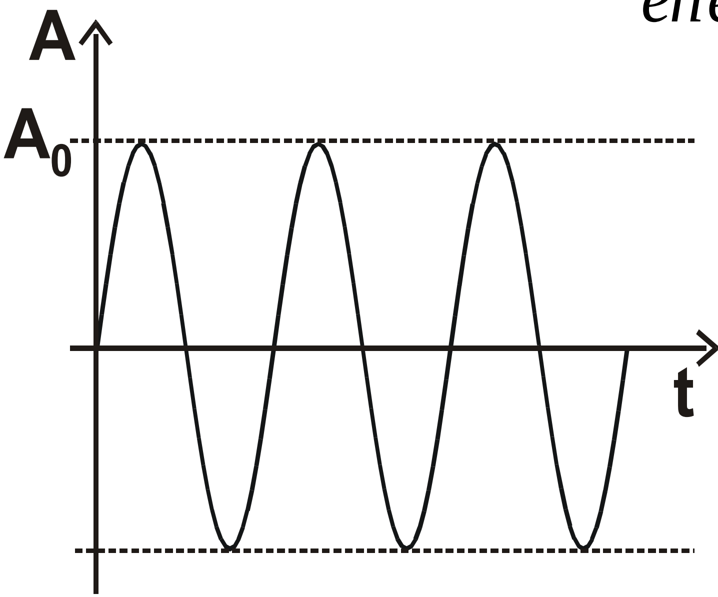
$$\Delta_{las}(1\text{mod}) = \frac{2\pi h\nu_0}{P}\left(\frac{\nu_0}{Q}\right)\frac{N_2}{N_2 - N_1\frac{g_2}{g_1}}$$

g_1, g_2 – degeneration degree

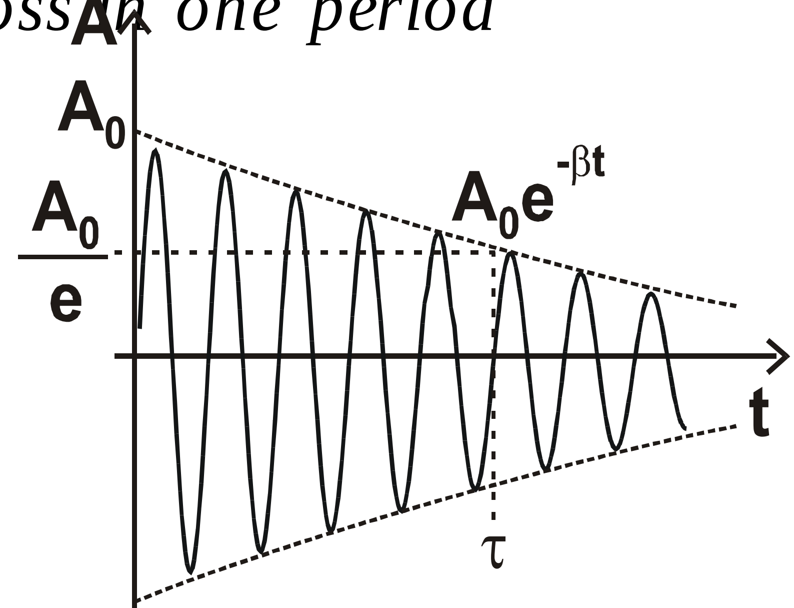
- Q- of resonator is usually lower than a theoretical value
- Energy losses from diffraction
- Heating of a medium (by pumping excitation)
- Mechanical nonstabilities

RESONATOR QUALITY Q

$$Q = 2\pi \frac{energy\_in\_the\_resonator}{energy\_loss\_in\_one\_period}$$



Harmonic oscillator



Damped harmonic oscillator

The model of the damped oscillator can be applied to describe phenomena occurring in the optical resonator. As a result of diffraction, reflections and other system imperfections, the optical resonator loses the accumulated energy, and the standing wave does not held the constant amplitude.

$$m\ddot{x} = -kx - \gamma\dot{x}$$

$$ma = F + F_{dump}$$

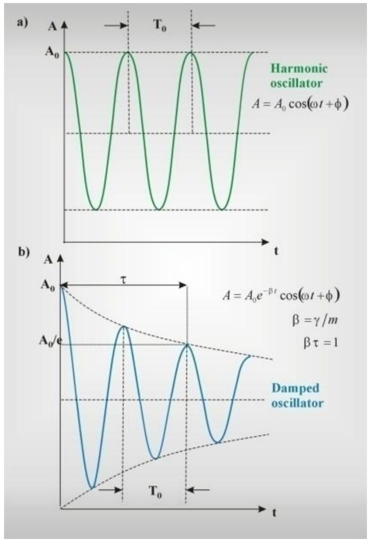
The quality factor of the system Q is defined by:

$$Q = 2\pi \frac{\text{energy gained in system}}{\text{energy lost during 1 cycle}}$$

so

$$Q = 2\pi \frac{A_0^2}{A_0^2 - A_0^2 e^{-2\beta T_0}}$$

where A_0 is the amplitude at t=0.



$$Q = 2\pi \frac{A_0^2}{A_0^2 - A_0^2 e^{-2\beta T_0}} = \frac{2\pi}{1 - e^{-2T_0/\tau}}$$

.

where the relationship $\beta\tau = 1$ between the damping factor β and the time τ has been employed.
The time τ is a time after which the amplitude A_0 of the damped oscillators decays by e=2.718 with respect to the initial time

Assuming that $T_0 << \tau$, the term $e^{-2T_0/\tau}$ can be expanded in series

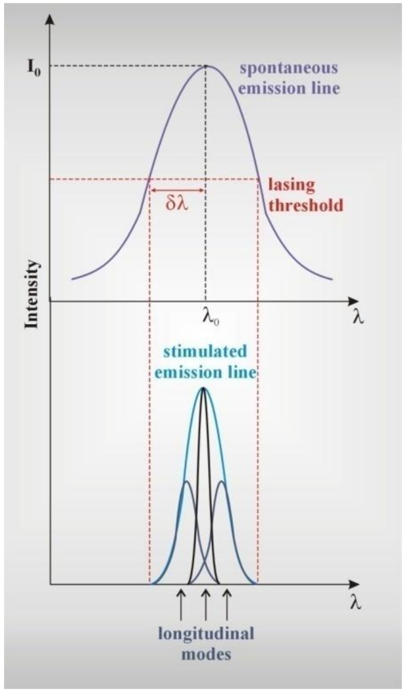
$$e^{-2T_0/\tau} = 1 - 2T_0/\tau + \ldots$$

Inserting into we get

$$Q = 2\pi \frac{\tau}{2T_0} = \pi \nu_0 \tau = \frac{\omega_0 \tau}{2} = \frac{\omega_0}{2\beta}$$

when $\beta \nearrow$ $Q \searrow$

$$Q \nearrow E(loss) \searrow \Rightarrow \beta \searrow$$



What is the relation to the bandwith of a single mode ?

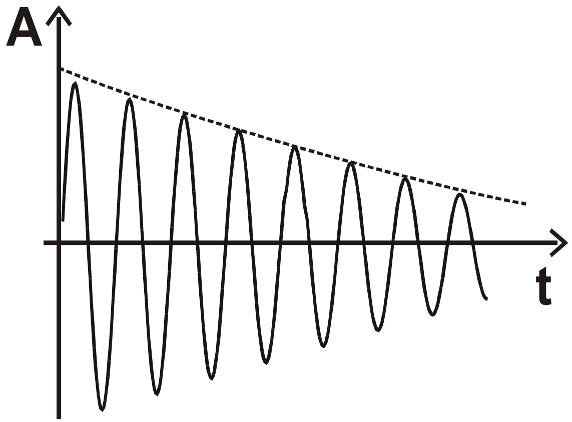
$$\Delta_{\frac{1}{2}}$$?

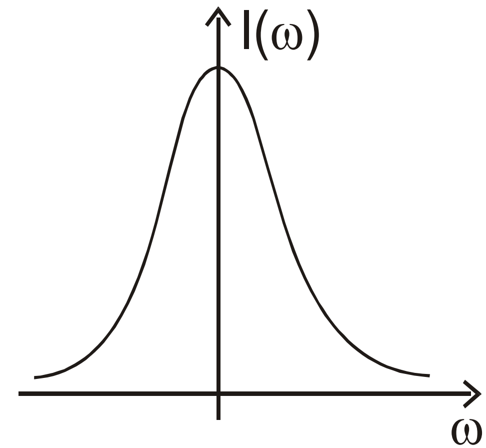
FOURIER TRANSFORM

Time domain

$$\Delta t \Delta E \geq \hbar$$

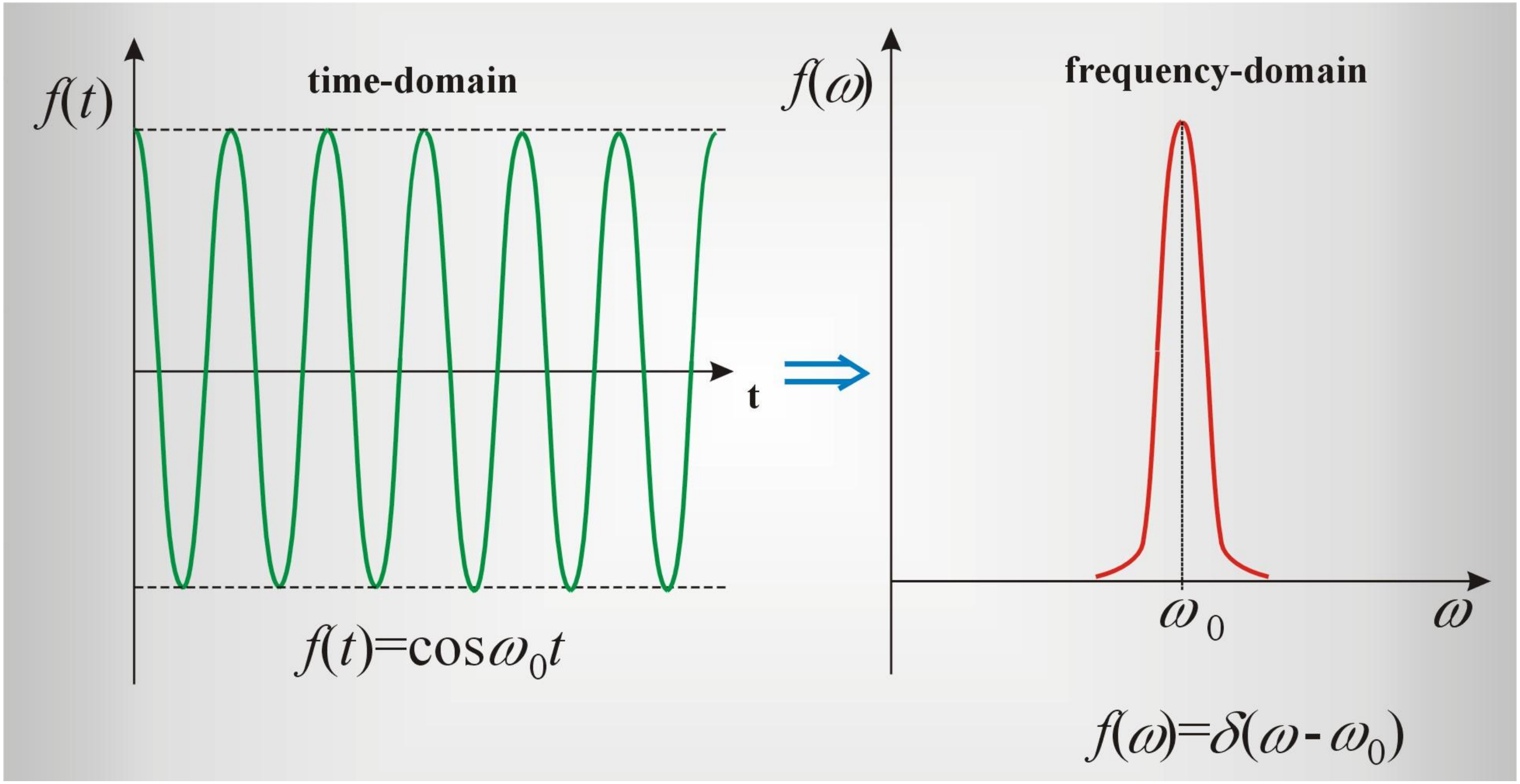
Frequency domain





$$f(t) = \sum_{n} f(\omega) \sin(n\omega t)$$

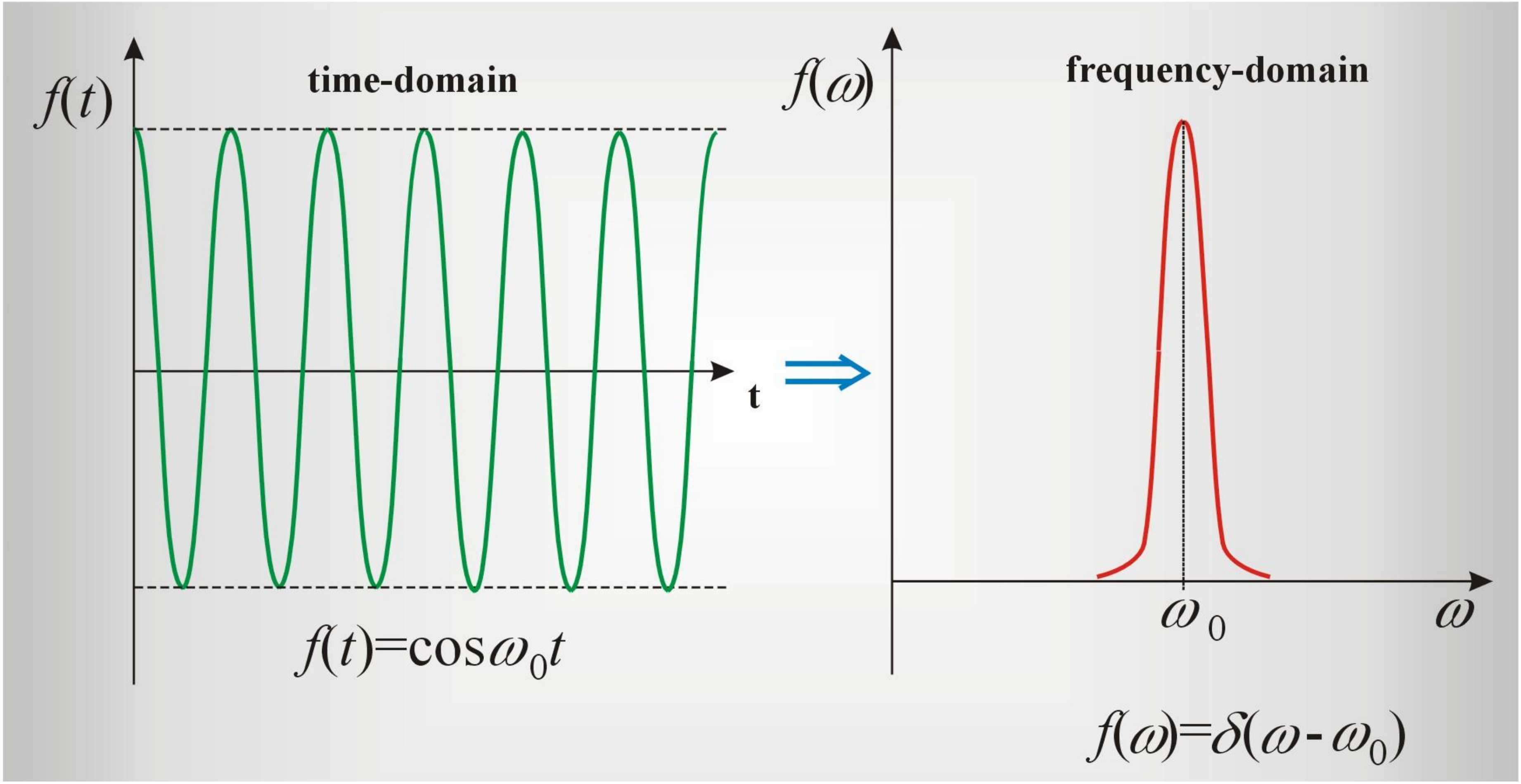
$$f(t) = \int_{-\infty}^{+\infty} f(\omega) e^{i\omega t} d\omega$$

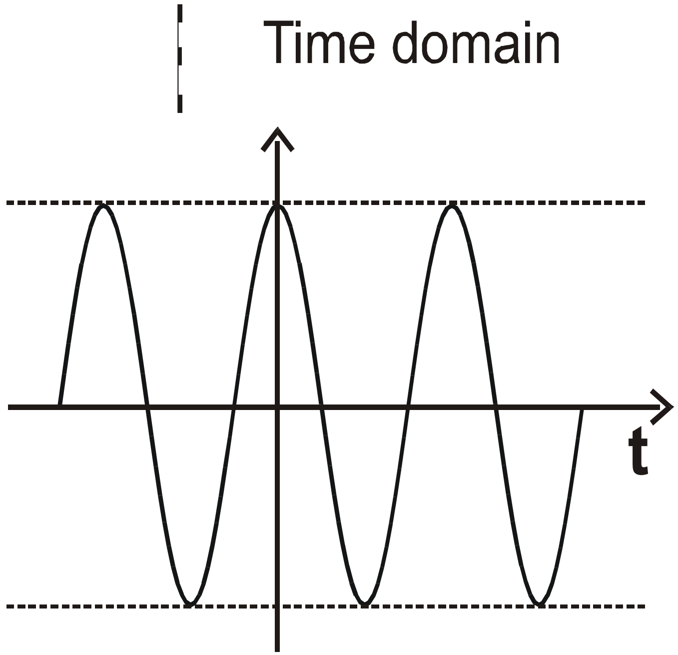


$$f(t) = \sum_{n} f(\omega)\sin(n\omega t)$$

$$f(t) = \int_{-\infty}^{+\infty} f(\omega) e^{i\omega t} d\omega$$

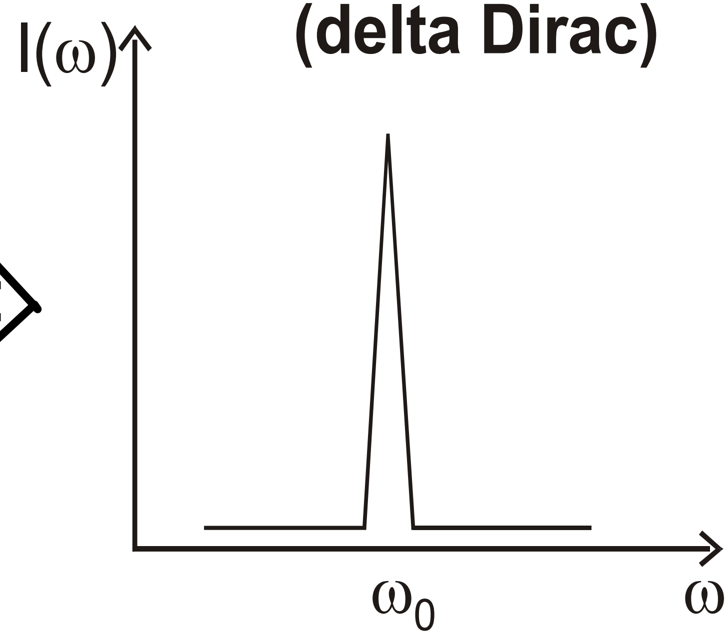
Fourier transform





$\cos \omega_0 t$

$\Rightarrow$



$\delta(\omega - \omega_0)$

$$\cos(\omega_0 t) = \int \delta(\omega - \omega_0) e^{i\omega t} d\omega = \cos \omega_0 t$$

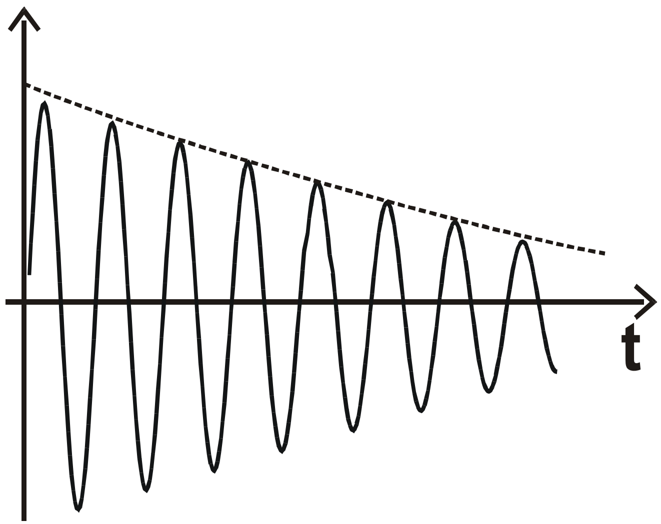
because

$$\int \delta(x - x_0) f(x) dx = f(x_0)$$

Properties of Dirac function $\delta(x - x_0)$

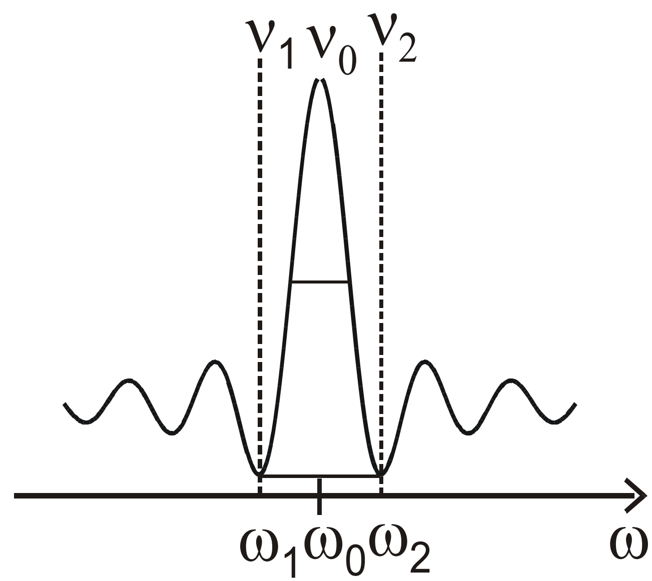
$$\delta(x - x_0) = \begin{cases} \infty & x = x_0 \\ 0 & x \neq x_0 \\ \delta(x - x_0) f(x) dx = f(x_0) \end{cases}$$

Time domain



$$\cos\omega_0 t e^{-\beta t}$$

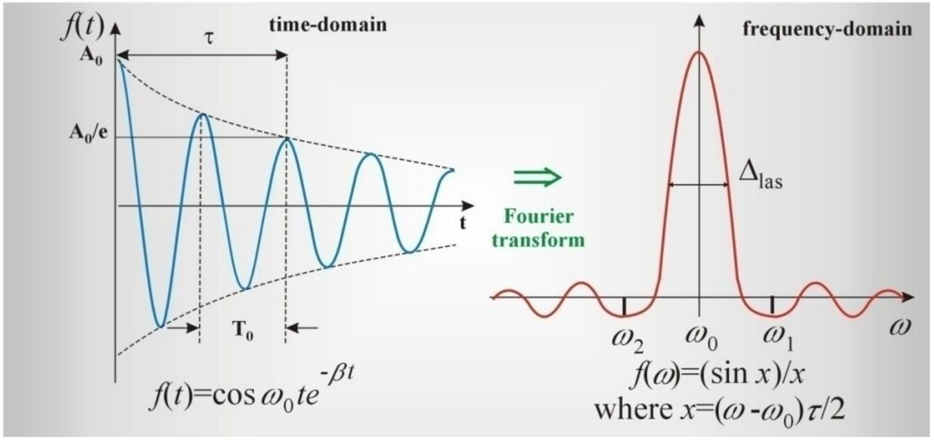
Frequency domain



$$\frac{\sin x}{x}$$

$$\nu = \nu_0$$

where $x = \pi(\nu - \nu_0)\tau$



f(t)
time-domain
τ
A0
A0/e
t
T0
f(t)=cos ω0te^-βt
Fourier transform
frequency-domain
Δlas
ω2
ω0
ω1
ω
f(ω)=(sin x)/x
where x=(ω -ω0)τ/2

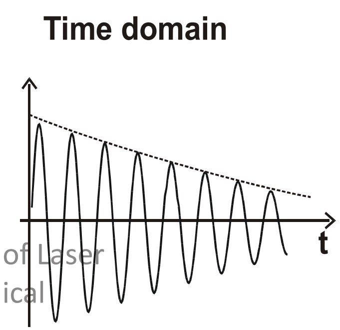
Function has a maximum for x=0, which means $\omega=\omega_0$ ($\nu=\nu_0$), becuase

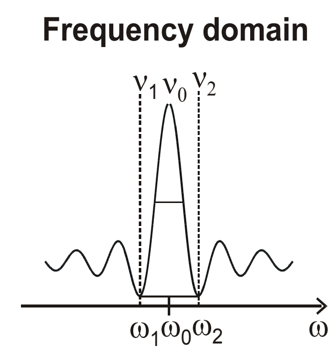
$$\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{\sin x'}{x'}=\lim_{x\to 0}\frac{\cos x}{1}=1$$

$$\lim_{x\to\frac{\pi}{2}}\cos x/1$$

$$\Delta_{\frac{1}{2}}=\nu_1-\nu_2\Rightarrow\tfrac{\pi}{2}=\pi\left(\nu_1-\nu_0\right)\tau \quad ; \quad -\tfrac{\pi}{2}=\pi\left(\nu_2-\nu_0\right)\tau$$

$$\Delta_{\frac{1}{2}}=\frac{1}{\tau}=\beta$$





$Q \nearrow \qquad \beta \searrow \qquad \Delta_{\frac{1}{2}} \searrow$

The larger Q-of the resonator, the narrower the bandwidth of a single mode

CONCLUSIONS

Bandwith of spantaneous emission depends on relaxation processes characterized by the energy relaxation T_1 and phase relaxation T_2

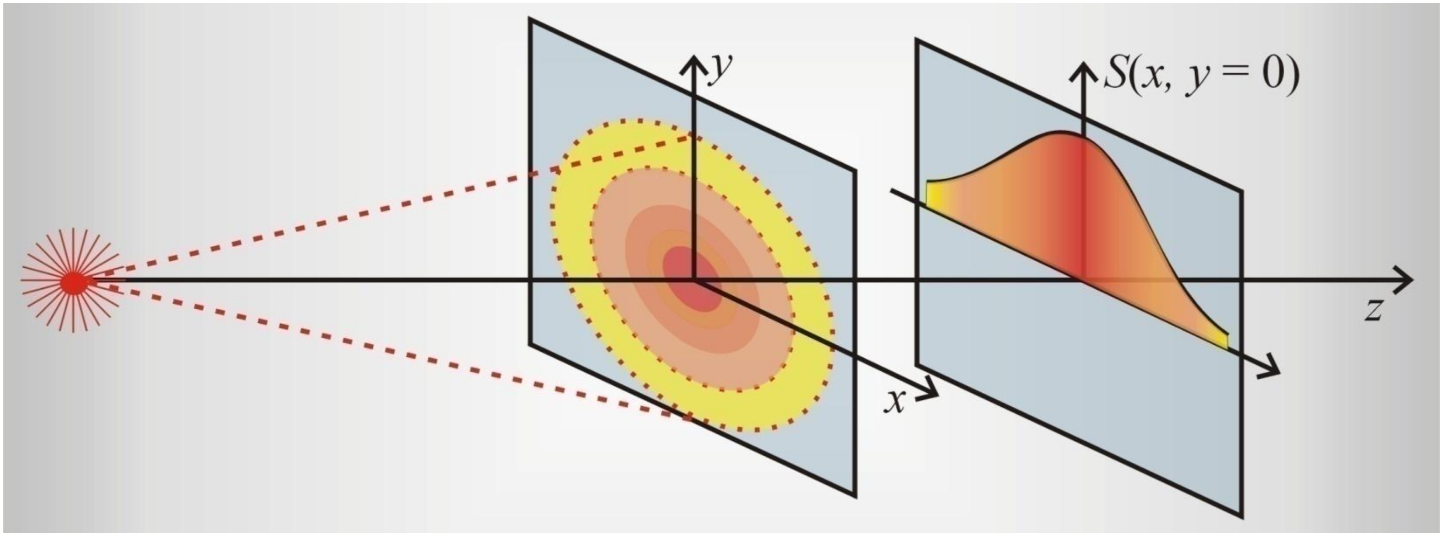
Bandwith of stimulated emission

1. **Quality of resonatorQ**
2. **Population inversion**
3. **Laser power**
4. **Number of modes** $N = \frac{4L\delta\lambda}{\lambda_0^2}$

(1–3) → **Bandwith of a single mode**

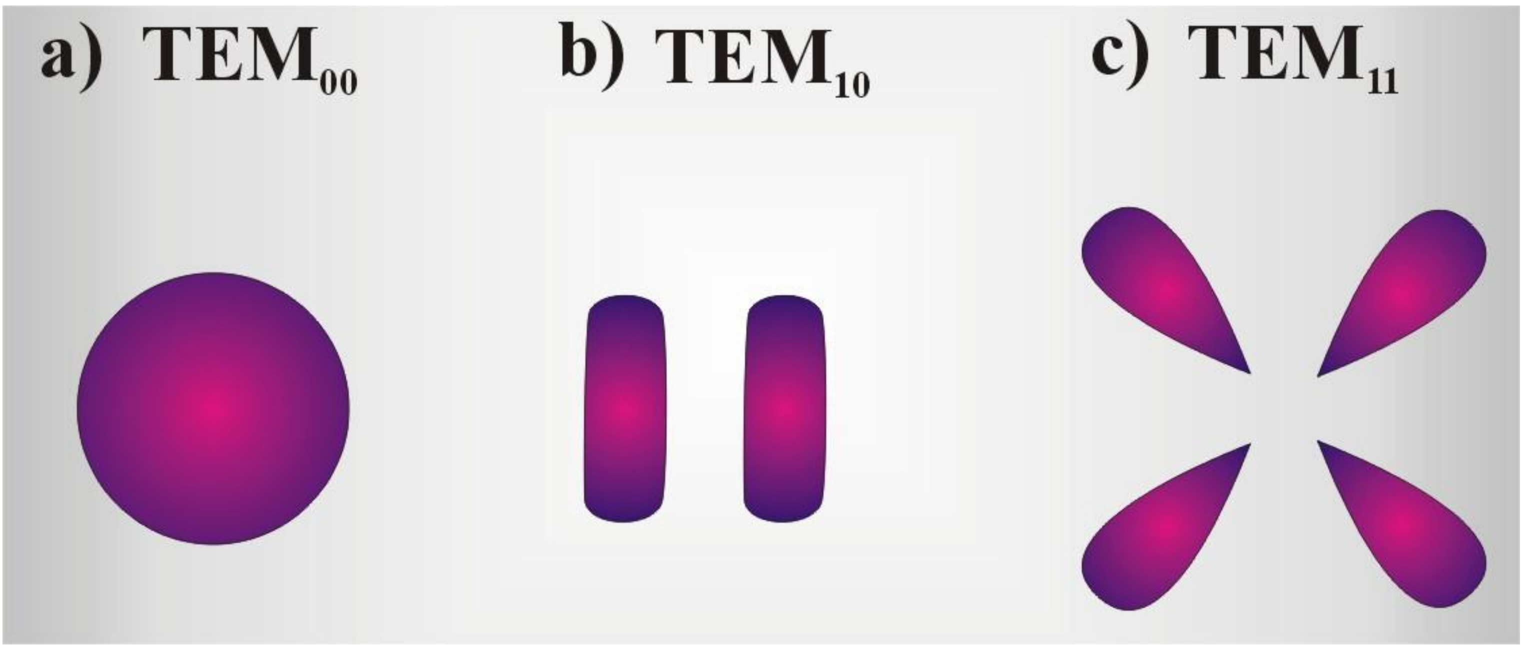
(1–4) → **Total bandwith of stimulated emission**

Transverse modes

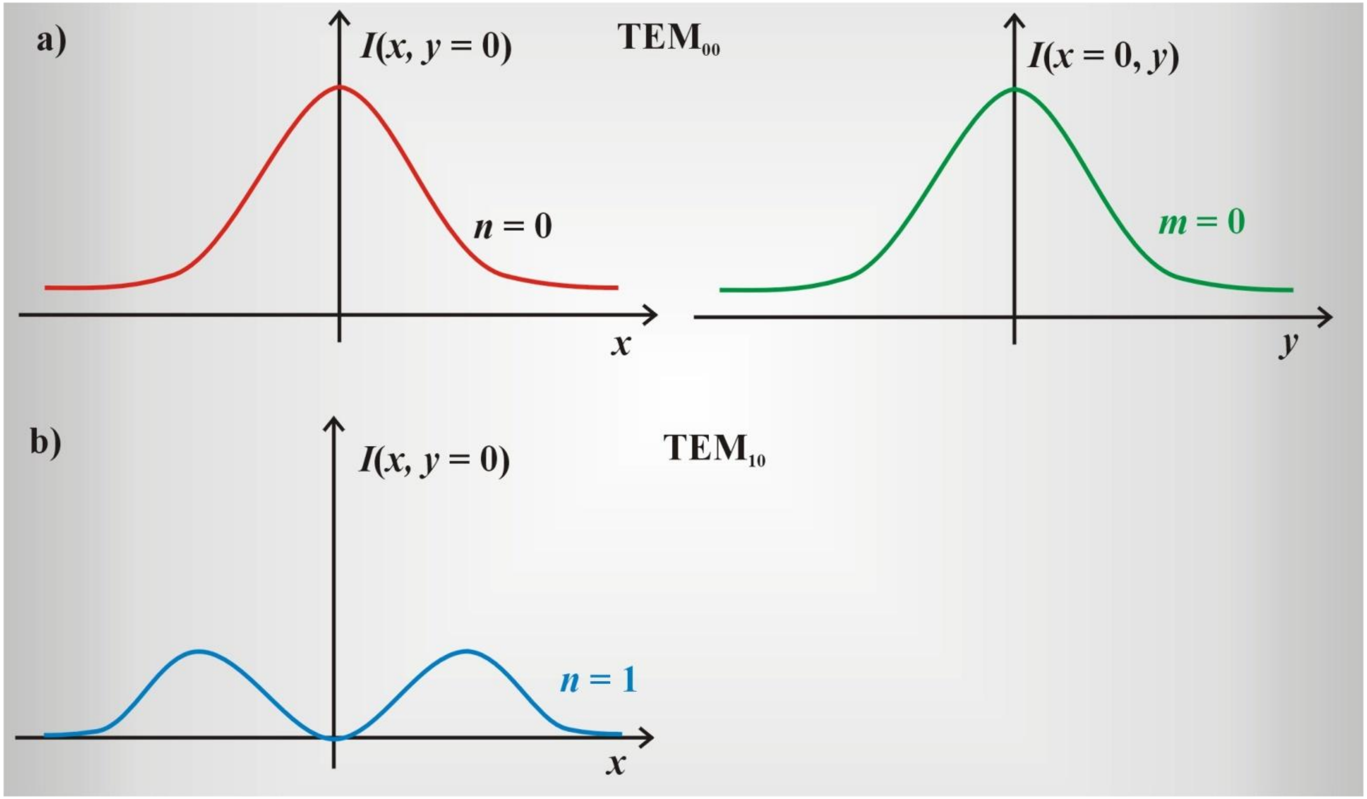


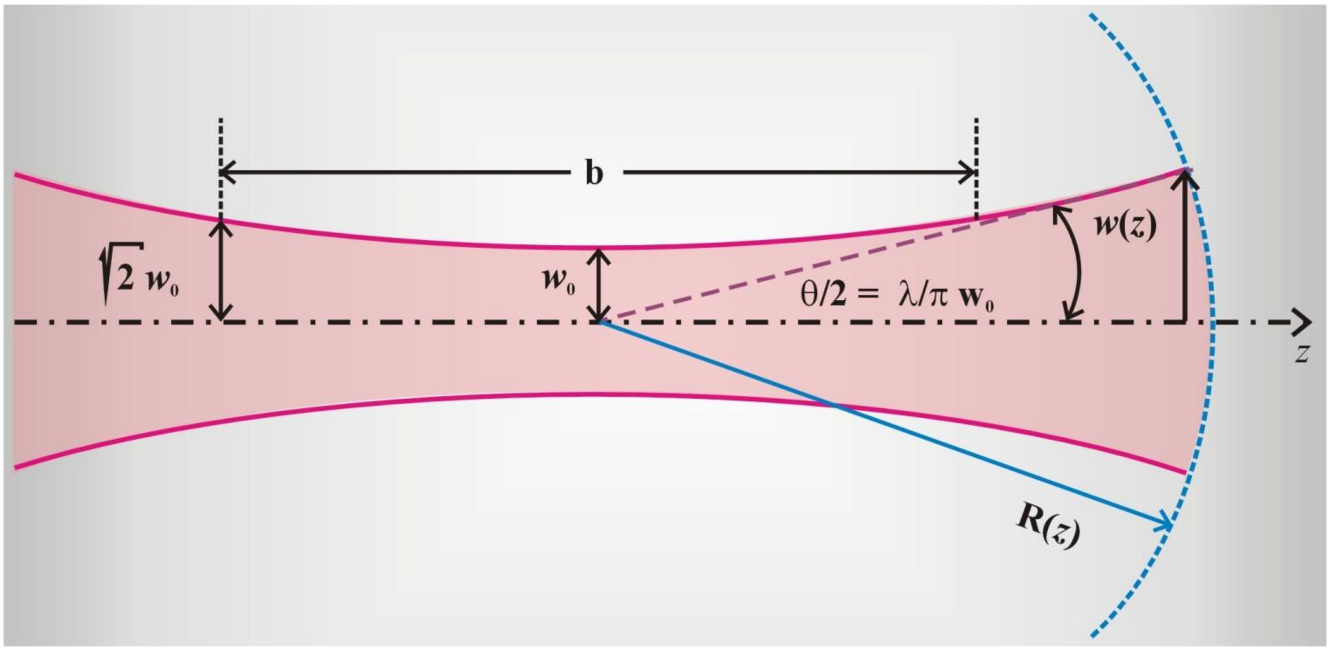
We observe an intensity distribution not only along the resonator axis, but also in the plane perpendicular to the direction of the laser beam propagation. The longitudinal modes are responsible for the spectral characteristics of a laser such as bandwidth and coherence length whereas the beam divergence, beam diameter, and energy distribution in the plane perpendicular to the beam propagation are governed by the transverse modes.

Transverse modes



Transverse modes




b
$\sqrt{2}\, w_0$
w_0
$w(z)$
$\theta/2 = \lambda/\pi w_0$
z
$R(z)$